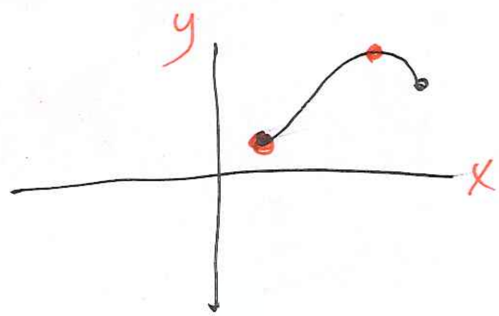
Absolute Maxima and Minima (over a region)

Recall: <u>local</u> extrema occur @ critical points
max/min

To find <u>absolute</u> max/min on a closed region, check CPs <u>and</u> boundaries

Recall (last semester)

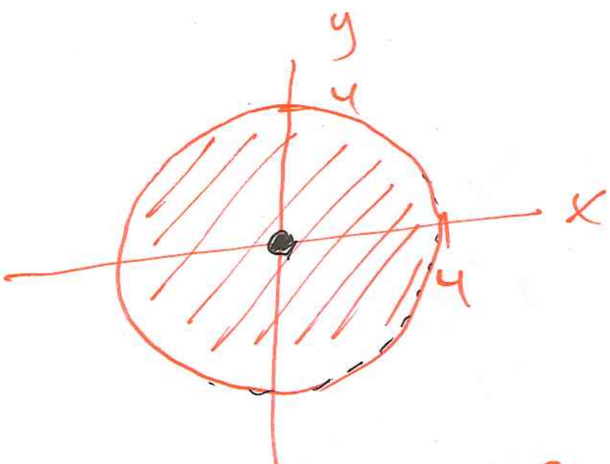


Max/min occur @ CPs or Endpoints

(ex) Find absolute max & min of

$$f(x,y) = 2x^2+y^2$$

in the region $R = \{\underbrace{(x,y)}_{\text{all points } (x,y)} \overset{\text{such that}}{:} \underbrace{x^2+y^2 \le 16}_{\text{this is true}}\}$



Need to check: CPs, boundary.

(CPs) $f_x = 4x = 0 \quad x=0$

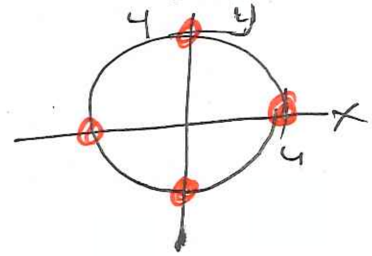
$f_y = 2y = 0 \quad y=0$

Only CP: $\boxed{(0,0)}$

Check Boundary:

If a point (x,y) is on boundary of R,

$x^2+y^2=16$



For these points, how does our function look?

$f(x,y) = 2x^2+\underbrace{y^2}_{16-x^2}$

$= 2x^2+16-x^2$

$= x^2+16$

On bdry: $x^2+y^2=16$

$\Rightarrow y^2=16-x^2$

Familar function

In R: $-4 \le x \le 4$

x^2+16 biggest: $x=\pm 4$

x^3+16 smallest: $x=0$

On bdry: If $x=0$, $y=\pm 4$

$(0,\pm 4)$

If $x=\pm 4$, $y=0$

$(\pm 4, 0)$

Check:

$(0,0)$ (CP)

$(\pm 4, 0)$

$(0, \pm 4)$

$f(0,0) = 0$ ← abs min

$f(\pm 4, 0) = 2 \cdot 16 + 0 = 32$ ← abs max

$f(0, \pm 4) = 16$ (nothing in particular)

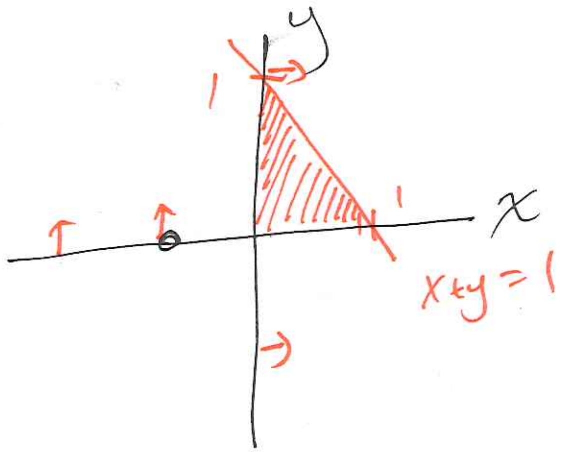
9x

$$f(x,y) = \frac{x+1}{x+y+1}$$

Find absolute extrema on region bounded by:

$x+y \le 1$, $x \ge 0$, $y \ge 0$

Check CPs & boundary



CPs: $f_x = \frac{(x+y+1)(1)-(x+1)(1)}{(x+y+1)^2} = \frac{y}{(x+y+1)^2} = 0 \quad y=0$

CP: $(-1,0)$

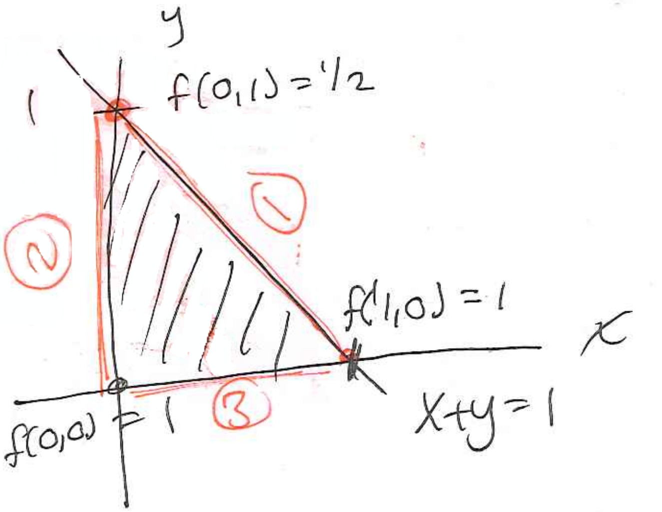
$f_y = \frac{(x+y+1)0-(x+1)(1)}{(x+y+1)^2} = -\frac{(x+1)}{(x+y+1)^2} = 0: \quad x=-1$

not in R
ignore it

No CPs in R

$x+1=0$
$x=-1$

Check boundaries:



① $x+y \le 1$

② $x \ge 0$

③ $y \ge 0$

① If $x+y=1$:

$$f(x,y) = \frac{x+1}{\underbrace{x+y}_{1}+1} = \frac{x+1}{2}$$

big as x big

small as x smaller

In R: $0 \le x \le 1$

So: highest pt on this edge is when $x=1$ ($x+y=1$)

$f(1,0)=1$

lowest: $x=0$

$f(0,1)=\frac{1}{2}$

$$f(x,y)=\frac{x+1}{x+y+1}$$

② If $x=0$:

$$f(x,y) = \frac{0+1}{0+y+1} = \frac{1}{y+1}$$

big as y small

small y big

$0 \le y \le 1$

$f(0,0)=1$

$f(0,1)=1/2$

③ If $y=0$:

$$f(x,y)=\frac{x+1}{x+0+1}=1$$

Largest value of f: $\boxed{1}$ ← pts in R with $y=0$ (many places)

Smallest: $1/2$ at $(0,1)$