Update: Ch 9.1 has also been cut. See main webpage for more info on final exam.
In Chapter 9, we only cover section 9.2.

Ch 9.2: Properties of Power Series

Geometric Series:

$$\sum_{n=0}^{\infty} r^n = \begin{cases} \frac{1}{1-r} & \text{if } |r|<1 \\ \text{DIV} & \text{otherwise} \end{cases}$$

We can think of this as a <u>function</u> of r:

$f(r) = \sum_{n=0}^{\infty} r^n$: Domain of $f(r)$: $-1<r<1$

eg $f(1/2) = \frac{1}{1-1/2} = 2$

$f(0) = \frac{1}{1-0} = 1$

$f(-0.75) = \frac{1}{1-(-3/4)} = \frac{1}{7/4} = 4/7$

$f(2) =$ DNE
2 not in domain of f

Power series: infinite analogue of polynomial

Polynomial: $c_1 + c_2 x + c_2 x^2 + c_3 x^3 + \cdots + c_N x^N$

A power series has the form:

$$\sum_{k=0}^{\infty} c_k (x-a)^k$$ where $\{c_k\}$ a constants, x is a variable

$\downarrow$

$$c_0 + c_1 (x-a)^1 + c_2 (x-a)^2 + c_3 (x-a)^3 + \cdots$$

a : "centre" of power series

c_k : "coefficients"

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eg $\sum_{k=0}^{\infty} 2x^k$ Power Series

Centre: $(x)^k = (x-0)^k$

$\boxed{a=0}$

Coefficients $c_k = 2$

eg $\displaystyle\sum_{k=0}^{\infty} 2^{k+1}(x-3)^k$

Centre:
$a=3$
Coeff's: $c_k = 2^{k+1}$

$= 2 + 2^2(x-3) + 2^3(x-3)^2 + \cdots$

eg $\displaystyle\sum \frac{k}{x!}$ not a power series

$= 0 + \frac{1}{x!} + \frac{2}{x!} + \frac{3}{x!} \cdots$

eg $\displaystyle\sum \frac{x^k}{x^{2k}+1}$ not a power series

Power Series:

$\displaystyle\sum_{k=0}^{\infty} c_k(x-a)^k$

no x's

Def: The set of values x for which a power series converges is its <u>interval of convergence</u> (similar to domain)

The <u>radius of convergence</u> is distance from centre of PS to the edge of its interval of convergence.

eg $\quad f(x) = \sum_{n=0}^{\infty} x^n$ (geometric)

Conv: $|x| < 1$

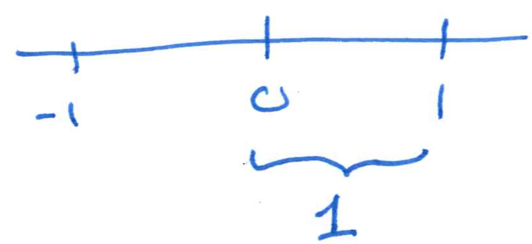
Div: otherwise

$f(r) = \sum_{n=0}^{\infty} r^n$

$S_N = \frac{1 - r^{N+1}}{1-r}$

Interval of Conv: $\boxed{(-1, 1)}$

Centre: 0



Radius of Conv

$\boxed{R = 1}$

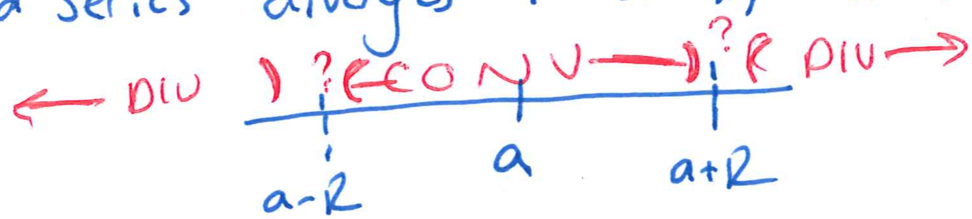
Power series <u>always</u> converges at its centre $(x=a)$

R: how far from centre (safe! definitely converges!) can we push x before we leave domain of fcn?

<u>Theorem</u> A power series centred at a ($\sum c_k (x-a)^k$) converges in one of three ways:

1. It converges for all x. Interval of Conv: $(-\infty, \infty)$
   Radius " " : $R = \infty$

2. There is a real number R such that series converges for all x, $|x-a| < R$ and series diverges for all x, $|x-a| > R$



3. The series converges only at centre
Interval of Conv: $[a, a]$
Radius of Conv: $R = 0$

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ex $\sum_{k=0}^{\infty} 2^{k+1}(x-3)^k = 2 + 4(x-3) + 8(x-3)^2 + 16(x-3)^3 + \dots$

If $x = 3$: $2 + 0 + 0 + 0 + \dots = 2$

So series converges at $x = 3$ (centre)

ex $\sum_{k=0}^{\infty} 2x^k = \sum_{k=0}^{\infty} 2(x-0)^k$ Centre: 0

$= 2 + 2x + 2x^2 + 2x^3 + \dots$

$x = 0$: $2 + 0 + 0 + 0 + \dots$
$= 2$ converges when $x = 0$

Syllabus: won't have to find interval of conv; do have to find radius of conv (Fri)