Information about your upcoming quiz is on our section web page.

Warmup: Your grade for this course is calculated as:

$$C = 0.1W + 0.06Q + 0.34M + 0.5F$$

where W is your webwork score, etc and all scores are measured out of 100.

Interpret the partial derivatives C_W, C_Q, etc.

$C_W = 0.1$ $\qquad \frac{\Delta C}{\Delta W} = \frac{1}{10}$ $\qquad$ ex If $W \uparrow 10$, $C \uparrow 1$

$\Rightarrow \Delta C = \frac{1}{10} \Delta W$

More complicated: $C = xy + x^2 + \sin y$

Higher-order derivatives

$$f(x,y)$$

$$f_x(x,y) \qquad f_y(x,y)$$

Two first-order partial derivatives

$$(f_x)_x \quad (f_x)_y \quad (f_y)_x \quad (f_y)_y$$

$$f_{xx} \quad f_{xy} \quad f_{yx} \quad f_{yy}$$

Four second-order partial derivatives

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ex $f(x,y) = x \sin y$

① $f_x = \sin y$

$f_{xx} = 0$ $\quad (f_x)_y = f_{xy} = \cos y$

$f_y = x \cos y$

$f_{yx} = \cos y \quad f_{yy} = -x \sin y$

Notation:

f_{xyxx}

1st 2nd 3rd 4th

$f(x,y)$

$\frac{\partial f}{\partial x}$: diff wrt x (1ST)

$$= \frac{\partial}{\partial x}[f]$$

$$\frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x}\right] = \frac{\partial^2 f}{\partial y \partial x}$$

1ST: x
2ND: y

$$\frac{\partial}{\partial y}\left[\frac{\partial^2 f}{\partial y \partial x}\right] = \frac{\partial^3 f}{\partial y \partial y \partial x} = \frac{\partial^3 f}{\partial y^2 \partial x}$$

① x
② y
③ y

Clairaut's Theorem (Equality of Mixed Partials)

Assume that f is defined on an open set of $\mathbb{R}^2$ and that f_{xy} and f_{yx} are continuous throught that set. Then $f_{xy} = f_{yx}$

ex) Is it possible that a function $f(x,y)$ exists:
- defined on all reals
- $f_x = 3x$, $f_y = 3y$?

← continuous functions

Clairaut's:
$f_{xy} = f_{yx}$ must be true

Possible!

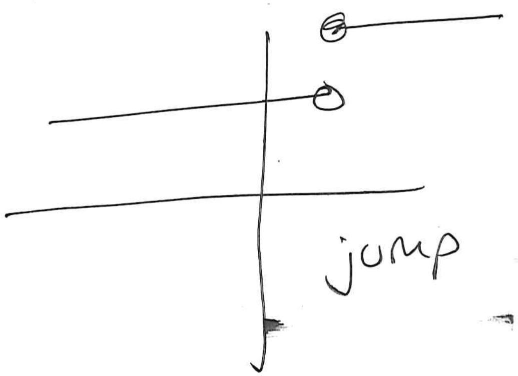
Check: $f_{xy} = 0$ $f_{yx} = 0$

$$f(x,y) = \frac{3}{2}x^2 + \frac{3}{2}y^2$$

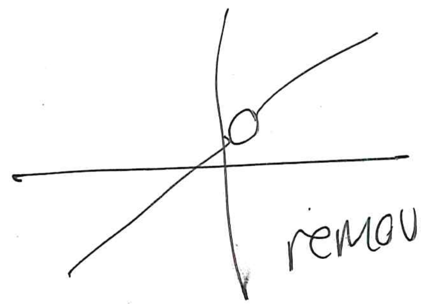
It does exist — we found it (by inspection)

A function f is continuous at a point a if

$$\lim_{x \to a} f(x) = f(a)$$



jump



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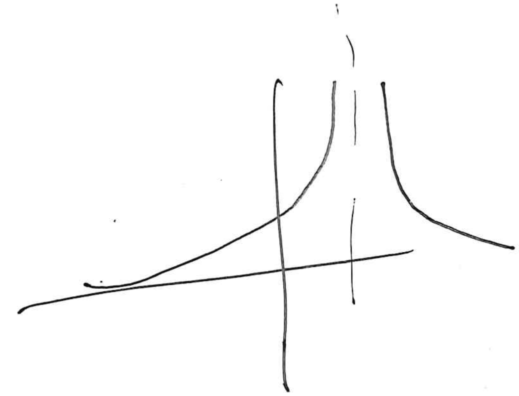


infinite

IS it possible $f(x,y)$ defined everywhere,

$f_x = 3x \qquad f_y = 3x$?

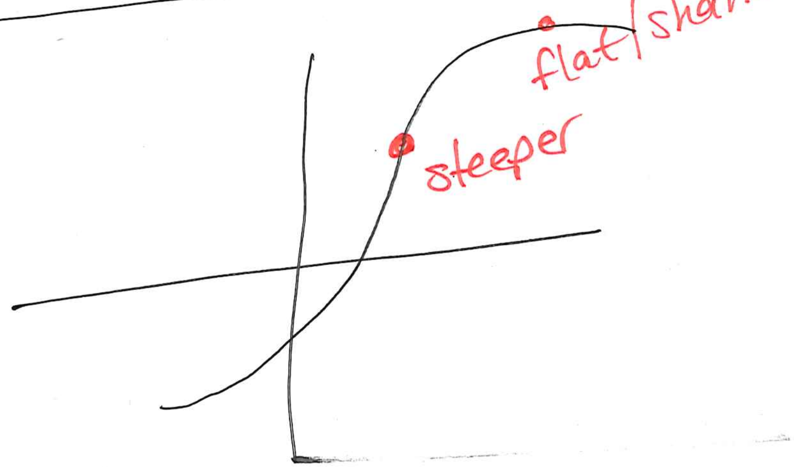
Clairaut: $f_{xy} = f_{yx}$

$f_{xy} = 0 \qquad f_{yx} = 3$

NO – not possible

Ch 12.8

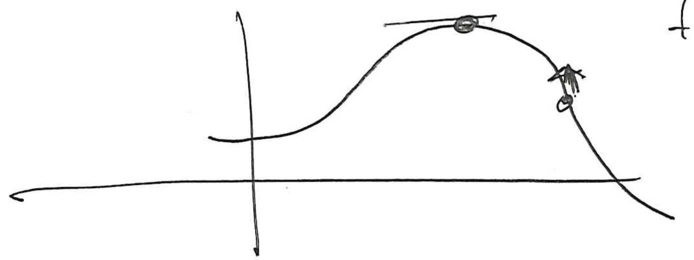
Recall $\mathbb{R}^2$:



deriv - slope

In $\mathbb{R}^3$: direction matters

In $\mathbb{R}^2$:



In $\mathbb{R}^3$: local maxima and minima occur at critical pts, ie

- $f_x = 0$ AND $f_y = 0$

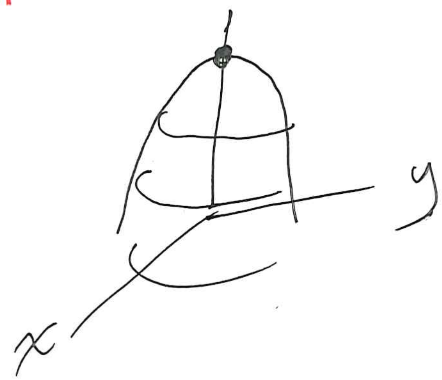
OR

- one (or both) partials DNE

ex $\quad z = 16 - 4x^2 - y^2$

$z_x = -8x = 0$ $\quad x = 0$ AND $y = 0$

$z_y = -2\cancel{x}y = 0$ $\quad (0,0)$ CP



ex $\quad f(x,y) = x^2 + xy + y^2 + 6x + 9$

$f_x = 2x + y + 6 = 0$

$f_y = x + 2y = 0 \longrightarrow \boxed{x = -2y}$

$2(-2y) + y + 6 = 0$

$-4y + y + 6 = 0$

$-3y + 6 = 0$

$\boxed{y = 2}$

$x = -4$

$\boxed{CP: (-4, 2)}$

Check: $f_x = 2(-4) + 2 + 6 = 0$ ✓

$f_y = -4 + 2(2) = 0$ ✓

ex $f(x,y) = x^2 + y^2 + xy + 2x + y$

$\boxed{CP: (-1, 0)}$

$f_x = 2x + y + 2 = 0$

$f_y = 2y + x + 1 = 0$

$x = -2y - 1$

$\boxed{x = -1}$

$0 = 2(-2y-1) + y + 2$

$0 = -4y - \cancel{2} + y + \cancel{2}$

$0 = -3y$

$\boxed{y = 0}$

ex $f(x,y) = x^2 + y^3 + xy + 5x$

$f_x = 2x + y + 5 = 0$

$f_y = 3y^2 + x = 0$

$\cancel{x = -3y^2}$

$0 = 2(-3y^2) + y + 5$

$0 = -6y^2 + y + 5$

$\cancel{= (y-1)(-6y+5)}$

lol oops

$\cancel{= (y+1)(-6y+5)}$

$\cancel{= (y+1)(-6y+5)}$

$\boxed{-(y+1)(6y+5)}$

$y = -1, \quad y = -5/6$

$(-3, -1)$

$\left(-3\left(\frac{25}{36}\right), -5/6\right)$

If $f_x(x,y) = f_y(x,y) = 0$ ← <u>CP</u>

AND there is some point close by
with $f(a,b) > f(x,y)$ ← not @ max
and $f(c,d) < f(x,y)$ ← not @ min

then (x,y) is a <u>saddle point</u>.

CP, <u>not</u> max/min