Remember from Last Time:

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

To integrate the product of two functions, choose one to be "u" and the other to be "dv." Integrate dv to get v, and differentiate u to get du.

ex $\int x^2 \ln x \, dx$

Let $u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = x^2 dx \rightarrow v = \frac{1}{3}x^3$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

ex

$$\int x e^x \, dx$$

$$\int u \, dv = uv - \int v \, du$$

$$u = x \xrightarrow{\text{diff}} du = dx$$

$$dv = e^x \, dx \xrightarrow[\text{anti}]{} v = e^x$$

$$\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv} = xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$$

Method: no name in literature

"integrating around in a circle"

Only work when int by parts changes integrand little

ex $\int e^x \cos x \, dx$

$u = e^x \longrightarrow du = e^x dx$

$dv = \cos x \, dx \longrightarrow v = \sin x$

$= e^x \sin x - \int e^x \sin x \, dx$

$u = e^x \rightarrow du = e^x dx$

$dv = \sin x \, dx \rightarrow v = -\cos x$

$= e^x \sin x - \left[-e^x \cos x - \int e^x(-\cos x) \, dx \right]$

$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

So:

$$\underbrace{\int e^x\cos x\,dx}_{+\int e^x\cos x\,dx} = e^x\sin x + e^x\cos x \underbrace{\cancel{-\int e^x\cos x\,dx}}_{\cancel{+\int e^x\cos x\,dx}} + C$$

$$2\int e^x\cos x\,dx = e^x\sin x + e^x\cos x + C$$

$$\boxed{\int e^x\cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x) + C}$$

I never really antidifferentiated!

ex $\int e^{2x} \sin x \, dx$

IBP:
$\int u\,dv = uv - \int v\,du$

$u = e^{2x} \longrightarrow du = 2e^{2x}\,dx$

$dv = \sin x\,dx \longrightarrow v = -\cos x$

$-\int 2e^{2x}(-\cos x)\,dx$

$= -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$

$u = 2e^{2x} \longrightarrow du = 4e^{2x}\,dx$

$dv = \cos x\,dx \longrightarrow v = \sin x$

$= -e^{2x}\cos x + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$

that is:

$\int e^{2x}\sin x\,dx = e^{2x}(2\sin x - \cos x) - \cancel{4\int e^{2x}\sin x\,dx}$

$+4\int e^{2x}\sin x\,dx$ $\quad$ $\cancel{+4\int e^{2x}\sin x\,dx}$

$$5\int e^{2x}\sin x\,dx = e^{2x}(2\sin x - \cos x) + C$$

$$\boxed{\int e^{2x}\sin x\,dx = \frac{1}{5}e^{2x}(2\sin x - \cos x) + C}$$

Ch 7.3 Trig Integrals

No new concepts: we apply substitution rule to specific class of functions

These functions will show up a lot in Ch 7.4

Recall: $\boxed{\sin^2 x + \cos^2 x = 1}$ divide by $\cos^2 x$

$$\left(\frac{\sin x}{\cos x}\right)^2 + \left(\frac{\cos x}{\cos x}\right)^2 = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

$$\boxed{\begin{aligned} \sin^2 x &= \frac{1-\cos 2x}{2} \\ \cos^2 x &= \frac{1+\cos 2x}{2} \end{aligned}}$$

ex

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] + C$$