Midterm info is on course webpage

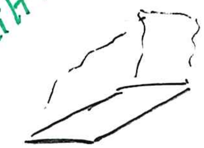
Expect questions of a variety of difficulties, roughly ordered simpler to more complex.

No formula sheet.

2018 01 24

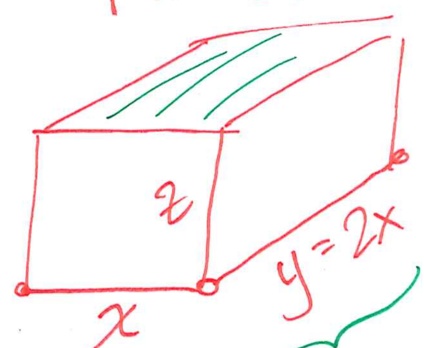
(ex) A rectangular box has:

- volume 72 cubic cm ← constraint
- width twice its length. ← builtin



What dimensions give <u>minimum</u> surface area.

Objective fcn: what we want to optimize
surface area



Area: $x \cdot 2x = 2x^2$

Area: xz

Area: $yz = 2x^2$

$$f(x,z) = 2xz + 2\cdot 2xz + 2\cdot 2x^2$$
$$= 6xz + 4x^2$$

Easier to look at:

$$f(x,y) = 6xy + 4x^2$$

Constraint:

$$(x)(2x)y = 72$$
$$2x^2y = 72$$

$$\left.\begin{aligned} f(x,y) &= 6xy + 4x^2 \\ g(x,y) &= 2x^2y - 72 = 0 \end{aligned}\right\}$$

Also know: $x, y > 0$

$$\left.\begin{aligned}
&① \quad \underbrace{6y+8x}_{f_x} = \lambda(\underbrace{4xy}_{g_x}) \\
&② \quad \underbrace{6x}_{f_y} = \lambda\,(\underbrace{2x^2}_{g_y}) \\
&③ \quad 2x^2y-72=0
\end{aligned}\right\}$$

$\lambda = \dfrac{6y+8x}{4xy}$ or $\left.\begin{array}{l}4xy=0\\ ③ \text{ fails}\end{array}\right]$ ignore

$$\lambda = \frac{6y+8x}{4xy} = \frac{3y+4x}{2xy}$$ makes ① true

② : $\cancel{6x} = \underbrace{\lambda \cdot 2x^2}_{} = \underbrace{\left(\dfrac{3y+4x}{\cancel{2x}y}\right)}_{\lambda}\cancel{2x^2}$

$$6 = \frac{3y+4x}{y}$$

$$6y = 3y+4x$$

$$3y = 4x$$

$$\boxed{y = \tfrac{4}{3}x}$$

③

$$2x^2(\underbrace{\tfrac{4}{3}x}_{y}) = 72$$

$$\frac{8}{3}x^3 = 8\cdot 9$$

$$x^3 = 27$$

$\boxed{x=3}$

$\boxed{y=4}$

other dimension:

$2x$: $\boxed{6}$

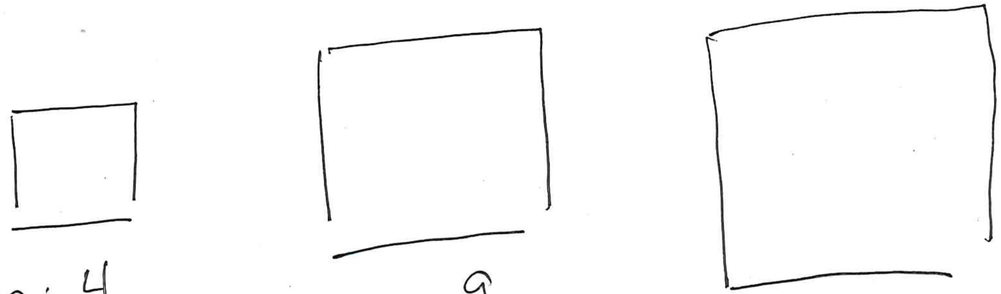
Box: $3\times4\times6$

SA: $6(3)(4)+4(3)^2$ smallest SA

$= 4\cdot9(2+1) = 12\cdot9 = \boxed{108}$

Observation

squares



Area: 4
$x^2 = 4$

Area: 9
$x^2 = 9$

Area: 16
$x^2 = 16$

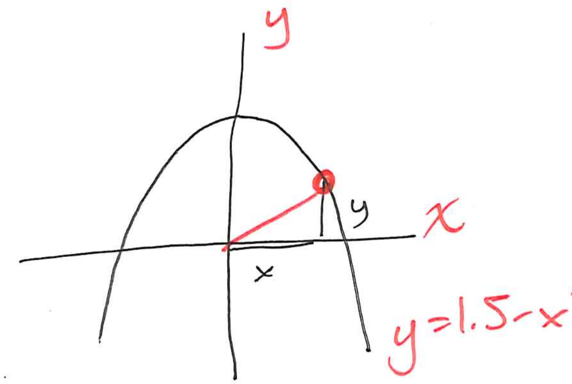
Q: which has smallest side length?

USEFUL: To find min value of x, we can find min value of x^2, then $\sqrt{\ }$ it.

Ex: $D(x,y) = \sqrt{x^2+y^2}$ ← ~~hard~~ annoying to differentiate

$D^2 = x^2+y^2$ ← nice to differentiate

(ex) Find the point(s) on $y = 1.5 - x^2$ closest to origin (use Lagrange multipliers).



Thing we want to minimize:
Distance from (x,y) to $(0,0)$

$$D(x,y) = \sqrt{x^2+y^2}$$

Constraint:
(x,y) should be on parabola

$$y + x^2 - 1.5 = 0$$

Observation: find (x,y) that minimizes

$$f(x,y) = x^2+y^2 \qquad \text{(nicer!)}$$

$$g(x,y) = y + x^2 - 1.5 = 0$$

① $2x = \lambda 2x$
② $2y = \lambda(1)$
③ $y + x^2 - 1.5 = 0$

$\rightarrow \boxed{\lambda = \frac{2x}{2x} = 1}$ or $\boxed{x = 0}$

If $\boxed{\lambda = 1}$

① true
② $2y = 1$
$y = 1/2$
③ $\frac{1}{2} + x^2 - \frac{3}{2} = 0$
$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

Points to Considers

$(1, 1/2)$

$(-1, 1/2)$

$(0, 3/2)$

If $\boxed{x = 0}$

① true (for any λ)
② $2y = \lambda$ → we can choose λ to be whatever!
③ $y + 0^2 - 1.5 = 0$
$y = 1.5$

In this case: $2(1.5) = \lambda$
$\lambda = 3$

$x = 0, \quad y = 3/2, \quad \lambda = 3$

① $2 \cdot 0 = 3 \cdot 2 \cdot 0$ ✓
② $2 \cdot \frac{3}{2} = 3 \cdot 1$ ✓
③ $\frac{3}{2} + 0^2 - 1.5 = 0$ ✓

Ch 12.9

$f(x,y) = x^2 + y^2$ (dist^2)

$$\left.\begin{aligned} f(1, 1/2) &= 1 + \tfrac{1}{4} = 5/4 \\ f(-1, 1/2) &= 1 + \tfrac{1}{4} = 5/4 \end{aligned}\right\}$$

$f(0, 3/2) = 0 + \frac{9}{4} = 9/4$

Min distance:

$\frac{\sqrt{5}}{2}$

at $(\pm 1, 1/2)$