Announcement:

Ch 9.3 and 9.4 are being cut from this year's syllabus due to time constraints

Recall from Last Time:

- Divergence Test
- Integral Test ← positive terms
- p-test $\sum \frac{1}{n^2}$
- Geometric Series $\sum (\frac{1}{2})^n$

particular series

- Direct Comparison Test
- Limit Comparison Test
- Ratio Test

positive terms

Tests for series convergence / divergence

Absolute Convergence

$\{a_n\}$ series consider: $\{|a_n|\}$

Theorem: If $\sum |a_n|$ converges, then

$\sum a_n$ converges as well

In this case, we say $\sum a_n$ "converges absolutely"

Even if we take out negatives, still converges.

(ex) $\sum \frac{\cos n}{n^2}$ Conv or Div?

Some terms +, some −

Consider related series: $\sum \frac{|\cos n|}{n^2}$

No negative terms — can use other tests

$|\cos n| \le 1$, so

$\dfrac{|\cos n|}{n^2} \le \dfrac{1}{n^2}$ By p-test

$\sum \frac{1}{n^2}$ converges

By comparison test, ← need series to have only nonnegative terms

$\sum \dfrac{|\cos n|}{n^2}$ converges as well

Theorem: $\sum \dfrac{\cos n}{n^2}$ converges, too.

$\sum a_n$	$\sum \lvert a_n \rvert$	$\sum a_n$ is called:
conv	conv	"absolutely conv"
conv	div	"conditionally conv" (needs its negatives to converge)
div	div	"divergent"
div	conv	NEVER HAPPENS

Telescoping Series

Terms cancel nicely

ex $\sum_{k=1}^{\infty}\left(\frac{1}{k+2}-\frac{1}{k+3}\right) = \lim_{N\to\infty} \underbrace{S_N}_{\text{partial sums}} = 1/3$

$k=1 \quad \frac{1}{3} - \cancel{\frac{1}{4}}$

$k=2 \quad + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}$

$k=3 \quad + \cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}$

$k=4 \quad + \cancel{\frac{1}{6}} - \frac{1}{7}$

$S_1 = \frac{1}{3} - \frac{1}{4}$

$S_2 = \frac{1}{3} - \frac{1}{5}$

$S_3 = \frac{1}{3} - \frac{1}{6}$

$S_4 = \frac{1}{3} - \frac{1}{7}$

$S_N = \frac{1}{3} - \frac{1}{N+3}$

$\lim_{N\to\infty} S_N = \frac{1}{3}$

ex $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$

Remember:

$\ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln n$

$n=1 \quad \cancel{\ln 2} - \cancel{\ln 1}^{0}$

$n=2 \quad \cancel{\ln 3} - \cancel{\ln 2}$

$n=3 \quad \cancel{\ln 4} - \cancel{\ln 3}$

$n=4 \quad \ln 5 - \cancel{\ln 4}$

$\vdots$

$S_1 = \ln 2$

$S_2 = \ln 3$

$S_3 = \ln 4$

$S_4 = \ln 5$

$\vdots$

$S_N = \ln(N+1)$

$$\sum \ln\left(\frac{n+1}{n}\right) = \lim_{N\to\infty} \ln(N+1) = \infty$$

Divergent series

ex Evaluate $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}-\frac{n+1}{n+2}\right) = \lim_{N\to\infty}\left(\frac{1}{2}-\frac{N+1}{N+2}\right)$

$= \frac{1}{2} - 1 = -\frac{1}{2}$

$n=1 \quad \frac{1}{2} - \cancel{\frac{2}{3}}$

$n=2 \quad \cancel{\frac{2}{3}} - \cancel{\frac{3}{4}}$

$n=3 \quad \cancel{\frac{3}{4}} - \cancel{\frac{4}{5}}$

$n=4 \quad \cancel{\frac{4}{5}} - \frac{5}{6}$

$S_1 = \frac{1}{2} - \frac{2}{3}$

$S_2 = \frac{1}{2} - \frac{3}{4}$

$S_3 = \frac{1}{2} - \frac{4}{5}$

$S_4 = \frac{1}{2} - \frac{5}{6}$

$S_N = \frac{1}{2} - \frac{N+1}{N+2}$