Ch 12.2 : Graphs and Level Curves

Idea: in $\mathbb{R}^3$, $z = f(x,y)$

domain + range are similar to $\mathbb{R}^2$, $y = f(x)$

(ex) $f(x,y) = e^{x^2+y^2}$

Domain: x, y can be any real #s

Range: $x^2+y^2 \geq 0$

so: $e^{x^2+y^2} \geq e^0 = 1$

Range: $[1, \infty)$

$f(0,0) = e^{0^2+0^2} = e^0 = 1$

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ex $f(x,y) = \sin\left(\frac{x}{\sqrt{y}}\right)$

Domain: Cant $\sqrt{neg}$: so $y \geq 0$

can't $\frac{x}{0}$: so $y \neq 0$

Domain: $y > 0$, x any real #

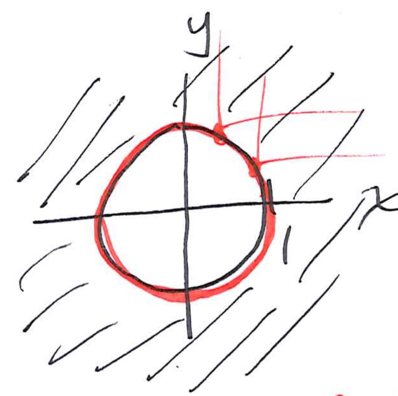
Range: $[-1, 1]$

ex $f(x,y) = \sqrt{x^2+y^2-1}$

Domain: $\boxed{x^2+y^2 \geq 1}$

Need: $x^2+y^2-1 \geq 0$

$\boxed{x^2+y^2 \geq 1}$



Unit circle: $x^2+y^2=1$

Every pair on or outside the unit circle

Range: $[0, \infty)$

Level curves: set z = constant, c

$c = \sqrt{x^2+y^2-1}$

$c^2 = x^2+y^2-1$

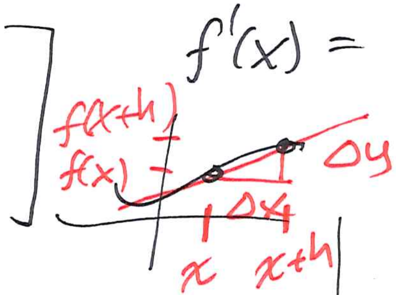
$\Rightarrow$ $x^2+y^2 = \boxed{1+c^2}$ const

$\boxed{x^2+y^2=d}$

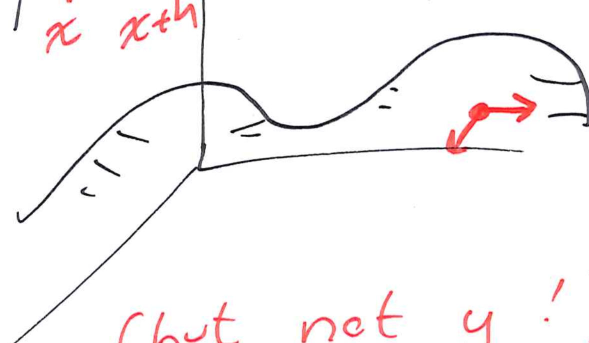
Ch 12.4 : Partial Derivatives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In $\mathbb{R}^2$:



$\frac{\Delta y}{\Delta x}$



In $\mathbb{R}^3$, direction matters

① If I move in x direction (but not y !)

Slope: $\frac{\Delta z}{\Delta x} = \frac{f(x+h, y) - f(x, y)}{h} = f_x(x, y) = \frac{\partial f}{\partial x}$

funny "d"

Partial derivative of f with respect to x

Insight: In this calculation, y is a constant

(ex) $f(x, y) = 2y^2 + 2x + xy$

$f_x(x, y) = 0 + 2 + y = 2 + y$

treat x as variable
y as constant

$\frac{d}{dx}(2x) = 2$

$\frac{d}{dx}(100x) = 100$

$\frac{d}{dx}(ax) = a$

$$f(x,y) = 2y^2 + 2x + (x)y$$

$$\frac{\partial f}{\partial y} = f_y(x,y) = 4y + 0 + x = 4y + x$$

treat y as variable, x as constant

Partial deriv of $f(x,y)$ with respect to y

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ex) $f(x,y) = 3xy^2 - 15x + \ln y$

$3xy^2 = \underbrace{(3y^2)}_{\text{const}}x$

$f_x(x,y) = 3y^2 - 15 + 0$

$f_y(x,y) = (3x)2y - 0 + \frac{1}{y} = 6xy + \frac{1}{y}$

ex) $f(x,y) = (2x)y^2 - (5x+3y)^3$

$f_x(x,y) = \frac{\partial f}{\partial x} = 2y^2 - 3(5x+3y)^2 \cdot (5+0) = 2y^2 - 15(5x+3y)^2$

$f_y(x,y) = 2(2x)y - 3\underbrace{(5x+3y)}(0+3)^2$

$= 4xy - 9(5x+3y)^2$

ex

$$\underbrace{x\cos z} + \underbrace{z^1\cos y} = 0 \qquad z = f(x,y)$$

Implicit differentiation

z: function of x / y

const

$y = x\cos(z)$

$y' = \cos z$

$\frac{\partial z}{\partial x}$: x - variable
y - const
z - fcn of x

$$\underbrace{x\left(-\sin z \cdot \frac{\partial z}{\partial x}\right) + \cos z(1)}_{\text{product}} + \underbrace{(\cos y)\frac{\partial z}{\partial x}} = 0$$

$x = y \cdot 13$

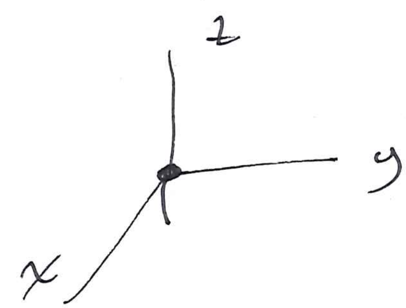
$1 = \frac{dy}{dx} \cdot 13$

$$\frac{\partial z}{\partial x}(-x\sin z + \cos y) = -\cos z$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-\cos z}{-x\sin z + \cos y}}$$

at point

$(0,0,0)$ $\left.\frac{\partial z}{\partial x}\right|_{x,y,z=0} =$



$$\frac{-\cos 0}{0 + \cos 0} = \frac{-1}{1} = -1$$

If I walk from (0,0,0)
in +x direction:
<u>downhill</u> b/c $\frac{\partial z}{\partial x} < 0$