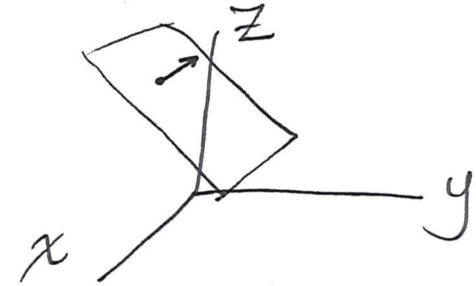
Last Time:



- In $\mathbb{R}^3$, we describe a plane using
  (1) any point on that plane, and
  (2) a normal vector to the plane
- Every line on the plane is perpendicular to its normal vector.
- Given a normal vector $\langle n_1, n_2, n_3 \rangle$ and a point (p_1, p_2, p_3), the equation of the corresponding plane is

$$n_1 x + n_2 y + n_3 z = \underbrace{p_1 n_1 + p_2 n_2 + p_3 n_3}_{\text{constant}}$$

If planes P and Q are ________,
what is the relationship of their normal vectors?

$$\text{Two planes are } \begin{pmatrix} \text{parallel} \\ \text{orthogonal} \end{pmatrix} \iff \text{their normal vectors are } \begin{pmatrix} \text{parallel} \\ \text{orthogonal} \end{pmatrix}$$

eg Plane P: $2x - 5y + z = 3$ ← normal vector

Last time: one normal vector to P is $\langle 2, -5, 1 \rangle$
(also: $\langle 4, -10, 2 \rangle$ or $\langle -2, 5, -1 \rangle$, etc)

write the equation of a plane parallel to P, passing through $(0, 0, 0)$. ← point

$$\boxed{2x - 5y + z = 0}$$

Q: For which value of c is $\langle 1, 1, c\rangle$ orthogonal (perpendicular) to $\langle 2, -5, 1\rangle$?

$\langle 1, 1, c\rangle \cdot \langle 2, -5, 1\rangle = 0$ ie $\langle 1, 1, c\rangle \perp \langle 2, -5, 1\rangle$

$2 - 5 + c = 0$

$\boxed{c = 3}$

Q: Give the equation of a plane orthogonal to P (P: $2x - 5y + z = 3$) passing through $\boxed{(1, 2, 3)}$

(Normal vector) $\perp$ $\langle 2, -5, 1\rangle$

Can use $\boxed{\langle 1, 1, 3\rangle}$ as normal vector ~~of new plane~~!

$x + y + 3z = 1 + 2 + 9$

set $x = 1$, $y = 2$, $z = 3$, plug into $x + y + 3z$

$\boxed{x + y + 3z = 12}$ ← orthogonal plane to P

Q: Find an equation for the plane parallel to

$$2x - 5y + 7z = 0$$ ← normal vector: $\langle 2, -5, 7 \rangle$

passing through the point $(0, 1, 0)$.

Plane: • normal vector : find
• point : given $(0,1,0)$

Parallel planes have parallel normal vectors

Eqn: $2x - 5y + 7z = 0 - 5 + 0 = -5$

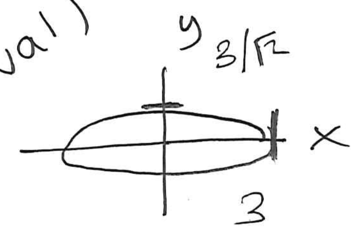
$$2x - 5y + 7z = -5$$

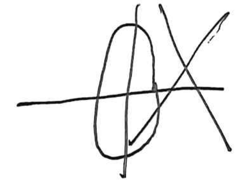
Drawing surfaces in $\mathbb{R}^3$ using traces

Review $\mathbb{R}^2$:

$x^2+y^2=4$ circle

$\boxed{x^2+2y^2=9 \quad \text{ellipse}}$ (oval)





$x=0, \quad 2y^2=9$
$y=\pm 3/\sqrt{2}$

$y=0, \quad x^2=9$
$x=\pm 3$

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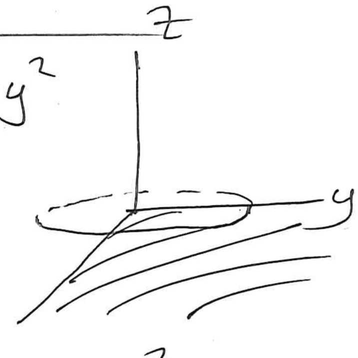
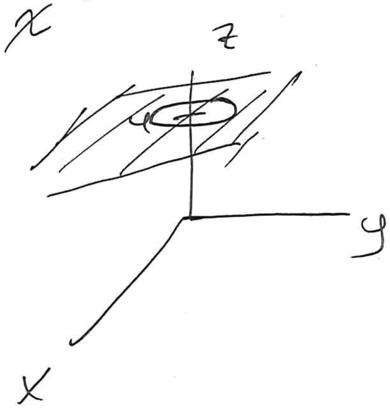
ex $\quad z = 16-4x^2-y^2$

What if $z=0$? $\quad 0=16-4x^2-y^2$
$\underline{16 = 4x^2+y^2}$
ellipse



Idea: consider horizontal slices

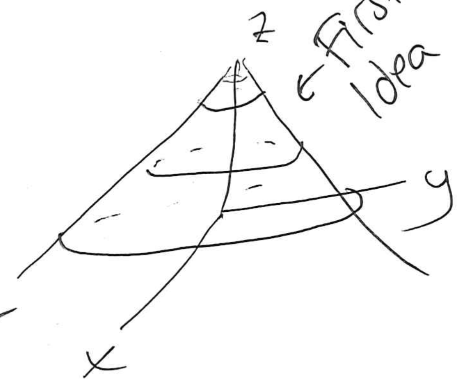
what if $z=4$? $\quad 4=16-4x^2-y^2$
$\underline{4x^2+y^2=12}$
ellipse



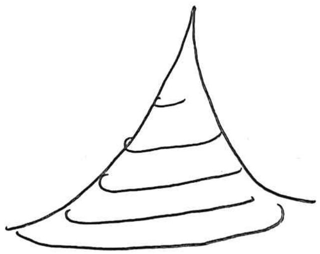
If z is a <u>constant</u>:

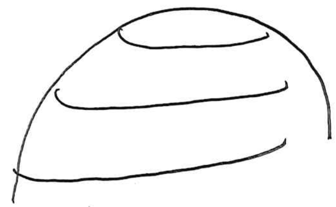
$$\underbrace{4x^2+y^2=\underbrace{(16-z)}_{\text{const}}}_{\text{ellipse}}$$

gets smaller as z gets bigger

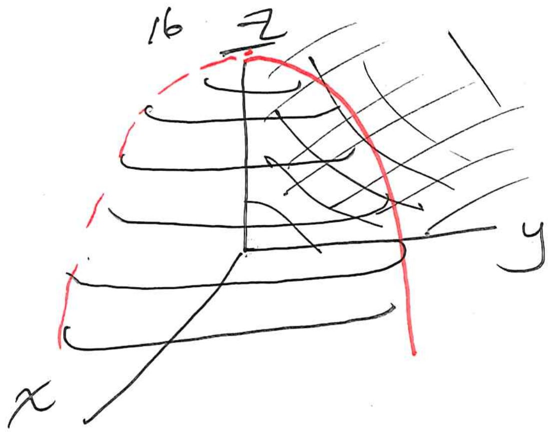


How do the ellipses stack?





What's the vertical cross-section?



Set $x=0$

$$z = 16 - 4x^2 - y^2$$

$$z = 16 - 4 \cdot 0^2 - y^2$$

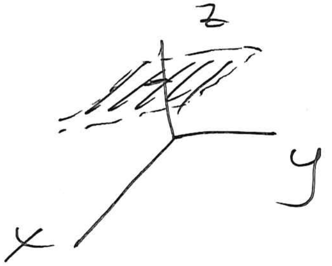
$$\boxed{z = 16 - y^2}$$

parabola pointing down

max $z=16$

← Think: $y = 16 - x^2$

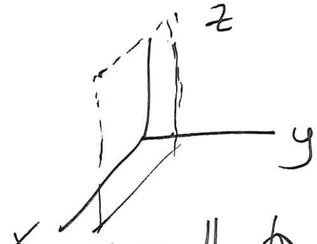
Def: A <u>level curve</u> of a surface is the intersection of that surface with some plane parallel to the xy-plane (horizontal cross-sections, $z = \text{const}$)



Def: A <u>trace</u> of a surface is the intersection of that surface with a plane parallel to one of the coordinate planes.

(A <u>level curve</u> is a special kind of trace)

Set $y = \text{const}$:



Plane $\parallel$ to xz-plane

Set $x = \text{const}$:

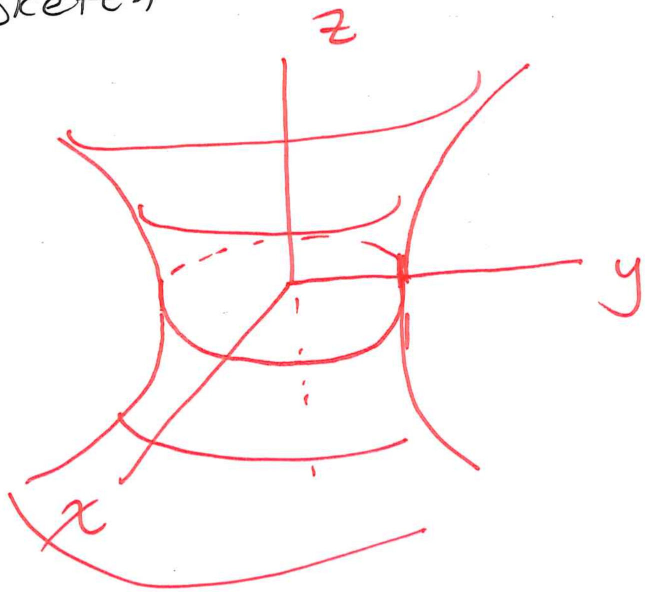
plane $\parallel$ to yz-plane

ex Sketch

$$x^2+y^2-z^2=1 \to x^2+y^2=\underbrace{1+z^2}_{\text{const}} \leftarrow \text{radius}^2$$

Using level curves.
(set z = constant, horizontal slices)

- If z constant, what shape? circle
- where is big/small? circles grow as $|z|$ grows ($1+z^2$)
- Sketch



~~p 896~~ Oops--should be 869

practice (ignore last)