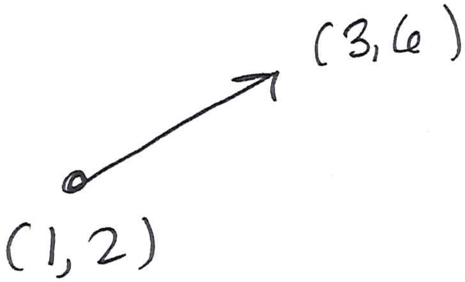
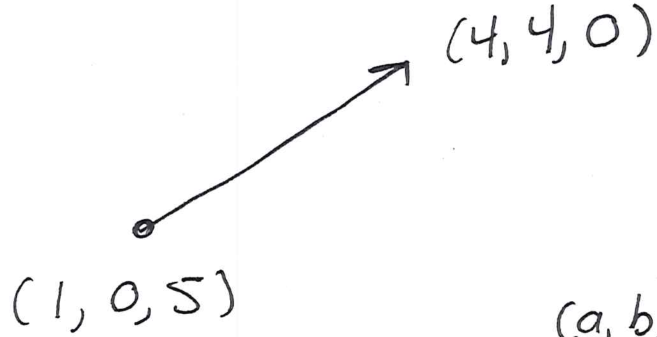
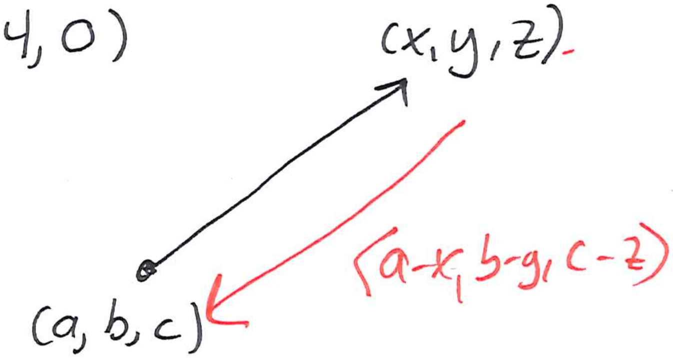
Warmup : Name the Vectors



$\boxed{\langle 2, 4 \rangle}$

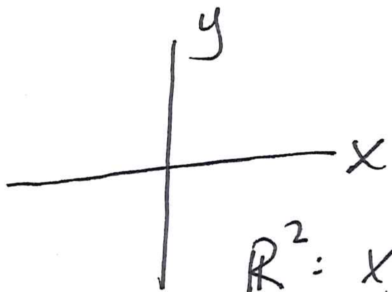


$\boxed{\langle 3, 4, -5 \rangle}$

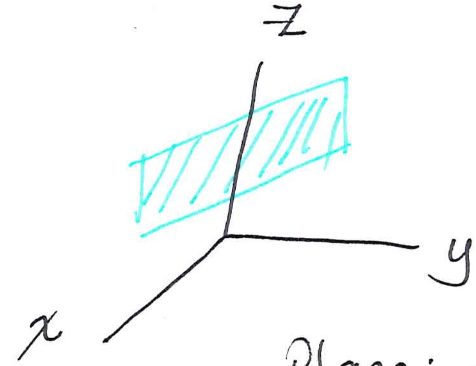


$\langle x-a, y-b, z-c \rangle$

Ch 12.1 : Planes & Surfaces



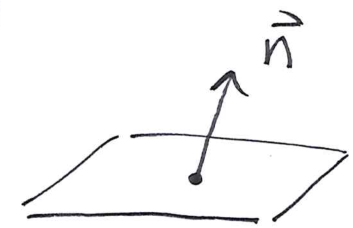
$\mathbb{R}^2$: xy-plane



$\mathbb{R}^3$

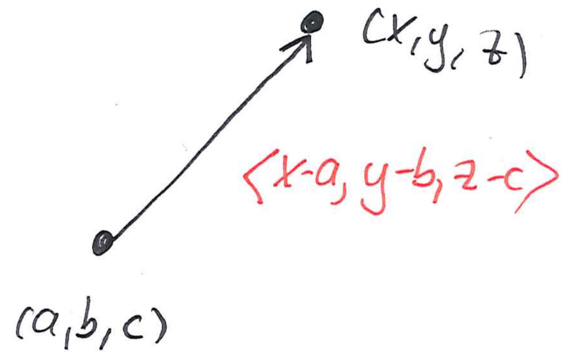
Plane:
- tilt
- position

Describe "tilt" of a plane (in $\mathbb{R}^3$) using a <u>normal vector</u>



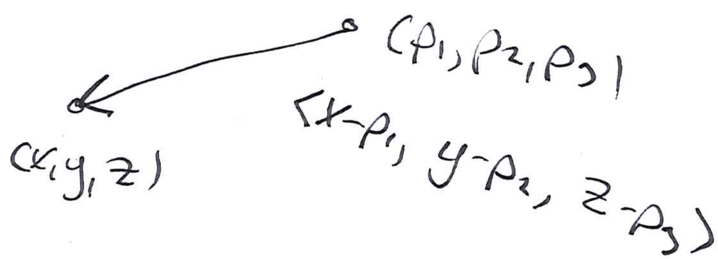
Then, if we know any single point on the plane, we know the whole plane

Given any two points on a plane, the vector between them is orthogonal (perpendicular) to a normal vector of the plane.



We want an <u>equation</u> for a plane with normal vector $\vec{n} = \langle n_1, n_2, n_3 \rangle$, including the point $P_0 = \langle p_1, p_2, p_3 \rangle$

If a point (x, y, z) is on the plane:



must be $\perp$ to $\vec{n}$

So: $\underbrace{\langle x-p_1, y-p_2, z-p_3\rangle}_{\text{vector in plane}} \cdot \underbrace{\langle n_1, n_2, n_3\rangle}_{\text{normal}} = 0$

$\Rightarrow \quad n_1(x-p_1) + n_2(y-p_2) + n_3(z-p_3) = 0$

$n_1 x + n_2 y + n_3 z = \underbrace{n_1 p_1 + n_2 p_2 + n_3 p_3}_{\text{constant}}$

$\boxed{\text{eg}}$ Plane P has normal vector $\langle 12, 0, -9\rangle$ and passes through $(\frac{1}{2}, 3, \frac{1}{3})$.

Its equation is:

$12x + 0y - 9z = 12(\frac{1}{2}) + 0(3) + (-9)(\frac{1}{3})$

$\boxed{12x - 9z = 3}$

ex $(0,0,0)$: $12(0) - 9(0) \neq 3$ not in plane
$x \; y \; z$

$(1, 10, 1)$: $12 - 9 = 3$ is in plane

ex) Give an equation for the plane in $\mathbb{R}^3$ with normal vector $\langle 3, 1, -2 \rangle$ passing through the point $(-5, 10, 15)$.

If $(\overset{x}{0}, \overset{y}{0}, \overset{z}{z})$ is in plane, what is z?

Equation:

$$3x + y - 2z = -15 + 10 - 30$$

$$\boxed{3x + y - 2z = -35}$$

$$0 + 0 - 2z = -35$$

$$\boxed{z = 35/2}$$

$(0, 0, 35/2)$ in plane

Parallel & Orthogonal Planes
(perpendicular)

Two planes are <u>parallel</u> if their normal vectors are <u>parallel</u>

Two planes are <u>orthogonal</u> if their normal vectors are <u>orthogonal</u> (dot prod is 0) ex P: plane with equation $\underline{2x - 5y + z = 3}$ $\vec{n} = \langle 2, -5, 1 \rangle$

(a) Write an equation for the plane parallel to P, passing through (1, 2, 3)

(b) If $\langle 1, 1, c \rangle \perp \langle 2, -5, 1 \rangle$, what is c ?

(c) Write an equation for a plane perpendicular to P, passing through (1, 2, 3).

(a) To define a plane, we need a normal vector and a point.

Point (given) (1, 2, 3)

normal vector: any vector parallel to $\langle 2, -5, 1 \rangle$

So: $2x - 5y + z = 2 - 10 + 3$

$$\boxed{2x - 5y + z = -5}$$

(b) $\langle 1, 1, c\rangle \perp \langle 2, -5, 1\rangle$

$\Rightarrow \quad \langle 1, 1, c\rangle \cdot \langle 2, -5, 1\rangle = 0$

$\Rightarrow \quad 2 - 5 + c = 0$

$\Rightarrow \quad \boxed{c = 3}$ so: $\langle 1, 1, 3\rangle \perp \langle 2, -5, 1\rangle$

(c) Need: point $\leftarrow$ given $(1, 2, 3)$

normal vector $\leftarrow$ any vector $\perp$ to $\langle 2, -5, 1\rangle$

One option: $\langle 1, 1, 3\rangle$

Then: $x + y + 3z = 1 + 2 + 9$

$\boxed{x + y + 3z = 12}$

Drawing surfaces in $\mathbb{R}^3$ using traces

Review: ($\mathbb{R}^2$)

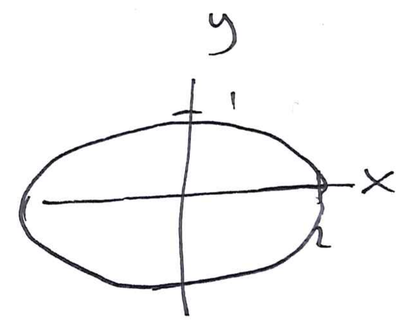
$x^2+y^2=4$: circle

$x^2+4y^2=4$: ellipse

sketch using intercepts

$(0, \pm 1)$

$(\pm 2, 0)$



ex $z = 16 - 4x^2 - y^2$

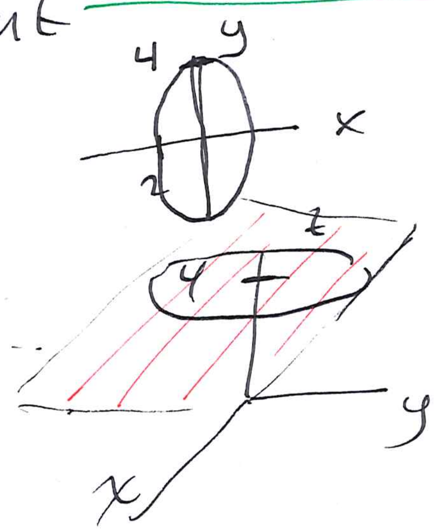
Take horizontal slices: set z = constant

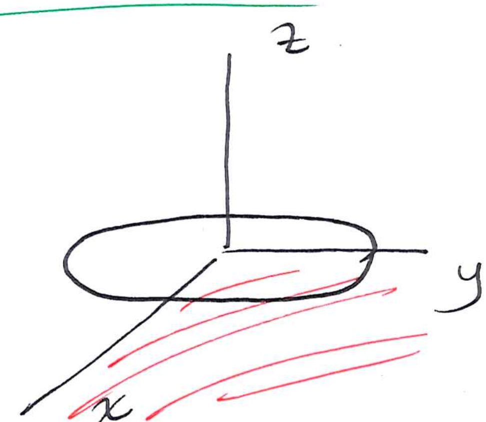
$\boxed{\text{If } z=0}$ $0 = 16 - 4x^2 - y^2$

$4x^2+y^2=16$: ellipse

$\boxed{\text{If } z=4}$ $4 = 16-4x^2-y^2$

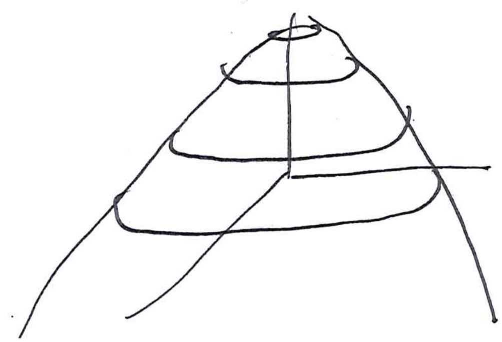
$4x^2+y^2=12$: ellipse





$\boxed{4x^2+y^2=16-z}$ Cross-sections: ellipses

as $z \uparrow$, smaller



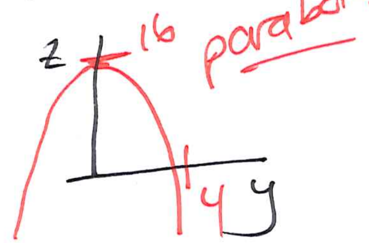
possibilities:

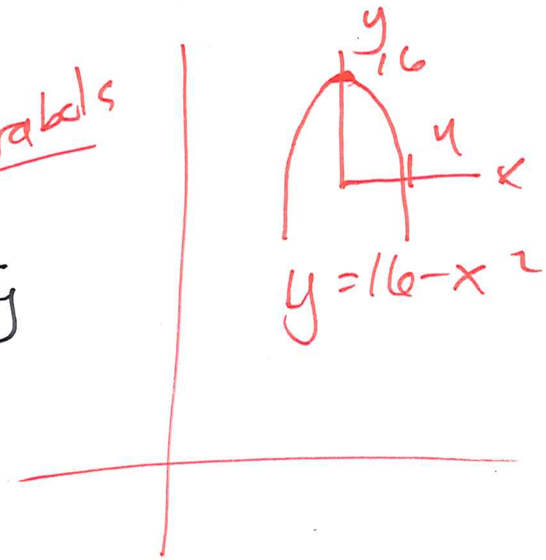
To get more info on shape: vertical slices

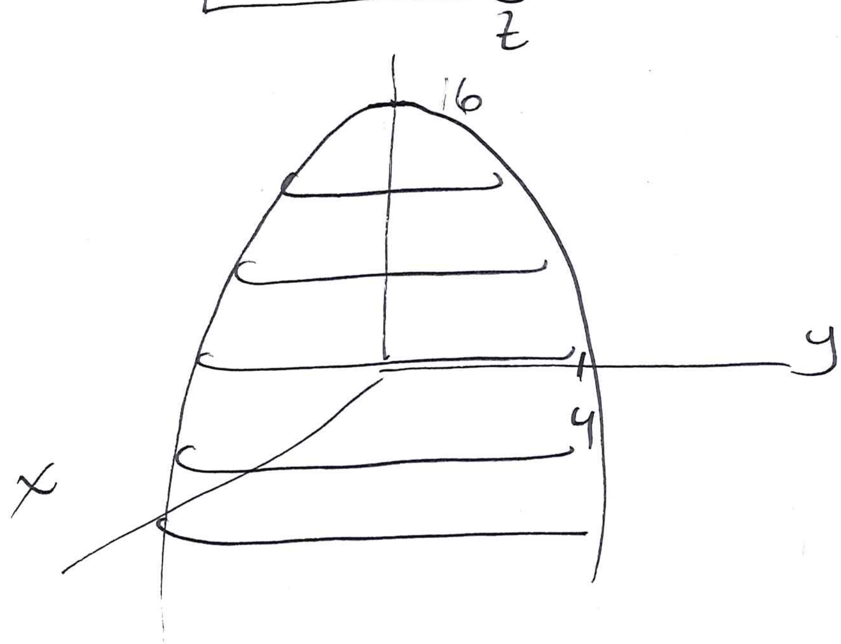
Say: set $x = \text{constant}$ $(x=0)$

$z = 16 - 4(0)^2 - y^2$

$\boxed{z = 16 - y^2}$



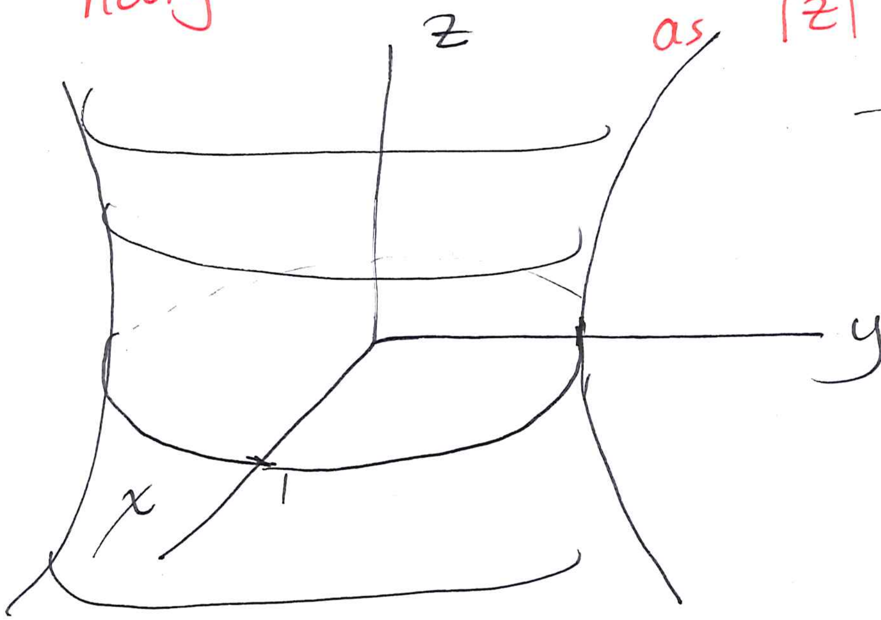




(ex) $x^2+y^2-z^2=1$

- Start with level curves
  (horizontal cross-section, setting $z=$ const)
  $x^2+y^2=(1+z^2)$ $\boxed{\text{circle}}$
- How do they change as z ↑ ↓
  symmetric in z
- Sketch As $(1+z^2)$ increases, circles grow as $|z|$ increases

hourglass



circle:

$x^2+y^2=$ constant

If $z=$constant, our eqn is

$x^2+y^2=(1+z^2)$ constant

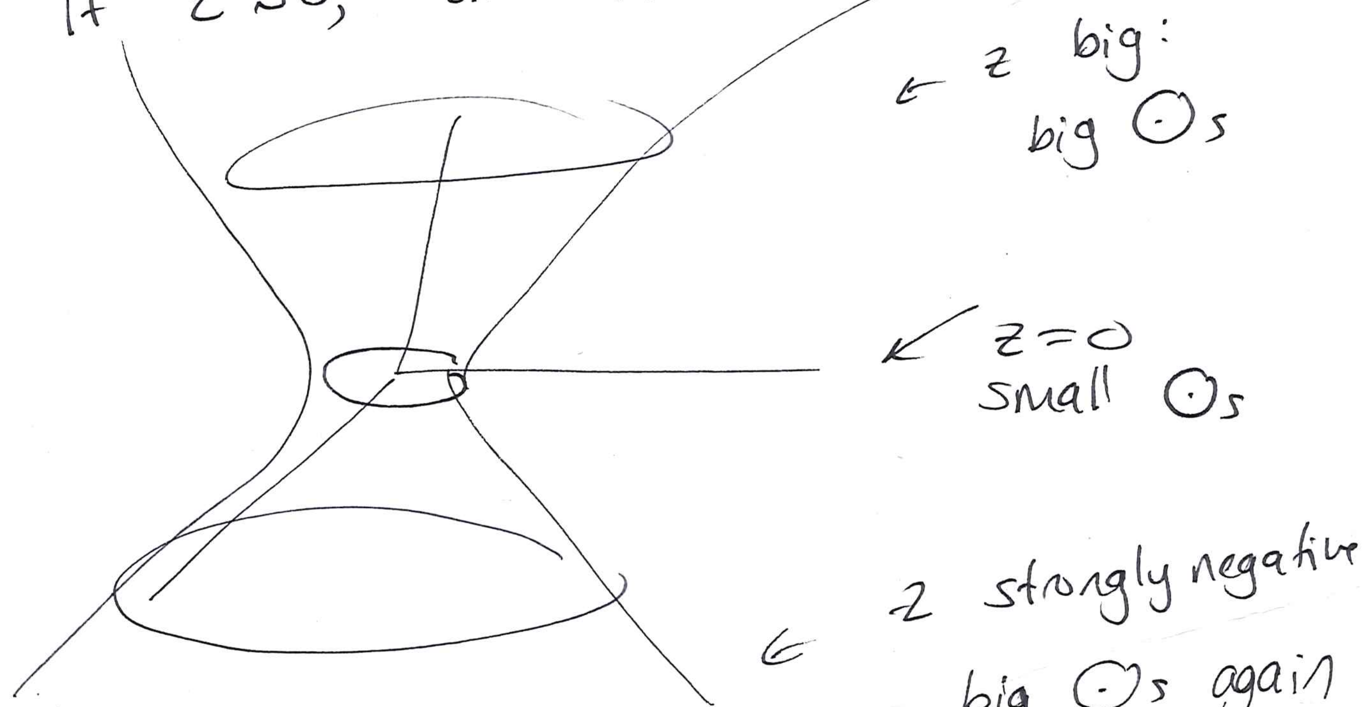
Radius of circular horiz cross sections:

$\sqrt{1+z^2}$

If z huge, then $1+z^2$ huge

If z hugely negative, then $1+z^2$ still huge (positive)

If $z \approx 0$, then $1+z^2$ small



You should recognize:

$x^2+y^2=c$: circle

$ax^2+by^2=c$ $(a\neq b$, positive$)$: ellipse

$ax^2+by=c$: parabola

$ax+by=c$: line

Practice: p. 869 of textbook
(skip last one)

Ch 12.2: Graphs & Level Curves

ex $f(x,y)=e^{x^2+y^2}$

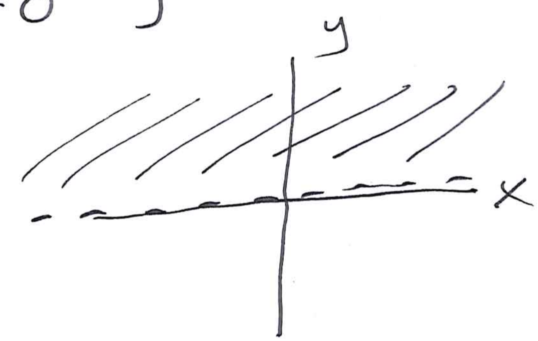
Domain: x,y any real numbers $(\mathbb{R})$

Range: Always $e^t>0$
<u>But</u> $x^2+y^2\geq 0$ So: $e^{x^2+y^2}\geq e^0=1$
Range: $[1,\infty)$

ex $f(x,y) = \sin\left(\frac{x}{\sqrt{y}}\right)$

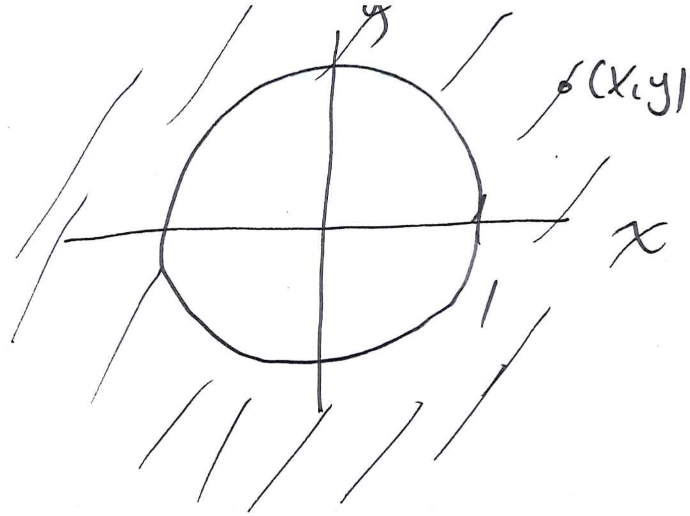
Domain: $\sqrt{y}$, so $y \geq 0$
dividing by $\sqrt{y}$, so $y \neq 0$] $\boxed{y > 0}$

$\boxed{x : \text{any real } \#}$



Range: $[-1, 1]$

ex $f(x,y) = \sqrt{x^2+y^2-1}$



Domain: $x^2+y^2 \geq 1$

(unit circle + everything outside)

Range: $[0, \infty)$

Level curves: $c = \sqrt{x^2+y^2-1}$

$c^2 = x^2+y^2-1$

$$\underbrace{(c^2-1)}_{\text{const}} = x^2+y^2$$