Remember from Last Class:

The substitution rule helps us reverse the chain rule.

$$\frac{d}{dx}\left[\cos(e^x+x^2)\right] = -\sin(e^x+x^2)\cdot(e^x+2x)$$

If we let our "inside function" be u, then we can re-write this as

$$\frac{d}{dx}\left[\cos(\underbrace{e^x+x^2}_{u})\right] = -\sin(u)\cdot\frac{du}{dx}$$

Antidifferentiating:

$$\int -\sin(e^x+x^2)\cdot(e^x+2x)\,dx = \int -\sin(u)\cdot du$$

$u = e^x+x^2$

$\frac{du}{dx} = e^x+2x$

$du = (e^x+2x)dx$

$$= \cos u + C$$

$$= \cos(e^x+x^2) + C$$

ex $\int \underbrace{\sin\theta}_{u} \, \boxed{\overset{du}{\cos\theta \, d\theta}} = \int u \, du = \frac{u^2}{2} + C$

$= \boxed{\frac{1}{2}\sin^2\theta + C}$

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta \, d\theta$

Chek: $\frac{d}{dx}\left(\frac{1}{2}\cdot(\sin\theta)^2 + C\right)$

$= \frac{1}{2}\cdot 2(\sin\theta)\cdot\cos\theta$

$= \sin\theta\cos\theta$

ex $\int \frac{t}{t-3} \, dt$

If we choose

$u = (t-3)^{-1}$

$\frac{du}{dt} = -1(t-3)^{-2} = \frac{-1}{(t-3)^2}$

$du = \frac{-1}{(t-3)^2} dt$

looks hard to pull off

If we choose $u = t$:

$\int \frac{u}{u-3} du$ not easier

u = variable : bad choice

If we choose $u = t-3 \longrightarrow u+3 = t$

$\frac{du}{dt} = 1$, so $du = dt$

$$\int \frac{u+3}{u}\,du = \int \left(\frac{u}{u} + \frac{3}{u}\right) du = \int \left(1 + \frac{3}{u}\right) du$$

$$= u + 3\ln|u| + C = t-3 + 3\ln|t-3| + C$$

write $\boxed{t + 3\ln|t-3| + C}$ (C arbitrary)

Ex $\int \tan x \, dx$

Hint: similar to our first example

$= \int \frac{\sin x}{\cos x} dx$

If we choose $u = \sin x$

then $du = \underbrace{\cos x \, dx}_{\text{multiplies}}$

we have in $\int$: $\left.\frac{1}{\cos x} dx \right\}$ divided

tough

Instead: $u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$= \int \frac{-1}{u} du$

$= -\ln|u| + C$

$= \boxed{-\ln|\cos x| + C}$

$= \ln|(\cos x)^{-1}| + C$

$= \ln\left|\frac{1}{\cos x}\right| + C$

$= \boxed{\ln|\sec x| + C}$

Recall: $a \ln(b) = \ln(b^a)$

(ex) $\displaystyle\int_0^{\pi/4} \tan x \, dx \underset{FTC}{=} \ln|\sec x| \Big]_0^{\pi/4}$

$= \ln|\sec(\pi/4)| - \ln|\sec 0|$

$\cos(\pi/4) = \frac{1}{\sqrt{2}}$

$= \ln\sqrt{2} - \ln 1$

$\cos(0) = 1$

$= \ln\sqrt{2} = \boxed{\frac{1}{2}\ln 2}$

(One way)

Another way:

$\displaystyle\int_0^{\pi/4} \tan x \, dx = \int_{0=x}^{\pi/4 = x} \frac{\sin x}{\cos x}\, dx$

$u = \cos x$

$-du = \sin x \, dx$

$\displaystyle = \int_1^{\frac{1}{\sqrt{2}}} \frac{-1}{u}\, du = -\ln|u| \Big]_1^{1/\sqrt{2}}$

If $x = \pi/4$

$u = \cos x = \cos(\pi/4) = \frac{1}{\sqrt{2}}$

If $x = 0$

$u = \cos x = \cos 0 = 1$

$$= -\ln\left(\frac{1}{\sqrt{2}}\right) - \underbrace{\left(-\ln(1)\right)}_{0}$$

$$= -\ln\left(\frac{1}{\sqrt{2}}\right) = -\ln\left(2^{-1/2}\right) = \boxed{\frac{1}{2}\ln 2}$$

(ex) $\displaystyle\int_0^2 \frac{2s}{s^2+1}\,ds$

$u = s^2+1$ ⊛

$du = 2s\,ds$

Bounds

$s=0$:

$u = s^2+1 = 0^2+1 = 1$

$s=2$:

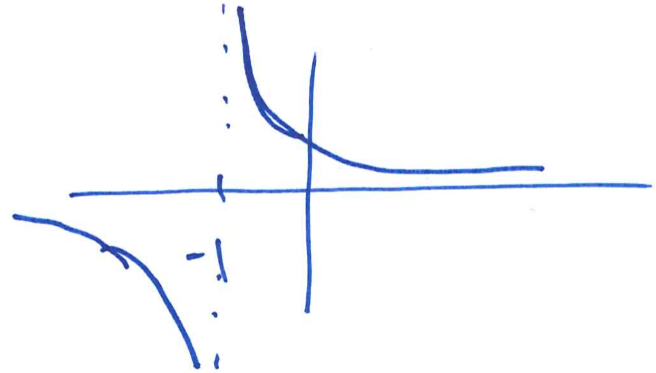
$u = s^2+1 = 4+1 = 5$

$$= \int_1^5 \frac{1}{u}\,du$$

$$= \ln|u| \Big|_{u=1}^{u=5}$$

$$= \ln 5 - \ln 1 = \ln 5$$

ex $\displaystyle\int_{-2}^{2} \frac{x^2}{x^3+1}\,dx$



FTC does not apply if $f(x)$ is discontinuous on $[a,b]$

can't evaluate (yet)

ex $\displaystyle\int_0^1 \frac{dx}{e^x+e^{-x}} = \int_0^1 \frac{1}{e^x+e^{-x}}\,dx$

try $u = e^x + e^{-x}$

then $du = (e^x - e^{-x})\,dx$

$\displaystyle\int_0^1 \frac{e^x}{e^x}\left(\frac{1}{e^x+e^{-x}}\right)dx = \int_0^1 \frac{e^x}{(\boxed{e^x})^2+1}\,dx$

$\boxed{u=e^x}$

$du = e^x\,dx$

If $x=0$ then $u=e^x=e^0=1$

If $x=1$ then $u=e^x=e^1=e$

$\displaystyle\int_1^e \frac{1}{u^2+1}\,du = \arctan u \Big|_1^e$

$= \arctan e - \arctan 1$

$= \boxed{\arctan e - \pi/4}$

ex $\int x^5\sqrt{x^3+1}\,dx$

$u = x^3+1$ ("inside fcn")

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$x^3 = u-1$

$$= \int x^3\sqrt{\boxed{x^3+1}}\;\underbrace{x^2\,dx}_{\frac{1}{3}du}$$

$$= \int (u-1)\sqrt{u}\cdot\frac{1}{3}\,du$$

$$= \frac{1}{3}\int \left(u^{3/2}-u^{1/2}\right)du = \frac{1}{3}\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]+C$$

$$= \boxed{\frac{1}{3}\left(\frac{2}{5}(x^3+1)^{5/2}-\frac{2}{3}(x^3+1)^{3/2}\right)+C}$$