Week 9 : Probability

Not in your textbook: Probability Appendix linked to main course website, in schedule at Week 9

Concepts aren't tough, but new notation and vocabulary. Make sure to review between classes.

Probability Basics

- A probability is a number between 0 and 1

  We interpret it as "likelihood"

  1 : will certainly happen

  0.5 : 50/50 happen or not

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- If (say) a probability is $1/3$: (limit)

  If we run the event many, many times, our desired outcome will happen in $1/3$ of trials

Notation

Events result in values

capital letters (usually)

lower-case (usually)

eg Event: "outcome of a dice roll" X

Value: "4", "5", "1"

$Pr(X=4)$ ← number : probability dice rolls 4

$= 1/6$

If x is 1, 2, 3, 4, 5, or 6, then

$Pr(X = x) = 1/6$

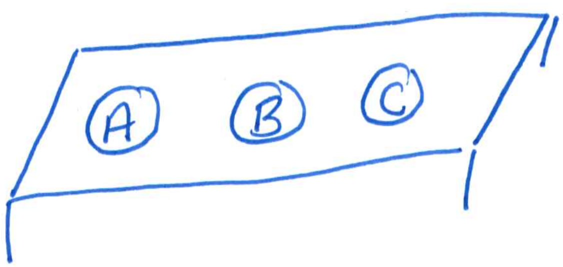
value

event
(roll)

$Pr(X = 1/2) = 0$ (can't roll $1/2$ on a dice)

A tester chooses one product. Event: X

eg.



Values X can take: A, B, C

What is the probability tester chooses our product (A)?

We write: $Pr(X = A)$

choice

our brand

Furthermore: Y : tester chooses a chair to sit in

Values of Y: L, R

$\Pr(X = B$ and $Y = R)$:

probability that the tester chooses product B AND sit in right chair

Note: "or" means "and/or"

- $\Pr(X = x$ or $X \neq x) = 1$

same event

100% probability something does or does not happen

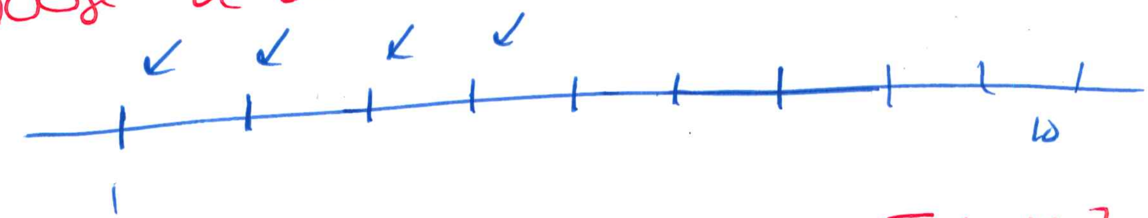
- $\underbrace{\Pr(X = x)}_{\text{value } x} = 1 - \underbrace{\Pr(X \neq x)}_{\text{other values}}$

Note: discrete vs continuous

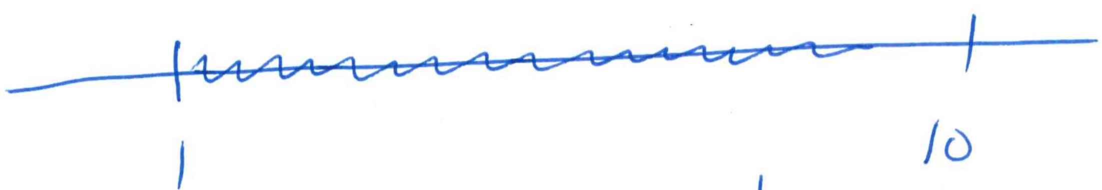
Discrete: things can be listed separately

Continuous: things exist on a continuum

(ex) Choose a whole number in $[1, 10]$ — discrete



(ex) Choose any real number in $[1, 10]$ — continuous



∞ many values

ex Roll 3 dice, add values — DISCRETE

$3, 4, 5, \dots, 18$

ex Number of pets you have — DISCRETE

$0, 1, 2, 3, \dots$

ex *Exact* age at noon today — CONTINUOUS

$[0, \dots]$

ex Volume of a box. — CONTINUOUS

$[0, \text{—}$

Syllabus: only accountable for *continuous* systems

Since discrete more familiar, we sometimes use them to illustrate concepts.

Cumulative Distribution Function

We can ask: T: temp tomorrow

$Pr(T \le 0)$ ← Prob freezing?

$Pr(T \le 10)$ ← Prob at most 10°?

$Pr(T \le 20)$ ← Prob at most 20°?

Define: $F(x) = Pr(T \le x)$ (useful later)

$F(100)$ high

$F(200)$ higher

$F(1\,000\,000) \approx 1$

$F(0)$ 1/10

$F(-100) \approx 0$

$F(-200)$

The cumulative distribution function for any random variable X, denoted $F(x)$, is:

$$F(x) = Pr(X \le x)$$

event value

Properties:

- $0 \le F(x) \le 1$ for all x
- $\lim_{x \to \infty} F(x) = 1$
- $\lim_{x \to -\infty} F(x) = 0$
- $F(x)$ nondecreasing

As $x \uparrow$, easier to be less than x

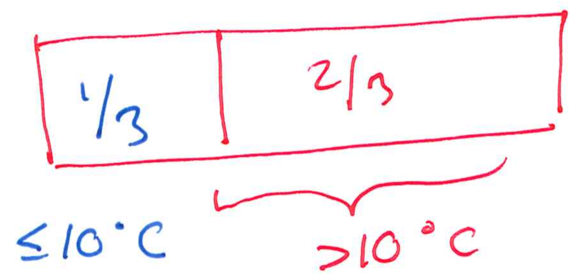
As $x \downarrow$, harder to be less than x

ex Half of days below (or =) 20 degrees,

2/3 of days were above 10 degrees

$F(20) = Pr(T \leq 20) = 1/2$

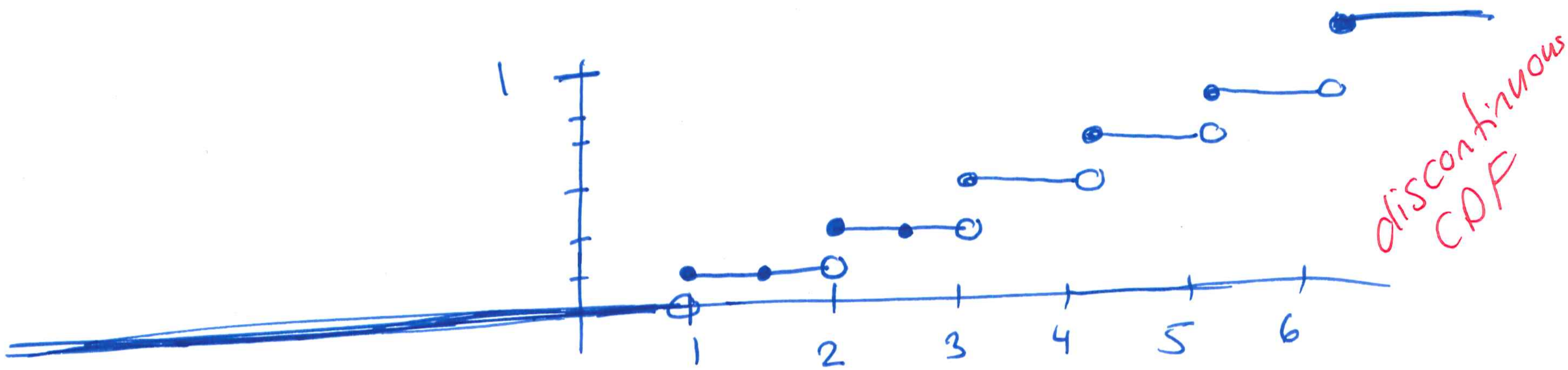
$F(10) = Pr(T \leq 10) = 1 - Pr(T > 10) = 1 - 2/3 = 1/3$



ex Sketch $F(x)$ for dice roll.

X : outcome

x : values



$F(1) = Pr(X \le 1) = Pr(X=1) = 1/6$

$F(1.5) = Pr(X \le 1.5) = Pr(X=1) = 1/6$

$F(2.5) = Pr(X \le 2.5) = Pr(X=1 \text{ or } X=2) = 2/6 = 1/3$

$F(2) = Pr(X \le 2) = Pr(X=1 \text{ or } X=2) = 1/3$

$F(0.9) = Pr(X \le 0.9) = 0$

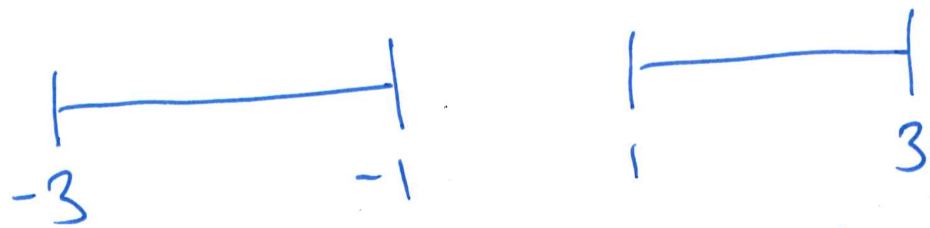
$F(10) = Pr(X \le 10) = 1$

(discrete)

ex Choose a number uniformly at random

no preference

from $[-3,-1] \cup [1,3]$



Cumulative Distribution Function (CDF)

$F(-1) = Pr(X \le -1) = 1/2$

$F(0) = Pr(X \le 0) = 1/2$

$[-3,-1]$

Sketch

$F(x)$

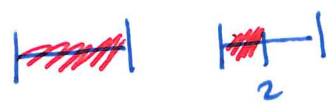
$F(-2) = Pr(X \le -2) = 1/4$

$F(0.9) = Pr(X \le 0.9) = 1/2$

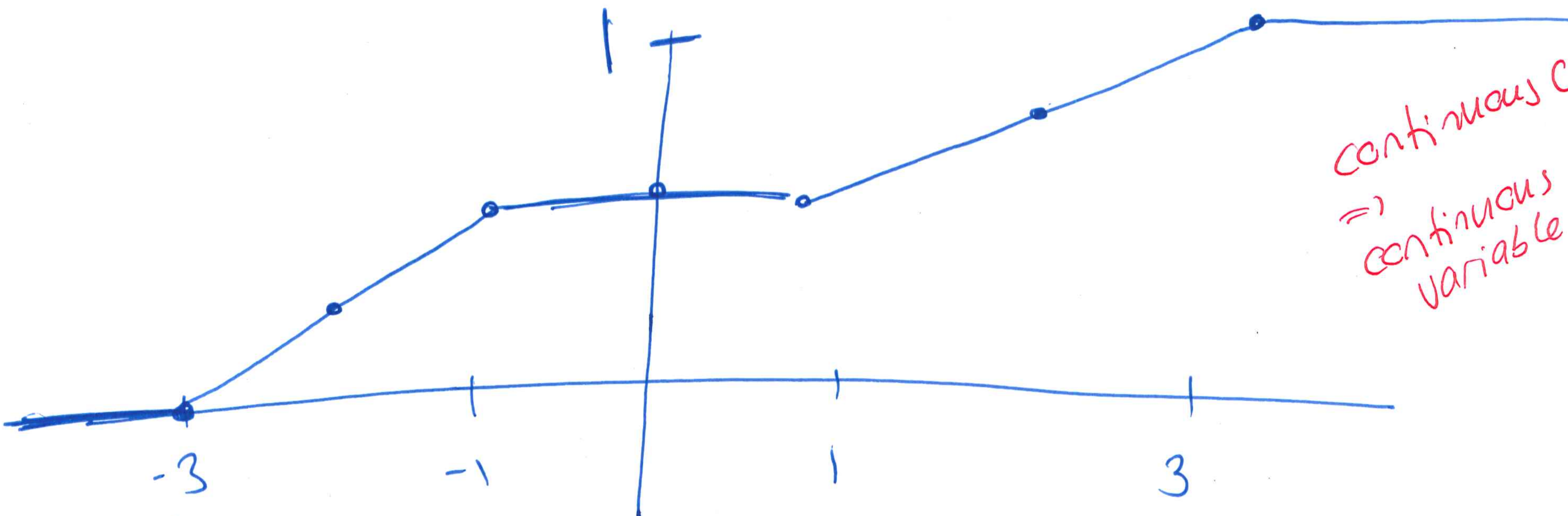
$\frac{1}{4}$ of total range

$\frac{1}{2}$ of total range

$F(2) = Pr(X \le 2) = 3/4$



$\frac{3}{4}$ of total range



continuous CDF
$\Rightarrow$ continuous variable

-3: one number out of infinitely many #s
odds of choosing -3: infinitely small
i.e. 0

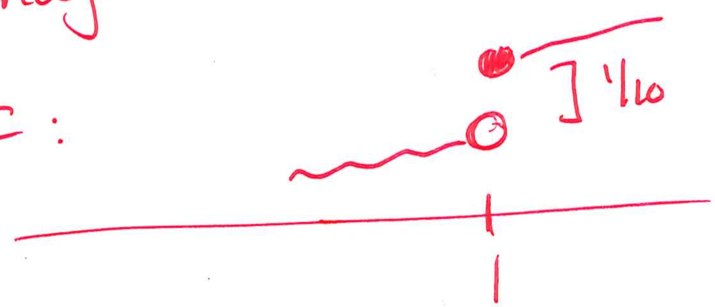
Def: <u>Continuous</u> random variable has a continuous CDF ($F(x)$)

Property: For any continuous random variable X and any value x,

$$Pr(X = x) = 0$$

Why? Imagine $Pr(X = 1) = 1/10$

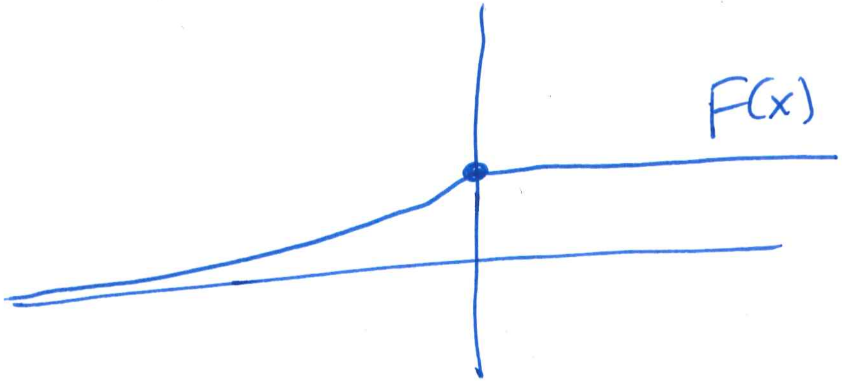
CDF:



jump discontinuity

$Pr(X \le 0.9999)$

vs $Pr(X \le 1)$ ← includes event of liklihood 1/10 not in other probability ex $F(x) = \begin{cases} e^x & \text{when } x \le 0 \\ 1 & \text{when } x > 0 \end{cases}$

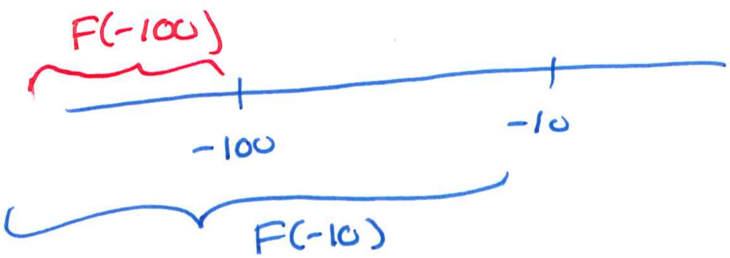


Compute:

$$\Pr(X<0) = P(X \le 0) = F(0) = e^0 = 1$$

$$\Pr(X \le -1) = F(-1) = e^{-1} = 1/e$$

$$\Pr(-100 \le X \le -10) =$$



$$\Pr(X \le -10 \text{ and } X > -100)$$

$$= \underbrace{F(-10)}_{\substack{\text{events with} \\ X \le -10: \\ \text{include}}} - \underbrace{F(-100)}_{\substack{\text{events with} \\ X \le -100: \\ \text{EXCLUDE}}} = e^{-10} - e^{-100}$$

If $F(x)$ is a CDF for X: (continuous)

$$Pr(a \le X \le b) = F(b) - F(a)$$

$Pr(X \le b)$

take out events w/ $X \le a$