Classify the origin $(0,0)$

$f(x,y) = \cos(xy)$

$f(0,0) = \cos 0 = 1$

Range of cosine: $[-1,1]$

This is as big as cosine ever gets!

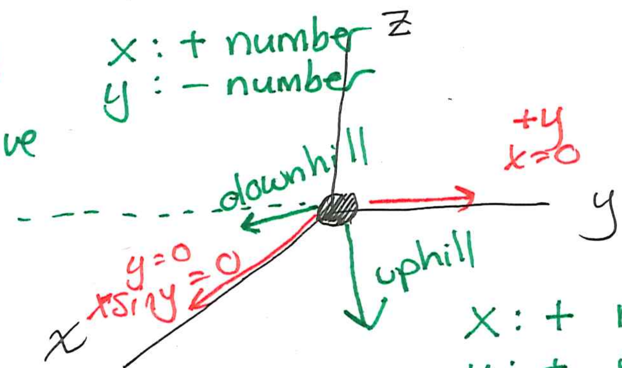
So $(0,0)$ is location of local max.

$\underbrace{x}_{+}\underbrace{\sin y}_{-}$ negative

$f(x,y) = x \sin y$

$f(0,0) = 0 \cdot \sin 0 = 0$

saddle point @ $(0,0)$



$x \sin y = 0$

x: + number
y: + number

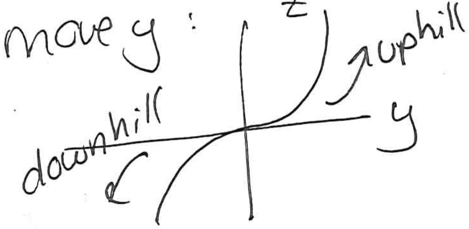
$\underbrace{x}_{+}\underbrace{\sin y}_{+} > 0$

$f(x,y) = x^2 + y^3$

$f(0,0) = 0$

saddle point

If I keep $x=0$ move y:



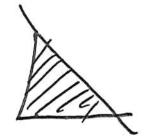
$f(x,y) = x^4 + y^6$

$f(0,0) = 0$

minimum

Always: $x^4 \geq 0$
$y^6 \geq 0$

So $x^4 + y^6 \geq 0$

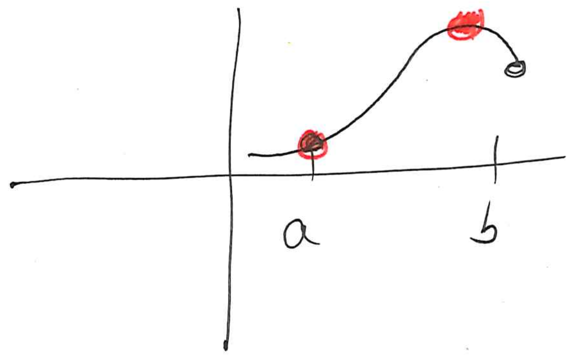
Absolute extrema over a closed, bounded region (ex: )

occur at CPs or along boundaries

Similar to last semester:

$y = f(x)$

$[a, b]$



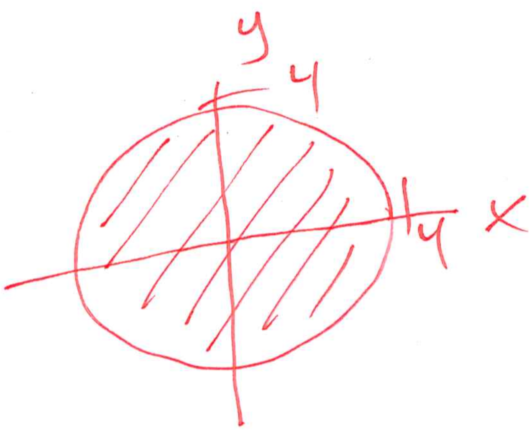
Check CPs + endpoints

ex) Find absolute extrema of

$$f(x,y) = 2x^2+y^2$$

over the region

$$R = \{ \underbrace{(x,y)}_{\text{all points } (x,y)} \ \underset{\text{such that}}{:}\ \underbrace{x^2+y^2 \le 16}_{\text{this is true}} \}$$



CPs : $f_x = 4x = 0 \quad x=0$

$f_y = 2y = 0 \quad y=0$

CP : $(0,0)$

Check boundary : "plugging in"

If (x,y) on bdry, that means $\underbrace{x^2+y^2=16}$

$y^2 = 16-x^2$

Then : $f(x,y) = 2x^2+\underbrace{y^2}$

$= 2x^2+(16-x^2)$

$= 16+x^2$

Looking at picture:

$-4 \le x \le 4$

On this parabola

$x=\pm 4$: highest

$x=0$: lowest

Points to check :

$(0, \pm 4)$ & $(\pm 4, 0)$

$f(0,0) = 0$ ← abs min of 0 at $(0,0)$

$f(0, \pm 4) = 0 + 16 = 16$ (nothing in particular)

$f(\pm 4, 0) = 2 \cdot 16 + 0 = 32$ ← abs max of 32 at $(4,0)$ $(-4,0)$

$16 + x^2$ min when $x = 0$

Considering (x,y) on bdry: $x^2 + y^2 = 16$

If $x = 0$:

$0^2 + y^2 = 16$

so $y = 4$ or $y = -4$

(ex) Find absolute extrema of

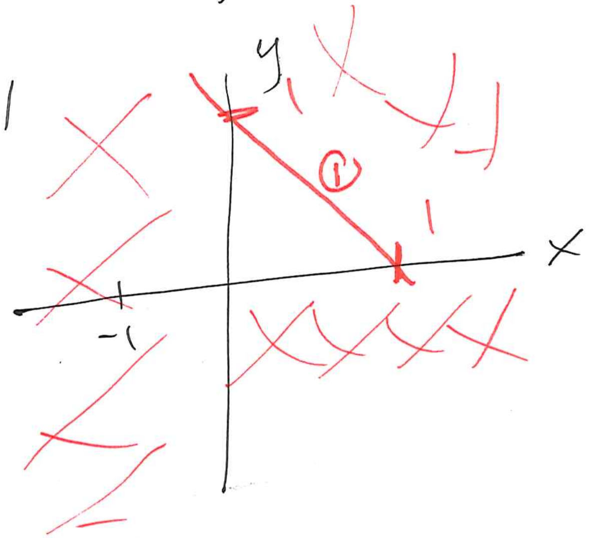
$$f(x,y) = \frac{x+1}{x+y+1}$$

over region:

$x+y \leq 1$

$x \geq 0$

$y \geq 0$



Method: Compare $f(x,y)$ at CPs & boundaries

In our region, partial derivatives exist.

$$f_x = \frac{(x+y+1)(1) - (x+1)(1)}{(x+y+1)^2} = \frac{y}{(x+y+1)^2} = 0$$

$$f_y = \frac{(x+y+1)0 - (x+1)(1)}{(x+y+1)^2} = \frac{-(x+1)}{(x+y+1)^2} = 0 \rightarrow -(x+1) = 0$$

$x = -1$

regardless of y: not in region

NO CPs in region.

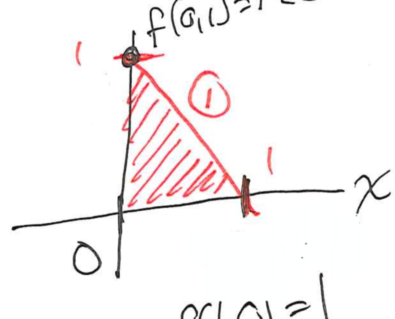
Abs extrema will be on boundary.

① $\underline{x+y=1}$ $\qquad f(x,y) = \dfrac{x+1}{x+y+1} = \dfrac{x+1}{2} = \dfrac{1}{2}x + \dfrac{1}{2}$

bigger for big x

smaller for small x

Look at picture:

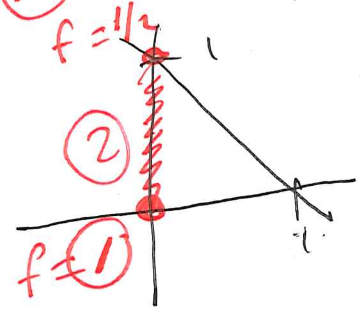


$0 \le x \le 1$

Max: If $x=1$, $f(x,y) = \frac{1}{2}+\frac{1}{2} = 1$

Min: If $x=0$, $f(x,y) = \frac{1}{2}$

② $x=0$



$f(x,y) = \dfrac{\cancel{x}+1}{\cancel{x}+y+1} = \dfrac{1}{y+1}$

Big for small y

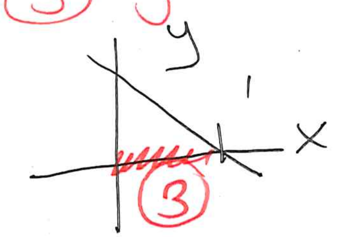
Small for large y

$0 \le y \le 1$

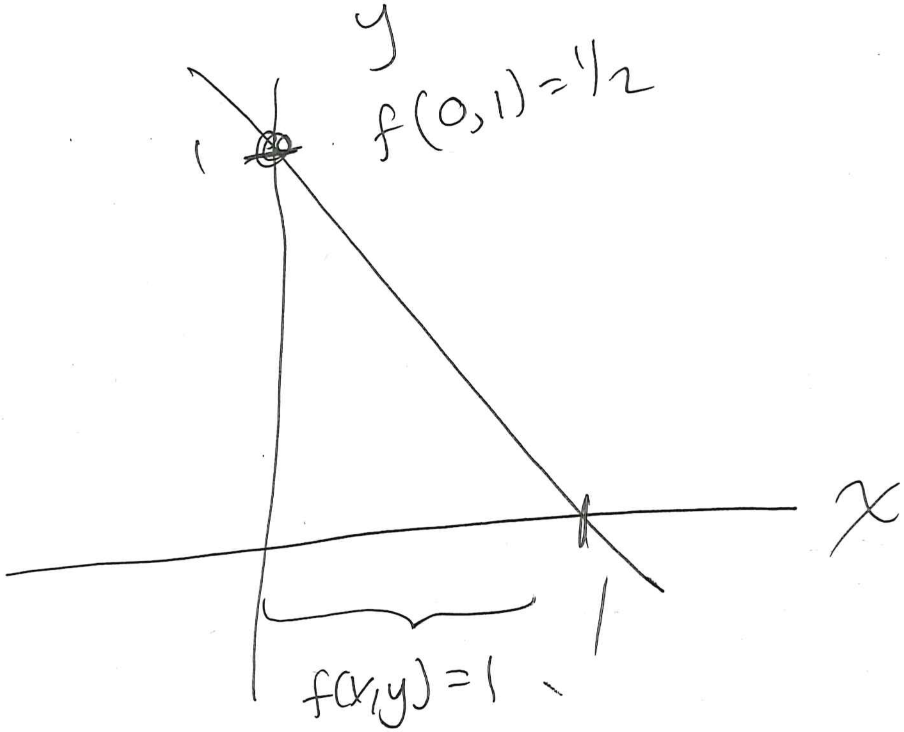
Max: $\dfrac{1}{0+1} = 1$ at $(0,0)$

Min: $\dfrac{1}{1+1} = \dfrac{1}{2}$ at $(0,1)$

③ $y=0$



$f(x,y) = \dfrac{x+1}{x+0+1} = 1$



Abs min: $1/2$
at $(0,1)$

Abs max: 1
when $y = 0$
$0 \le x \le 1$

ex Find all absolute extrema of

$$f(x,y) = (xy)e^{-x-y}$$

on the region $R = \{ (x,y) : x \geq 0,\ y \geq 0,\ x+y \leq 1 \}$

$$f_x = (xy)e^{-x-y}(-1) + e^{-x-y}(y)$$

$$= e^{-x-y}(y-xy) = y(1-x)\underbrace{e^{-x-y}}_{>0} = 0$$

$y=0$ or $1-x=0$

$x=1$

$$f_y = \underbrace{x(1-y)e^{-x-y} = 0}$$

If $y=0$: $x(1)e^{-x-y}=0$

$\boxed{x=0}$

CP: $(0,0)$ $\ast$

If $x=1$:

$1(1-y)e^{-x-y}=0$

$1-y=0$

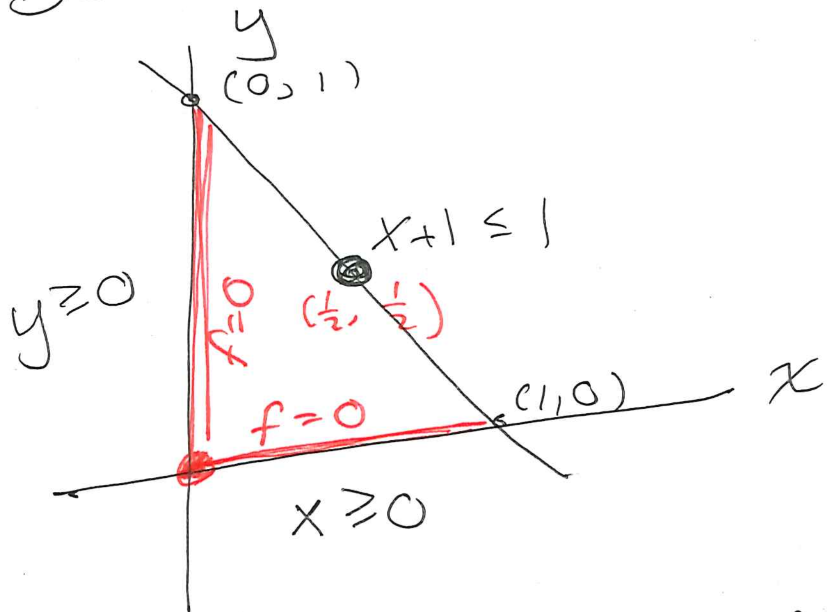
$y=1$

$\underbrace{\text{CP: } (1,1)}$ not in R

R: $x+y \leq 1$

IGNORE

Boundaries:



$\boxed{x=0}$

$f(x,y) = \underbrace{x}_{0}\, y\, e^{-x-y} = 0$

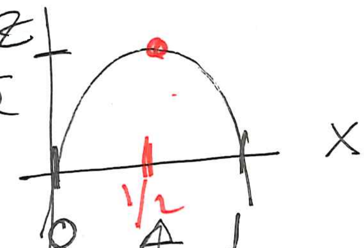
$\boxed{y=0}$ $f(x,y) = x\,\underbrace{y}_{0}\, e^{-x-y} = 0$

$\boxed{x+y=1}$

$y = 1-x$

$f(x,y) = (xy)e^{-(x+y)} = x(1-x)e^{-1} = \underbrace{\frac{1}{e}x(1-x)}_{\text{parabola}}$

$= \frac{1}{e}(-x^2+x)$



To find max find CP $g(x) = \frac{1}{e}(-x^2+x)$

$g'(x) = \frac{1}{e}(-2x+1) = 0$

$1 = 2x$

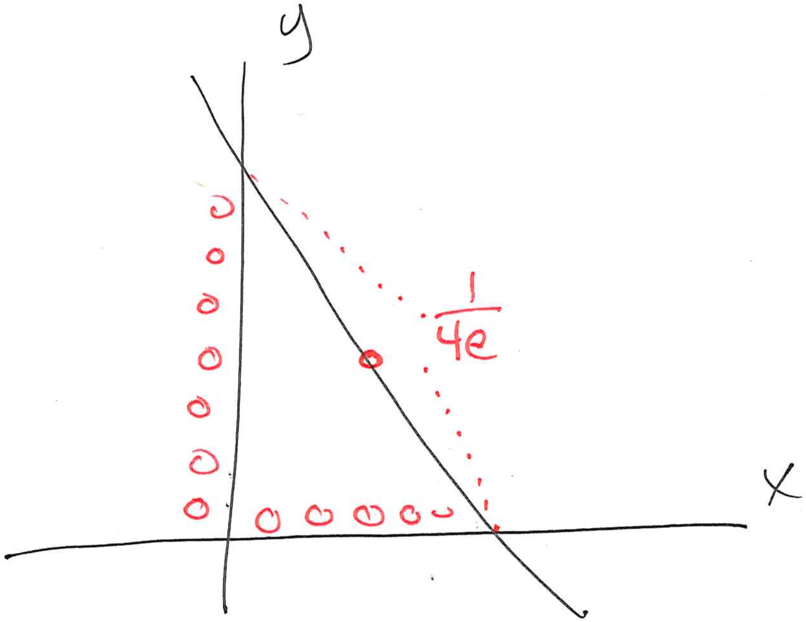
$x = 1/2$

From picture:

$0 \le x \le 1$

Since $x+y=1$, if $x = 1/2$, then $y = 1/2$

$f(1/2, 1/2) = \frac{1}{e}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4e}$



Abs Max on R:

$\frac{1}{4e}$ at $(1/2, 1/2)$

Abs Min: 0

occurs: when $x=0$ and/or $y=0$

Compare / Contrast

Finding:	Local extrema	Absolute extrema
Using:		
Find CPs	Y	Y
Test boundary	N	Y
2nd Deriv Test	Y	N

Find over closed, bounded region