Announcements:

- You should be able to log into webwork now (but no assignments were up)

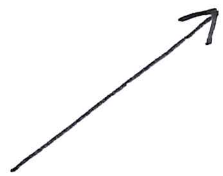
- We set up a Piazza page

  Access code: rosegarden

  (link on our section's webpage)

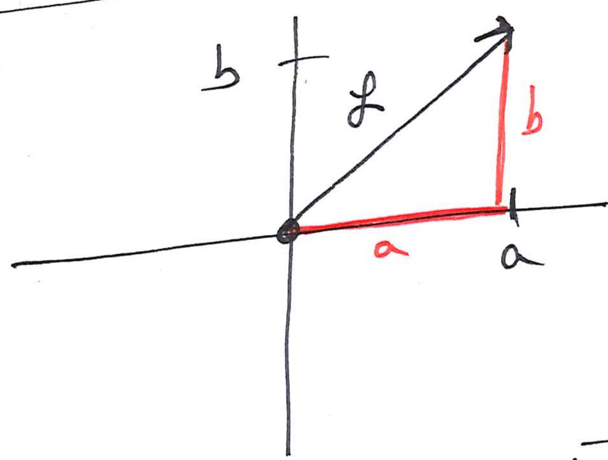
  piazza.com/ubc.ca/winterterm22017/105

Vectors: have direction & length (magnitude, norm)



$\langle 1, 2, 17 \rangle$ or whatever

$\mathbb{R}^2$



$\langle a, b \rangle$

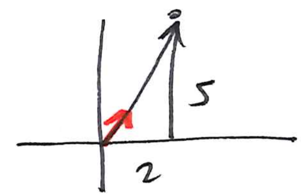
Length of $\langle a, b \rangle$:

$$|\langle a, b \rangle| = \|\langle a, b \rangle\| = \boxed{\sqrt{a^2+b^2}}$$

$\mathbb{R}^3$: $\underbrace{|\langle a, b, c \rangle|}_{\text{length}} = \sqrt{a^2+b^2+c^2}$

To describe the $\underline{\text{direction}}$ of a vector $\vec{v}$,

we use a $\underbrace{\text{unit vector}}_{\text{(length 1)}}$ in same direction as $\vec{v}$.



(ex) Say $\vec{v} = \langle 2, 5 \rangle$

- $|\vec{v}| = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$
- direction: $\vec{u} = \frac{1}{\sqrt{29}} \langle 2, 5 \rangle = \underbrace{\langle 2/\sqrt{29}, 5/\sqrt{29} \rangle}_{\text{direction}}$

Check: $\vec{u} = \vec{v} \underbrace{\left(\frac{1}{\sqrt{29}}\right)}_{\text{positive scalar } (\mathbb{R})}$

so: same direction

Check: $|\vec{u}| = \sqrt{\left(\frac{2}{\sqrt{29}}\right)^2 + \left(\frac{5}{\sqrt{29}}\right)^2} = \sqrt{\frac{4}{29} + \frac{25}{29}} = \sqrt{\frac{29}{29}} = 1$ ✓

ex Find a unit vector in the direction of:

$\langle 7, -5 \rangle$

Length: $\sqrt{49+25} = \sqrt{74}$

$$\vec{u} = \frac{1}{\sqrt{74}} \langle 7, -5 \rangle = \left\langle \frac{7}{\sqrt{74}}, \frac{-5}{\sqrt{74}} \right\rangle$$

$\langle a, b, c \rangle$

Length: $\sqrt{a^2+b^2+c^2}$

$$\vec{u} = \frac{1}{\sqrt{a^2+b^2+c^2}} \langle a, b, c \rangle$$

$$= \left\langle \frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}} \right\rangle$$

Dot Product

$$\langle a, b\rangle \cdot \langle x, y\rangle = ax + by$$

(vec) · (vec) = scalar

$$\langle a, b, c\rangle \cdot \langle x, y, z\rangle = ax + by + cz$$

eg. $\langle 3, 1, 5\rangle \cdot \langle 2, 8, -2\rangle = (3)(2) + (1)(8) + (5)(-2)$

$= 6 + 8 - 10 = \boxed{4}$

Why would anyone do this?

If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \cdot \vec{b} = 0$

eg. $\langle 14, 0, -3\rangle$, $\langle 0, 5, 10\rangle$ Are they $\perp$?

$0 + 0 - 30 \neq 0$ No not $\perp$

Properties of Dot Product

- $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (commutative)

- If s is a scalar,

$$\overbrace{s(\underset{\uparrow\,\text{vec}}{\vec{a}}\cdot\underset{\uparrow\,\text{vec}}{\vec{b}})}^{\text{scalar}} = \underbrace{(\underbrace{s\vec{a}}_{\text{vector}})\cdot\underset{\uparrow\,\text{vector}}{\vec{b}}}_{\text{scalar}}$$

(associative)

- $$\underbrace{\vec{a}}_{\text{vec}}\cdot\underbrace{(\vec{b}+\vec{c})}_{\text{vec}} = \underbrace{\vec{a}\cdot\vec{b}}_{\text{scalar}} + \underbrace{\vec{a}\cdot\vec{c}}_{\text{scalar}}$$

ex $\vec{a} = \langle 1, 0, 3\rangle$ $\quad \vec{b} = \langle 3, 0, -1\rangle$ $\quad \vec{c} = \langle -2, 0, -6\rangle$

$\vec{c} = (-2)\vec{a} \Rightarrow \vec{a} \parallel \vec{c}$

$\vec{a}\cdot\vec{b} = 3+0+(-3) = 0$ $\quad \vec{a}\perp\vec{b}$

$\vec{c}\perp\vec{b}$

$\vec{c}\cdot\vec{a} = -2+0-18 \neq 0$

Which are parallel?

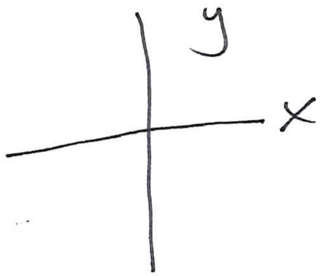
Which are perpendicular?

Compute: $(\vec{a}+\vec{b})\cdot\vec{c}$

$\langle 4, 0, 2\rangle\cdot\langle -2, 0, -6\rangle$

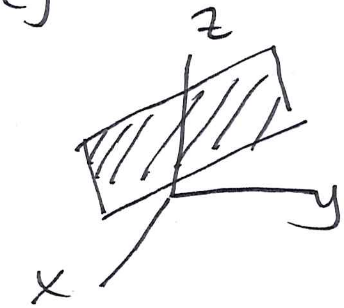
$= -8 + 0 - 12 = \boxed{-20}$

Ch 12: Planes & Surfaces



"the xy-plane"

$\mathbb{R}^2$



$\mathbb{R}^3$

We can describe any plane in $\mathbb{R}^3$ by its tilt and a point on it.

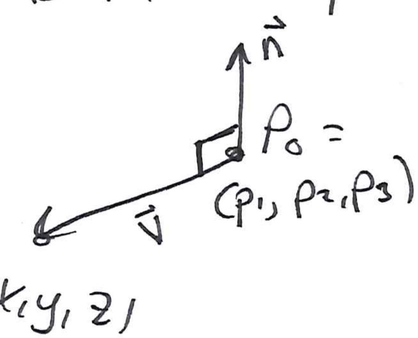
P: plane (in $\mathbb{R}^3$)

normal vector $\langle n_1, n_2, n_3 \rangle = \vec{n}$

contains the point $P_0 = (p_1, p_2, p_3)$

How do we know if a point (x, y, z) is in our plane P?

Geom:



the vector from P_0 to (x, y, z) $\perp$ to $\vec{n}$

(dot prod is 0)

$\vec{v}: \langle x - p_1, y - p_2, z - p_3 \rangle$

So: $\langle x - p_1, y - p_2, z - p_3 \rangle \cdot \langle n_1, n_2, n_3 \rangle = 0$

$$(x-p_1)n_1 + (y-p_2)n_2 + (z-p_3)n_3 = 0$$

$$\underbrace{n_1x} - n_1p_1 + \underbrace{n_2y} - n_2p_2 + \underbrace{n_3z} - n_3p_3 = 0$$

$$n_1x + n_2y + n_3z = n_1p_1 + n_2p_2 + n_3p_3$$

$$= \underbrace{\vec{n} \cdot \vec{P_0}}_{\text{constant}}$$

coords of $\vec{n}$

ex A plane passes through $(1,2,1)$ and has normal vector $\langle 5,-1,2\rangle$.

Its equation:

$$5x - y + 2z = 5 - 2 + 2$$

$$\boxed{5x - y + 2z = 5}$$

ex: Is $(0,0,0)$ in plane?

$$5(0) - (0) + 2(0) \neq 5$$

$\boxed{\text{NO}}$

Is $(1,0,0)$ in plane?

$$5(1) - 0 - 0 = 5 \quad \boxed{\text{YES}}$$