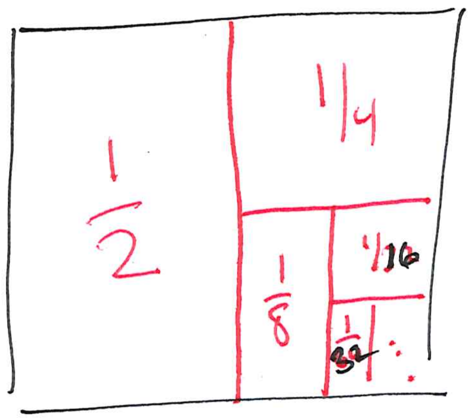
Sequences & Series (Ch 8)

(4)



Area of whole shape: 1

Areas of pieces: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

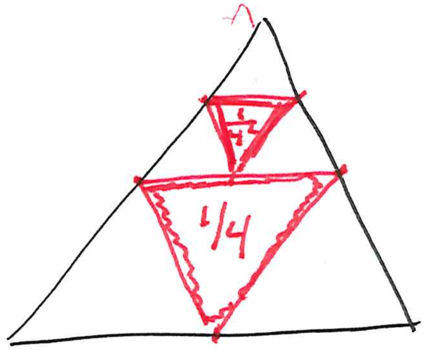
list of #s, in some order: "sequence"

Add up areas of pieces:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = 1$$

add up terms of a sequence: "series"

ex



(equilateral)

Total area: 1

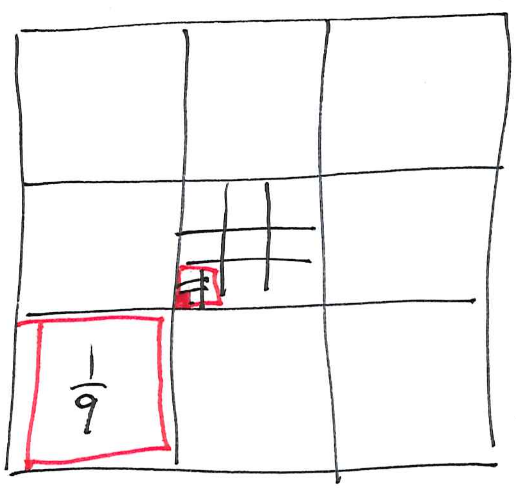
Pieces: $\frac{1}{4}, \frac{1}{4^2}, \frac{1}{4^3}, \frac{1}{4^4}, \ldots$

"sequence"

Sum of Pieces: $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots = \frac{1}{3}$

"series"

ex

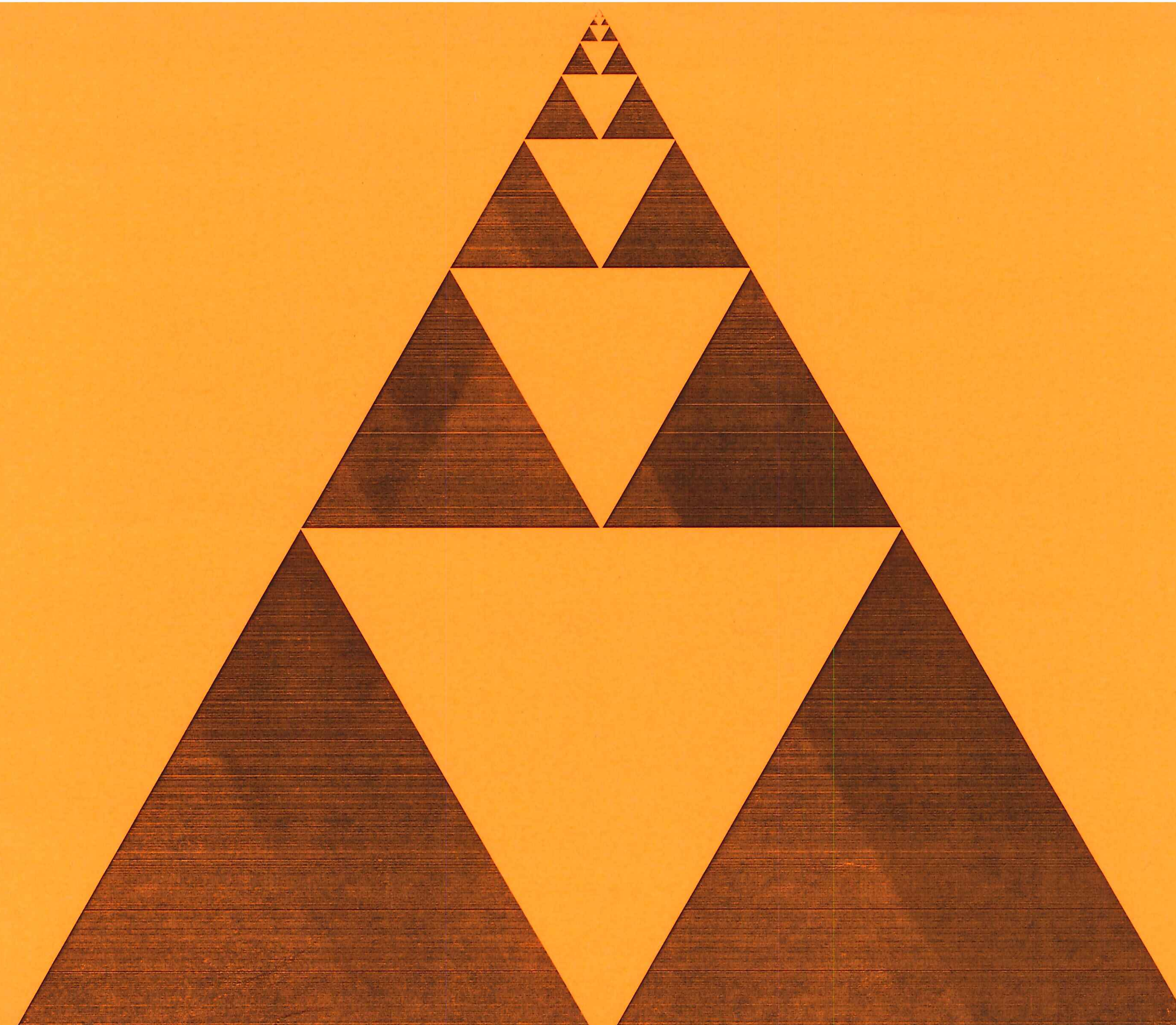


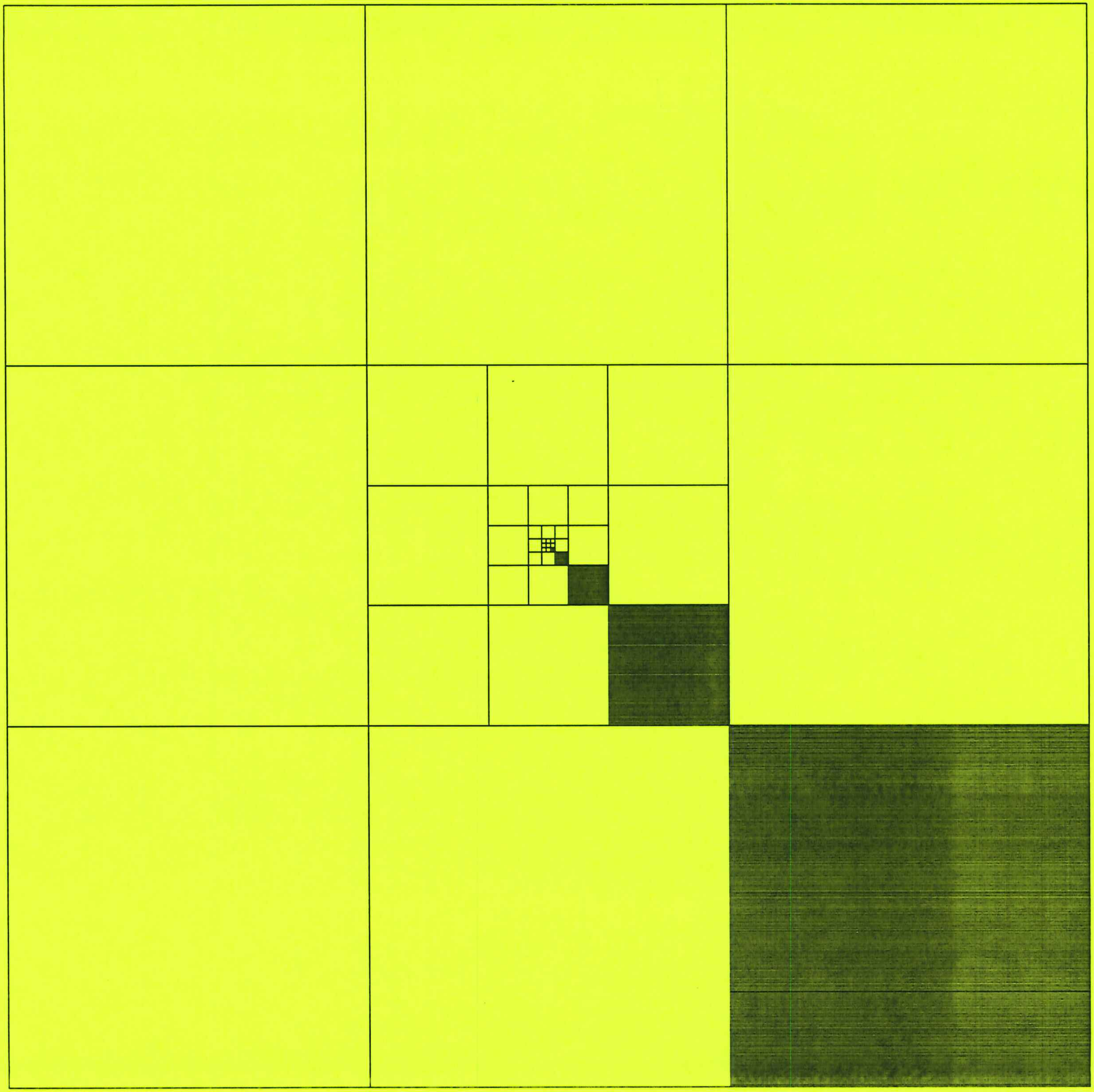
Total area: 1

Pieces: $\frac{1}{9}, \frac{1}{9^2}, \frac{1}{9^3}, \ldots$

"sequence"

Add: $\frac{1}{9} + \frac{1}{9^2} + \frac{1}{9^3} + \cdots = \frac{1}{8}$





3,689,257
4,324,810
4,833,239
5,371,315
7,206,643
8,787,949
10,374,196
11,506,655
14,009,429
18,238,247
21,568,311
22,992,604
24,343,181
25,309,331
27,296,859
28,846,761
30,007,094
31,612,897
33,476,688

1871	3,689,257	
1881	4,324,810	+17.2%
1891	4,833,239	+11.8%
1901	5,371,315	+11.1%
1911	7,206,643	+34.2%
1921	8,787,949	+21.9%
1931	10,374,196	+18.1%
1941	11,506,655	+10.9%
1951	14,009,429	+21.8%
1961	18,238,247	+30.2%
1971	21,568,311	+18.3%
1976	22,992,604	+6.6%
1981	24,343,181	+5.9%
1986	25,309,331	+4.0%
1991	27,296,859	+7.9%
1996	28,846,761	+5.7%
2001	30,007,094	+4.0%
2006	31,612,897	+5.4%
2011	33,476,688	

Ch 8.1 Sequences

A sequence is a list of #s with some order

We can express:

recursively

We need to know previous values to find next values.

or

explicitly

Given any n, I can calculate n^{th} term in sequence without calculating previous terms ex) I start with \$100 in a bank account

I add \$5 each month (no interest)

Recursively

$A(0) = 100$

If $t > 0$,

$A(t) = A(t-1) + 5$

amount this month

amount last month

Explictly

$A(t) = 100 + 5t$

Notation:

$$\{a_n\}_{n=1}^{b} = \{f(n)\}_{n=1}^{b}$$

Sequence: $a_1, a_2, a_3, \ldots, a_b$

$f(n)$: explicit function describing a_n

ex

$a_1 = 1/2$

$a_2 = 1/4$

$a_3 = 1/8$

$a_4 = 1/16$

$\vdots$

explicit: $\{a_n\}_1^{\infty} = \left\{\frac{1}{2^n}\right\}_1^{\infty}$

Lazy: we also write $\boxed{a_n = \frac{1}{2^n}}$

we understand n starts at 1

keep going

Recursive: $a_1 = 1/2$

If $n>1$, $a_n = \frac{1}{2} a_{n-1}$

$a_1 = 1/2$

$a_2 = \frac{1}{2} a_1 = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

$a_3 = \frac{1}{2} a_2 = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$

$\boxed{f(n) = \frac{1}{2^n}}$

$a_3 = \frac{1}{8}$

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ex $\quad \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{+1}{16}, \ldots$

recursive: $\boxed{\begin{array}{l} a_1 = -1/2 \\ \text{if } n > 1: \ a_n = \frac{-1}{2} a_{n-1} \end{array}}$

explicit:

$\{a_n\}_{n=1}^{\infty} = \left\{ \left(\frac{-1}{2}\right)^n \right\}_{n=1}^{\infty}$

ex explicit:

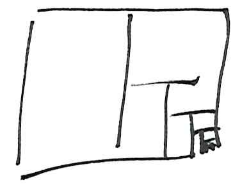
$\frac{1}{3}, \frac{1}{5}, \frac{1}{9}, \frac{1}{17}, \frac{1}{33}, \ldots$

$\{a_n\}_{n=1}^{\infty} = \left\{ \frac{1}{2^n + 1} \right\}_{n=1}^{\infty}$

ex $\quad a_n = \frac{1}{2^n}$

(Sequence: $\underset{\overset{\|}{a_1}}{\frac{1}{2}}, \underset{\overset{\|}{a_2}}{\frac{1}{2^2}}, \underset{\overset{\|}{a_3}}{\frac{1}{2^3}}, \underset{\overset{\|}{a_4}}{\frac{1}{2^4}}, \dots$)

$\lim_{n \to \infty} a_n = 0$



ex A ball falls from 1 m

each bounce: height is 80% of its last bounce.

h_n: height

$h_0 = 1$

$\underbrace{h_n = 0.8\, h_{n-1} \text{ if } n \geq 1}_{\text{recursive}}$

explicit:

$\boxed{h_n = 0.8^n}$

$\lim_{n \to \infty} h_n = 0$

$h_0 = 1$

$h_1 = 0.8$

$h_2 = (0.8)(0.8)$

$h_3 = (0.8)(0.8)(0.8)$

$h_4 = (0.8)(0.8)(0.8)(0.8)$

We can think of an infinite sequence as a function whose domain is the whole numbers

$f(0) = 1$

$f(1) = 0.8$

$f(2) = (0.8)^2$

$f(3) = (0.8)^3$

$f(1/2) \ldots ?$ no meaning to us

$f(-3) \ldots ?$ no meaning to us