New Topic – Integration

Course: Vectors ✓
Multivariable Functions ✓
Integration ←
Continuous Probability
Series

Ch 5.1 : Approximating areas under curves

Why is this useful?

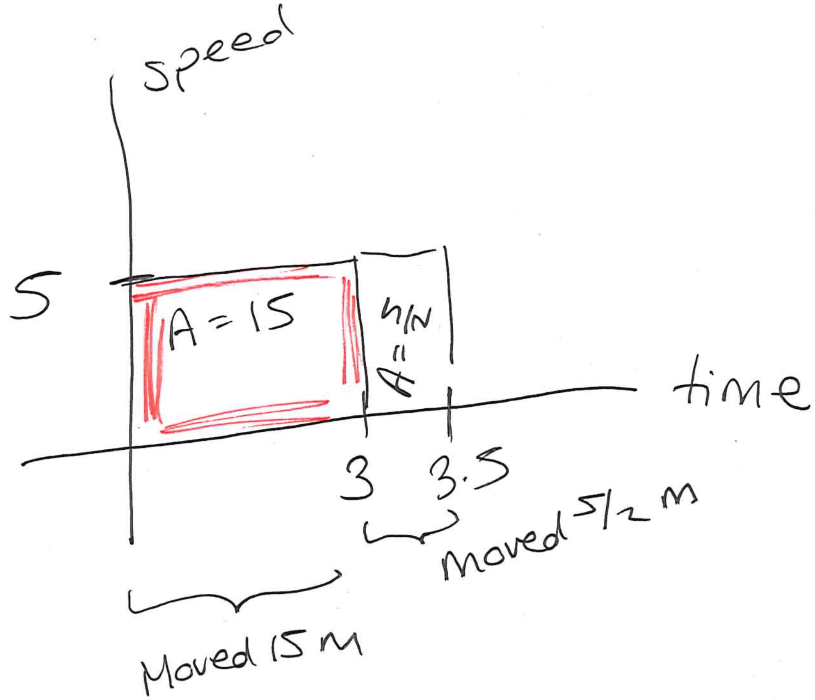
DIST = RATE × TIME

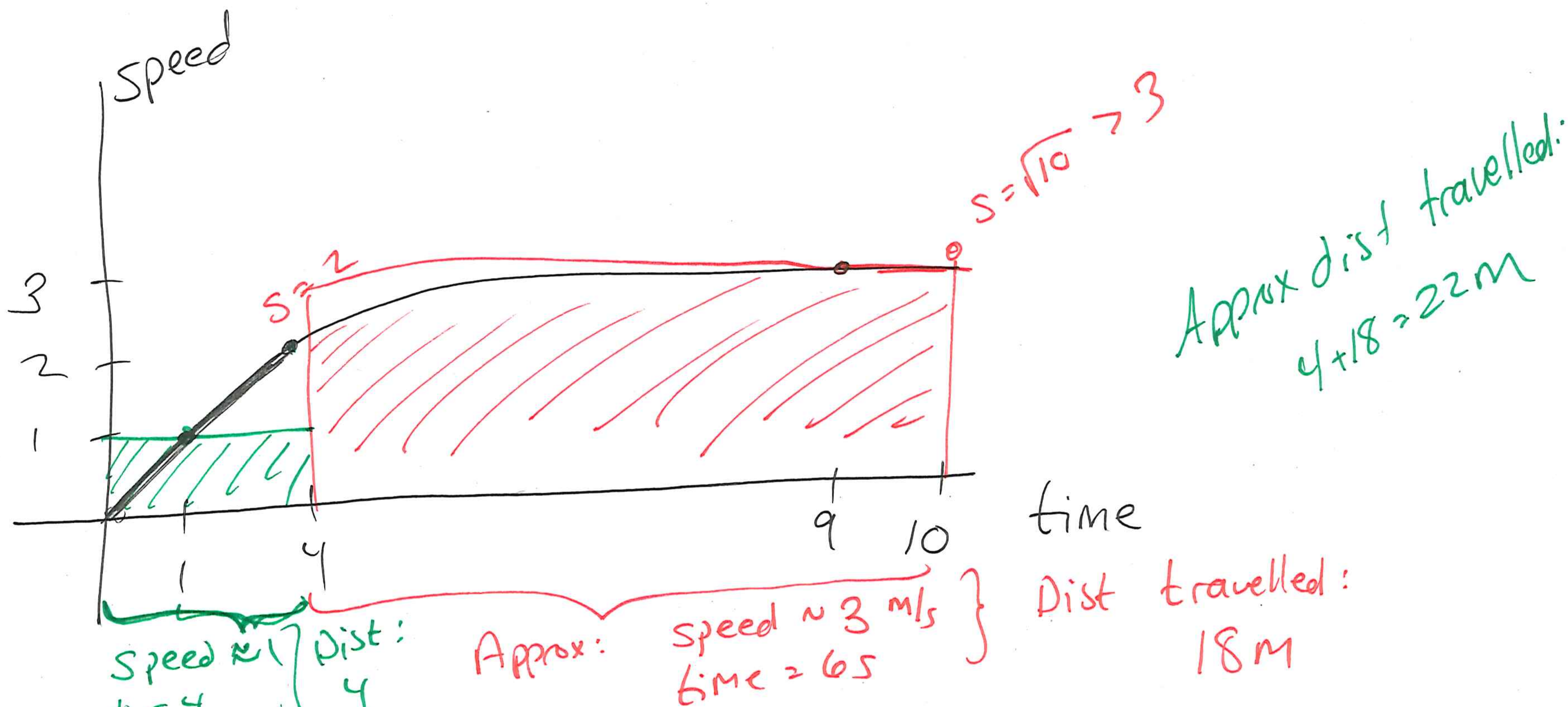
$\boxed{\text{example}}$

5 m/s, 3 sec : dist travelled 15 m

5 m/s, 1/2 sec : " " 2.5 m

At time t, $0 \le t \le 10$,
speed: $\sqrt{t}$ m/s


speed
5
A = 15
A = 5/2
time
3
3.5
Moved 5/2 m
Moved 15 m

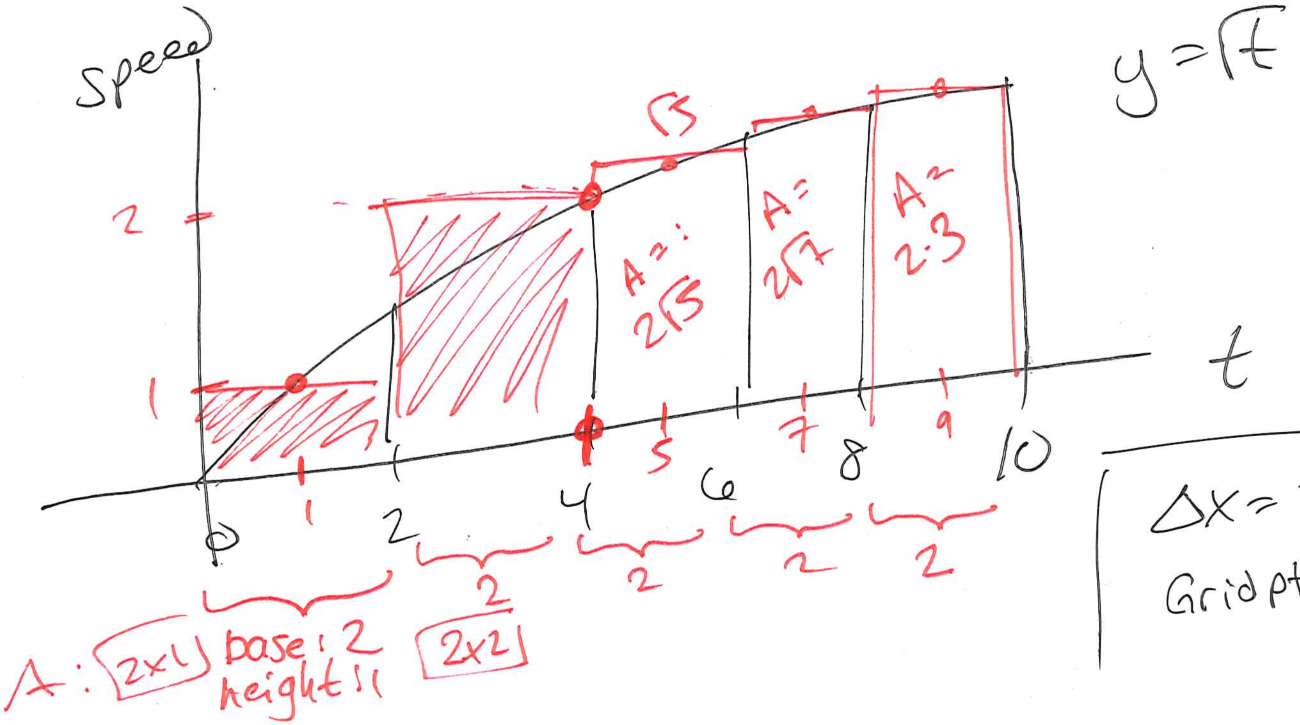

speed
s = √10 > 3
Approx dist travelled:
4+18 = 22m
s = 2
3
2
1
time
1
4
9
10
Speed ≈ 1
t = 4
Dist: 4
Approx: speed ≈ 3 m/s
time = 6s
Dist travelled: 18m

If we graph $y = f(x)$,
where $f(x)$ is rate of change,
then the area under curve is
amount of change

Approx: $0 \le t \le 10$, $f(t) = \sqrt{t}$
(rate of change e.g. speed)

Area: amount of change (eg distance moved)

5 rectangles
(say)



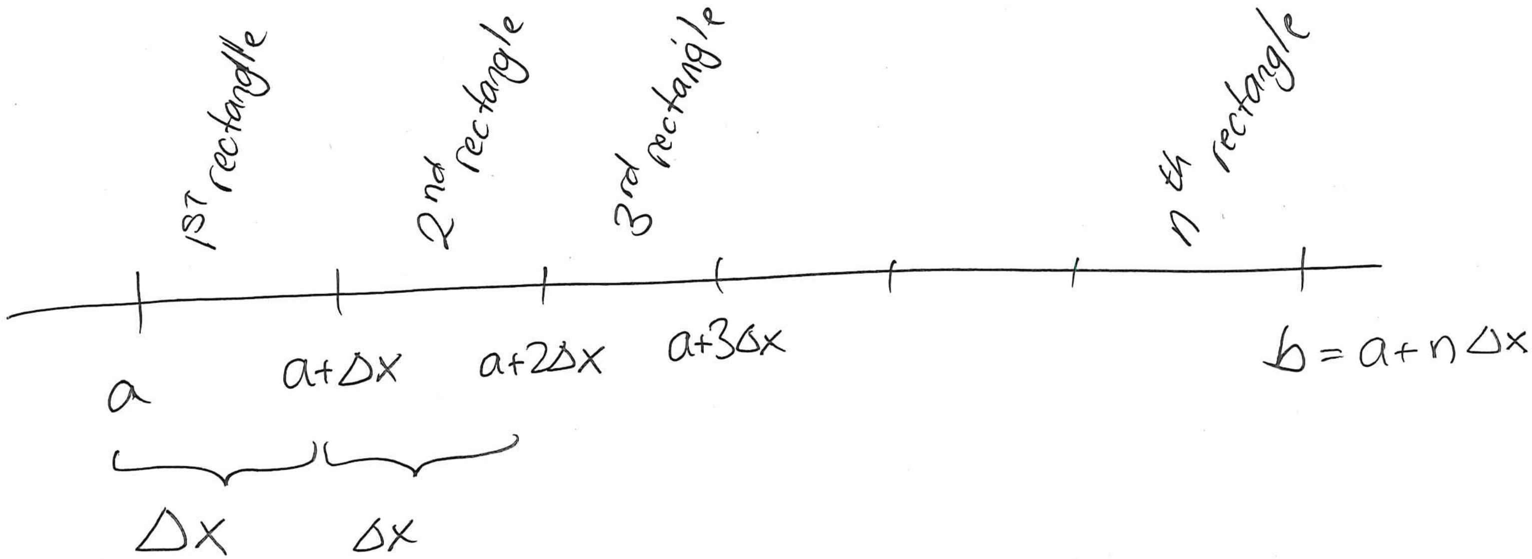
$A \approx \boxed{2\times1 + 2\cdot2 + 2\sqrt{5} + 2\sqrt{7} + 2\cdot3}$

$\Delta x = 2$ (bases of ▯)
Grid pts: 0, 2, 4, 6, 8, 10

Regular partition of $[a,b]$ into n pieces: cuts into equal pieces

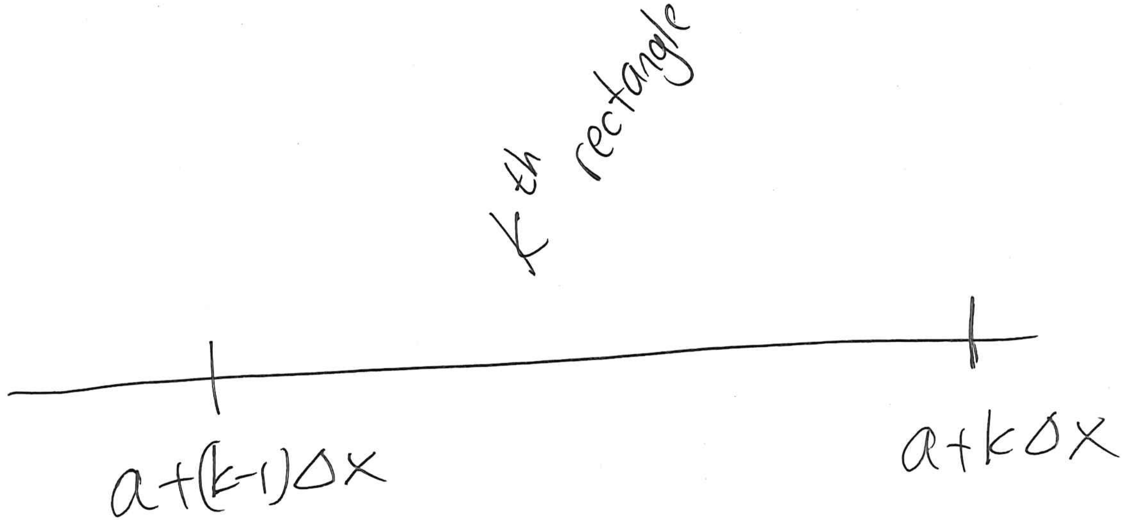
equal size

length of each piece $\Delta x = \dfrac{b-a}{n}$



"Grid points" borders of rectangles

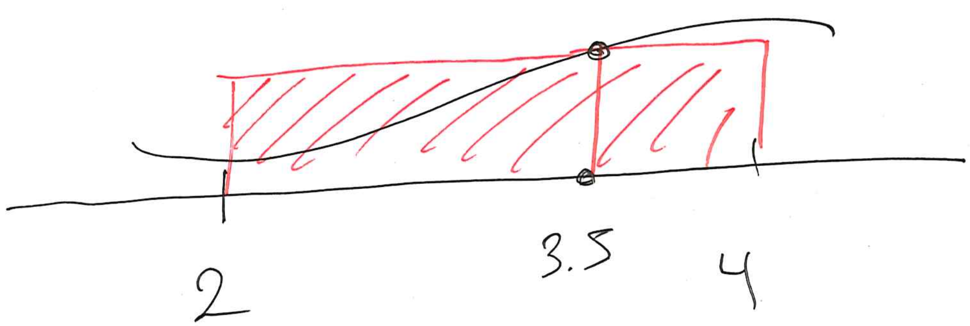
$$x_0 = a$$
$$x_1 = a + \Delta x$$
$$x_2 = a + 2\Delta x$$
$$\vdots$$
$$x_k = a + k\Delta x$$
$$\vdots$$
$$x_n = a + n\Delta x$$



Base of rectangle: Δx

Height of rectangle: height of function at <u>some</u> point in interval, $[a+(k-1)\Delta x, a+k\Delta x)$

We call that point: x_k^*



$x_k^* = 3.5$

$\Delta x = 2$ (base)

$f(x_k^*) = f(3.5)$ (height)

$A = 2 \cdot f(3.5)$

Riemann Sum

(sum of approximating []s)

$(b)(h) + (b)(h) + (b)(h) + \cdots + (b)(h)$

$\Delta x \cdot f(x_1^*) + \Delta x f(x_2^*) + \Delta x f(x_3^*) + \cdots + \Delta x f(x_n^*)$

where: $\Delta x = \frac{b-a}{n}$ (base)

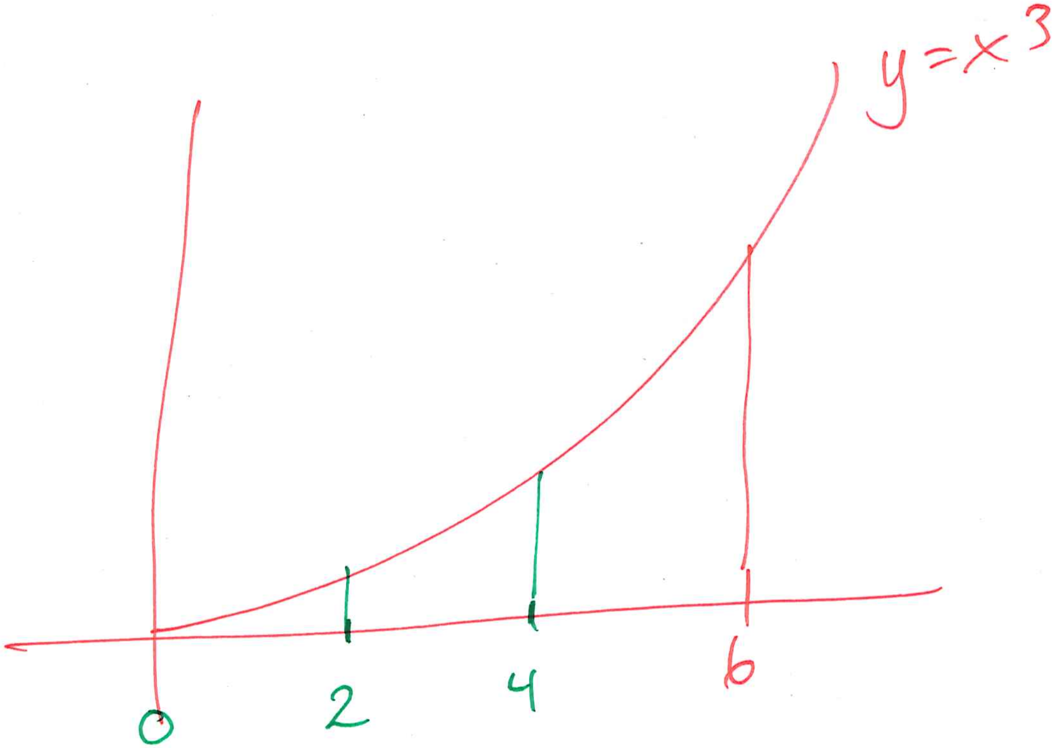
$f(x_k^*)$ = height of function at some point x_k^* in appropriate interval

If we choose (all intervals) x_k^* to be:
left / right / midpoint

we call our sum a
left / right / midpoint Riemann sum.

(ex) Approx area under $y = x^3$, $0 \le x \le 6$, using 3 intervals.

$f(x) = x^3$, $n = 3$, $a = 0$, $b = 6$



Vocab:

Grid Points

$x_0 = 0$

$x_1 = 2$

$x_2 = 4$

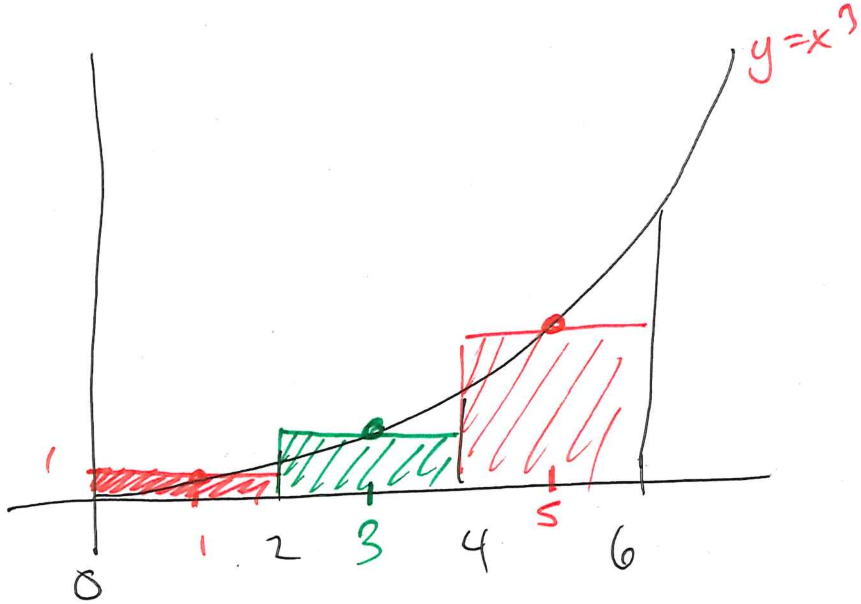
$x_3 = 6$

$$\Delta x = \frac{b-a}{n} = \frac{6}{3} = 2$$

base of ▯

Regular partition: all ▯ have same base length.

MIDPOINT RIEMANN SUM:



height: $f(x_k^*)$,

x_k^*: midpt of interval

1st $\square$: $x_1^* = 1$

$A = 2$ $\quad f(1) = 1$ height

$\Delta x = 2$ base

2nd $\square$: $x_2^* = 3$

$A = 54$ $\quad f(3) = 3^3 = 27$ height

$\Delta x = 2$ base

3rd $\square$: $\Delta x = 2$ base

$A = 250$ $\quad x_3^* = 5$

height: $f(5) = 5^3 = 125$

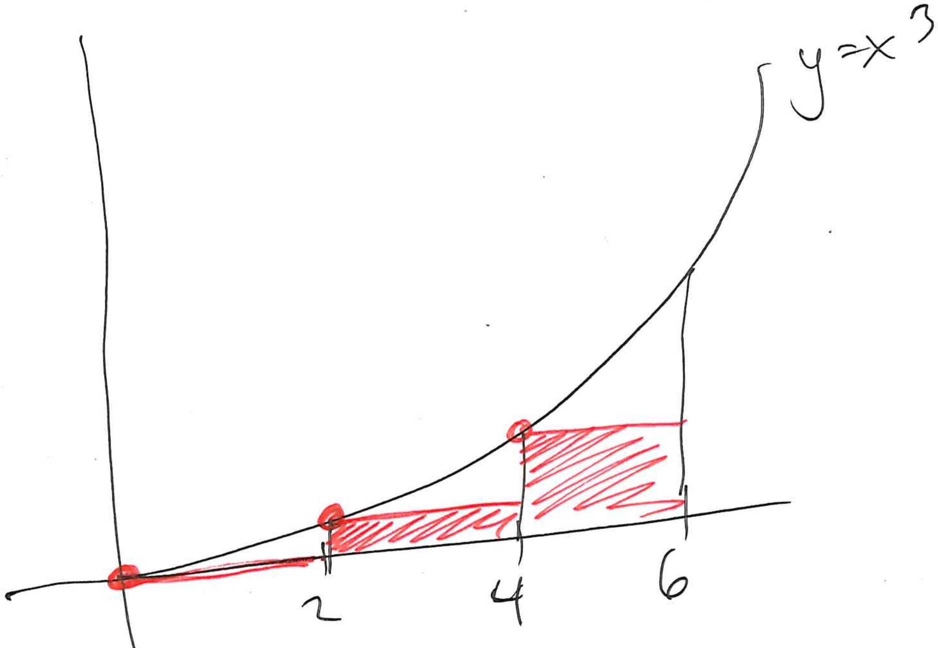
MP Rieman Sum:

Area under curve

$\approx 2 + 54 + 250$

$= \boxed{306}$

LEFT RIEMANN SUM



Area under curve $\approx 0 + 16 + 128$

$\Delta x = 2$ (bases)

$\boxed{1}$: $h = f(0) = 0$ $\quad x_1^* = 0$

$b = \Delta x = 2$

$A = 0$

$\boxed{2}$: $h = f(2) = 8$ $\quad x_2^* = 2$

$b = \Delta x = 2$

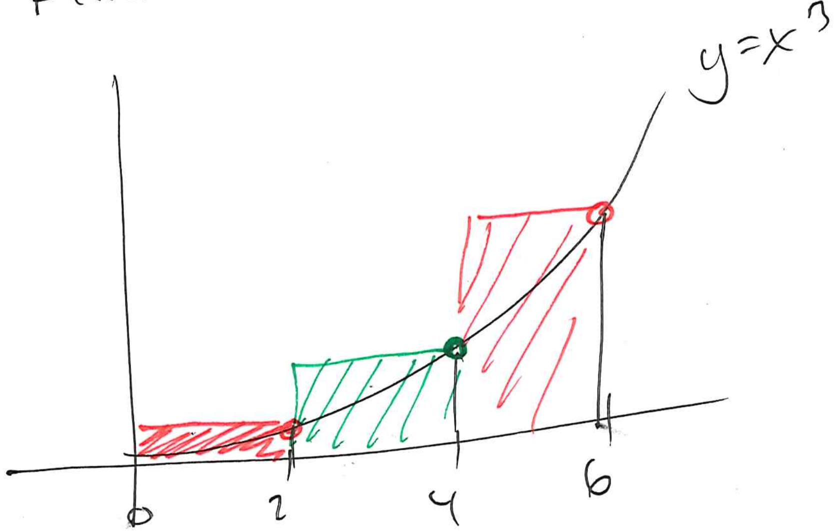
$A = 16$

$\boxed{3}$: $h = f(4) = 64$ $\quad x_3^* = 4$

$b = \Delta x = 2$

$A = 128$

RIGHT RS:



$x_1^* = 2$

$A_1 = 2 \times f(2)$
$= 2 \cdot 8$
$= 16$

$x_2^* = 4$

$A_2 = \Delta x \cdot f(4)$
$= 2 \cdot 64$
$= 128$

$x_3^* = 6$

$A_3 = \Delta x\, f(6)$
$= 2 \cdot 216$
$= 432$

Area $\approx 16 + 128 + 432$

ex

time	12:00	12:15	12:30	12:45	1:00
speed	60 kph	80	100	100	40

Q: Approx how far car travelled from 12 to 1

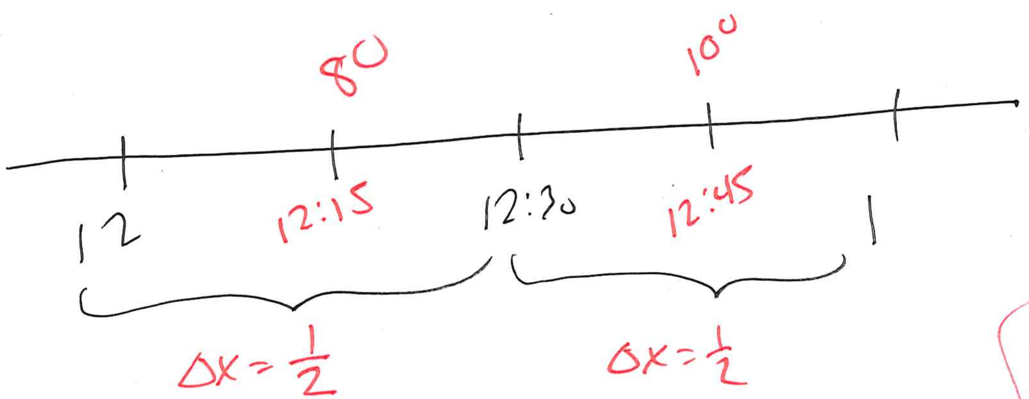
$\Delta x = \frac{1}{4}$ hr (15 min)

Using RS: # intervals $n=4$

Left RS: $\frac{1}{4}(60) + \frac{1}{4}(80) + \frac{1}{4}(100) + \frac{1}{4}(100)$

Right RS: $\frac{1}{4}(80) + \frac{1}{4}(100) + \frac{1}{4}(100) + \frac{1}{4}(40)$

100 kph
15 min → $\frac{1}{4}$ hr
dist ≠ 1500 km
= 25 km



MIDPOINT RS: $n=2$

$\boxed{80 \cdot \frac{1}{2} + 100 \cdot \frac{1}{2}}$