Week 9: Probability

Not in textbook: linked to course website by "week 9" in schedule

Basics

A probability is a number from 0 to 1

we interpret it as the likelihood of an event

0: won't happen   1: definitely happens

If the probability of an event happening is (say)

1/3 : we interpret this to mean

over many, many trials (limit)

the event happens in 1/3 of them

Notation

Event : usually capital letter, like X

eg. "outcome of a dice roll"

Value : usually lower case letter, like x

eg. "4"

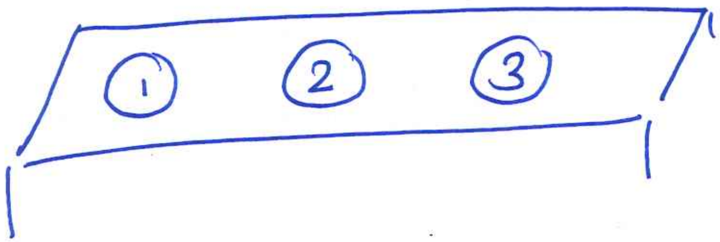
$Pr(X=x)$: probability that event X results in value x

eg $\underbrace{Pr(X=4)}$ If X is dice roll, this means: "probability the dice comes up 4"

a number in [0,1]

$Pr(X=4) = 1/6$

Table with 3 product brands



A consumer chooses one brand.

You want to know $\underbrace{Pr(X=1)}$

Event: choice
Value: 1

Probability the consumer chooses Brand 1

Two chairs
Event: Y which chair
Possible values of Y: L, R

$\underbrace{Pr(X=1 \text{ and } Y=L)}$

Probability the consumer chooses brand 1 AND sits in left chair ex $\Pr(X=x \text{ or } X\neq x) = 1$

any X, x

event, value

100% of the time, something happens or doesn't happen

"or" means "and/or"

$$\Rightarrow \underbrace{\Pr(X=x)} = 1 - \underbrace{\Pr(X\neq x)}$$

We'll talk about continuous probability – since discrete probability is more intuitive, we'll often use discrete examples to illustrate ideas.

discrete: separated, listable

continuous: exist on a continuum

(ex) Event: roll 3 dice, add values. (X)

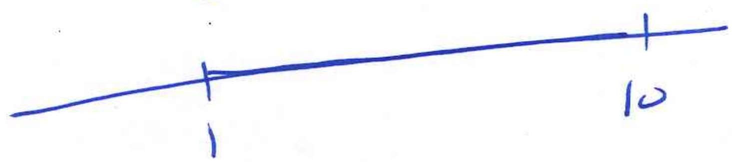
Values of X:

$3, 4, 5, \ldots, 18$ DISCRETE

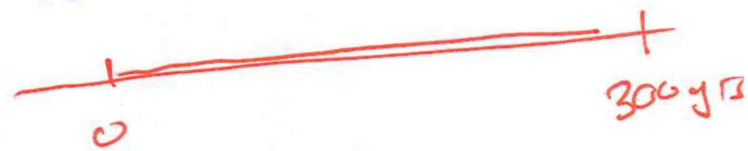
(ex) Event X: choosing any real number between 1 and 10

Values of X: $[1, 10]$ CONTINUOUS

Infinitely many choices



(ex) The exact age of a person at noon today CONTINUOUS



(ex) The number of pets a person has. DISCRETE

$0, 1, 2, 3, 4, \ldots$

Cumulative Distribution Function

ex (Motivation) We might want to ask:
what is the probability
the temp tomorrow is below freezing?
Below 10°C?
Below 20°C?

T: event, temp tomorrow

Asking:

$$Pr(T \leq 0) = F(0)$$
$$Pr(T < 10) = F(10)$$
$$Pr(T < 20) = F(20)$$

The cumulative distribution function for any random variable X, denoted by $F(x)$, is the probability that X assumes a value less than or equal to x:

$$F(x) = Pr(X \leq x)$$

It has the following properties:

- $0 \leq F(x) \leq 1$ for all x (it's a probability)
- $\lim_{x \to \infty} F(x) = 1$ (as x increases, it's more likely that $X \leq x$)
- $\lim_{x \to -\infty} F(x) = 0$ (as x decreases, it's <u>less</u> likely that $X \leq x$)
- $F(x)$ is a nondecreasing function

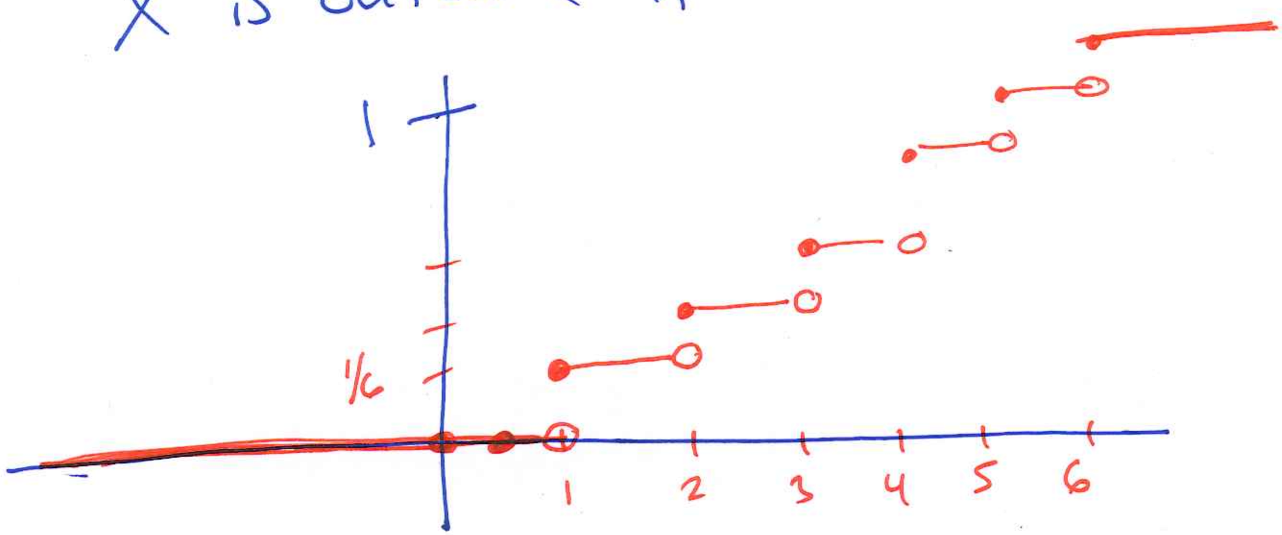
ex (a) Half of days were below or = 20 degrees,
(b) One-third of days were above 10 degrees.]
(continuous)

(a) $F(20) = Pr(T \leq 20) = 1/2$

(b) $F(10) = Pr(T \leq 10) = 1 - Pr(T > 10) = 2/3$

$1/3 : > 10°C$ $\quad 2/3 \leq 10°C$

ex (discrete ex) Sketch $F(X)$, where X is outcome of (fair) dice roll.



$F(1) = Pr(X \leq 1)$
$= Pr(X=1) = 1/6$

$F(1/2) = Pr(X \leq 1/2) = 0$

$F(3) = Pr(X \leq 3)$
$= Pr(X=1 \text{ or } X=2 \text{ or } X=3)$
$= 1/2$

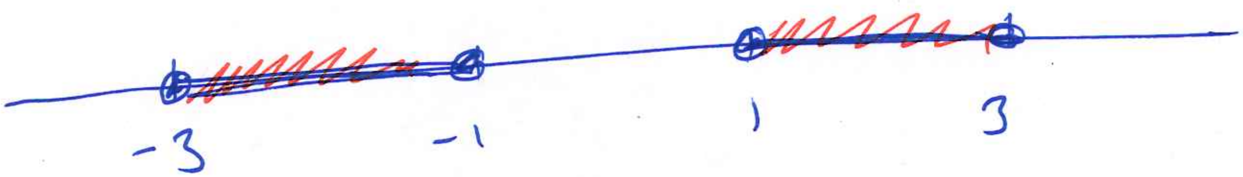
$F(6.1) = Pr(X \leq 6.1) = 1$

$F(1.5) = Pr(X \le 1.5) = Pr(X \le 1) = 1/6$

ex (continuous ex) choose a number

in: $[-3, -1] \cup [1, 3]$

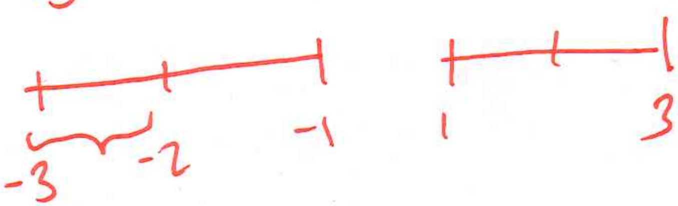
"uniformly" (no preference)



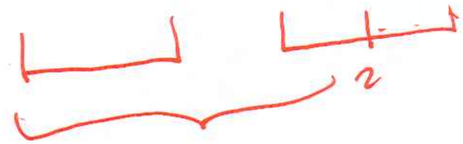
$F(0) = Pr(X \le 0) = 1/2$

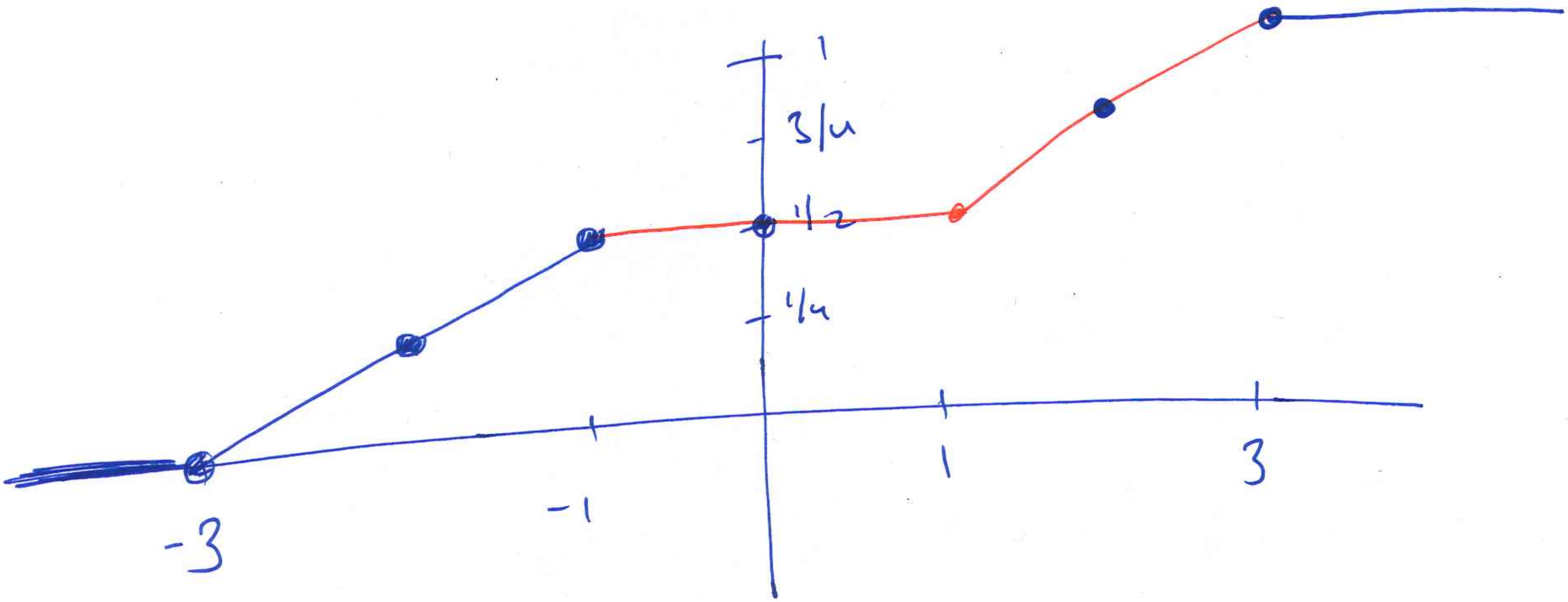
$F(-1) = Pr(X \le -1) = 1/2$

$F(-2) = Pr(X \le -2) = 1/4$



$F(2) = Pr(X \le 2) = 3/4$



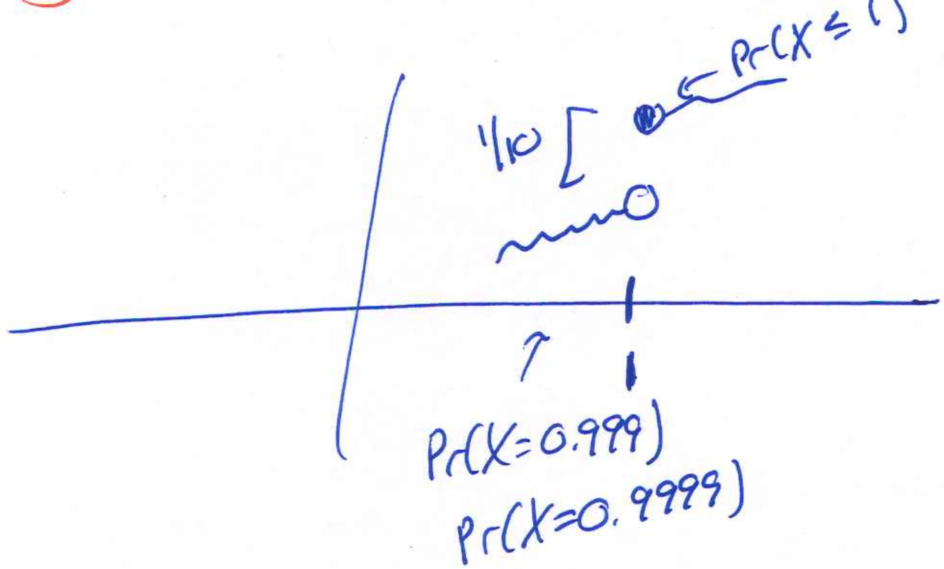


Def. of <u>Continuous</u> Random Variable: random variable whose Cumulative Distribution Function (CDF) is continuous.

If X is a continuous random variable, then $\Pr(X = x)$ for any single value x is 0

(Why is that true?)

Say $\Pr(X=1) = 1/10$



CDF:
jump discontinuity

This is <u>not</u> a continuous system.