Math 105, Section 204, Quiz 1

Wednesday, January 17

Communicating mathematics is an important skill to practice, so simplify and justify all answers unless otherwise directed. Show your work, and use proper notation.

1. Give any nonzero unit vector that lies in the plane $5x - 2y - z = 1$. $\boxed{5}$

Solution: We'll show two ways of finding a vector in the plane: we can take any vector that is orthogonal to the normal vector of the plane, or we can find two points in the plane, and take the vector between them. Once we have a vector in the plane, we multiply it by an appropriate scalar to make it a unit vector.

Option 1: The normal vector to our plane is $\langle 5, -2, -1\rangle$. There are infinitely many vectors in $\mathbb{R}^3$ that are orthogonal to this. Remember that vectors have a dot product equal to zero. One possible answer (again, out of many!) is $\langle 0, 1, -2\rangle$. This vector has length $\sqrt{0+1+5} = \sqrt{5}$. To make it a unit vector, we multiply by the scalar $\dfrac{1}{\sqrt{5}}$. So, one unit vector in the plane is $\langle 0, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\rangle$.

Option 2: The points in the plane are exactly those points (x, y, z) that make the equation $5x - 2y - z = 1$ true. Two such points (of infinitely many!) are $(0, 0, -1)$ and $(0, -1, 1)$. To find a vector between these two points, we subtract their coordinates: $\langle 0, 1, -2\rangle$. To make this a unit vector, we note that its length is $\sqrt{0+1+4} = \sqrt{5}$, so we multiply by the scalar $\dfrac{1}{\sqrt{5}}$. This gives us a unit vector in the plane of $\langle 0, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\rangle$.

Marking scheme: Everything correct: + 5 points
Vector in the plane, but not a unit vector: +3 points
Vector normal to the plane, but a unit vector: +2 points
Algebra mistake(s): -1 point

2. Find all critical points of the surface $f(x, y) = x^4 + y^4 - 18x^2 + 10x^2y^2$. You do not need to classify them. $\boxed{5}$

Solution: The critical points will be the points (x, y) where $f_x(x, y) = f_y(x, y) = 0$. So, we start by finding both partial derivatives of our function. Since we want to find their roots, we also factor them.

$f_x(x, y) = 4x^3 - 36x + 20xy^2 = 4x(x^2 - 9 + 5y^2)$
$f_y(x, y) = 4y^3 + 20x^2y = 4y(y^2 + 20x^2)$

Note that f_y is only zero when $y = 0$. So, all critical points will have y-coordinate zero.

With that in mind, we see (x, y) is a critical point if: $0 = f(x, 0) = 4x(x^2 - 9 + 0) = 4x(x - 3)(x + 3)$. That is, when x is 0, 3, or -3.

All together, our critical points are $(0, 0)$, $(3, 0)$, and $(-3, 0)$.

Marking scheme: Everything correct: $+5$ points
Missed $(-3, 0)$ but got $(3, 0)$: $+ 4$ points
Everything correct with differentiation, but only found the CP $(0, 0)$: $+3$ points
Partial derivatives correct: $+2$ points
Partial derivatives incorrect, mistake is more than an isolated incident: $+0$ points
Otherwise, if partial derivatives are incorrect, grading will depend on whether the mistake(s) made the problem easier or not.

3. Double or Nothing **(optional)**

Pick ONE of the questions on this quiz that you are confident you understood. If the grader gives you full marks for this question, its points will be DOUBLED. If the grader does not give you full marks for this question, you will receive NO POINTS on that question.

The purpose of this question is to encourage you to think critically about your own understanding, without relying on someone else's solutions.