

# PROBABILITY

## Extreme Basics

A probability is a number btw 0 + 1  
giving how likely something is to happen

ex: If an event has probability 0:  
even after many trials, event never happens

If an event has probability 1:  $\rightarrow 100\%$   
even after many trials, event always happens

If an event has probability  $\frac{1}{3}$ :  $\leftarrow 33\frac{1}{3}\%$

If we do lots & lots & lots of trials,  
 $\sim \frac{1}{3}$  of the time, event happens

$$\lim_{\# \text{ tries} \rightarrow \infty} \left( \frac{\# \text{ times when event happens}}{\# \text{ times I've tried}} \right) = \frac{1}{3}$$

Notation:

Event: like capital letters  
 $X$


eg.  $X = \text{"roll dice"}$

Value: like + uk lower-  
case letters  
 $x$

eg.  $x = 4$

$\Pr(X=4)$   
↑ probability that  
↑ dice roll  
is 4

or  $\Pr(X=x)$

ex: rolling dice 

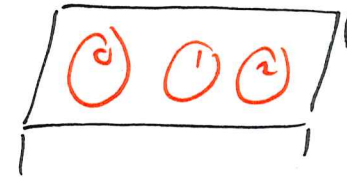
$$\Pr(X=1) = \frac{1}{6}$$

$$\Pr(X=1 \text{ or } X=2) = \frac{1}{3}$$

(ex)

Event:

$X$  person picks



Value:

$x$  0, 1, or 2

What is the probability person picks our product?

$$\Pr(X=0)$$

eg  $\Pr(X=0) = 1/3$   
then  $\Pr(X \neq 0) = 2/3$

@  $\Pr(X=x \text{ or } X \neq x) = 1$   
 $\Pr(X=x) = 1 - \Pr(X \neq x)$

In a discrete process, we can list all possible values of an event.

- Roll 3 dice, add values  
Possible outcomes: 3, 4, ..., 18

DISCRETE

- Choosing a whole # between 1 + 10  
Possible outcomes: 1, 2, ..., 10

DISCRETE

- Choosing any # between 1 + 10  
Possible outcomes:  $[1, 10]$

NOT DISCRETE

exist along a continuum



- Exact age of a person at noon today

$[0, 200]$  yrs



NOT  
DISCRETE

- Amount of oil in an oil spill

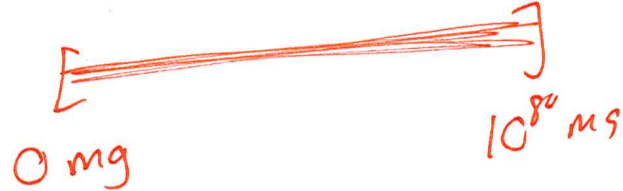
AMBIGUOUS

Some # molecules

$0, 1, 2, \dots, 10^{80}$

DISCRETE

weight or volume  
not discrete:



Realistically: we can't measure exactly

ex Choose a # from  $\left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \right\}$

Discrete Options:  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

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## Cumulative Distribution Function

ex T: temp tomorrow

Reasonable Question  
what are.

$$F(0) = \Pr(T \leq 0)$$

← Prob it's below freezing

$$F(20) = \Pr(T \leq 20)$$

← Prob I need to bring a jacket

$$F(50) = \Pr(T \leq 50)$$

← Prob I don't have to worry about dying

Want:  
Function

$$F(x) = \Pr(T \leq x)$$

$$F(x) = \Pr(T \leq x)$$

$F(x)$  is a probability, so

$$0 \leq F(x) \leq 1$$

$$F(1000) = \Pr(T \leq 1000) \approx 1$$

$$\lim_{x \rightarrow \infty} F(x) = 1$$

$$F(-1000) = \Pr(T \leq -1000) = 0$$

$$\lim_{x \rightarrow -\infty} F(x) = 0$$

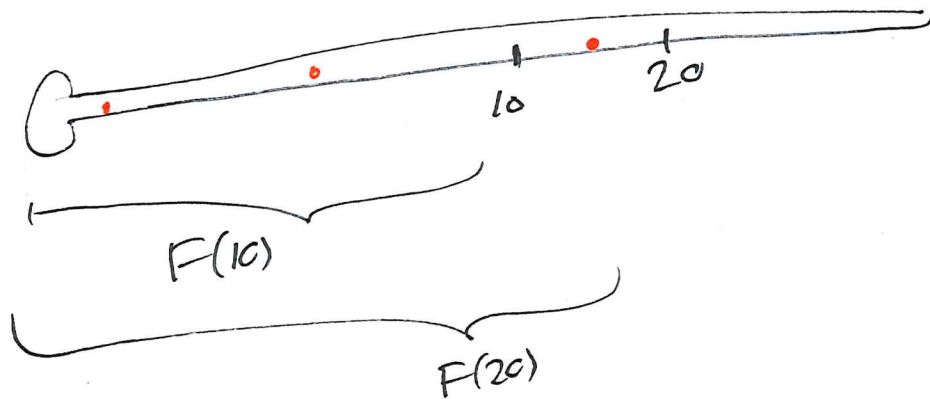
Compare  $F(10)$  &  $F(20)$

$$F(10) = \Pr(T \leq 10)$$

$$F(20) = \Pr(T \leq 20)$$

ALWAYS:

$$F(10) \leq F(20)$$



Def: Let  $X$  be a random variable (event)

define  $F(x) = \Pr(X \leq x)$

$F(x)$  is called the cumulative distribution function of  $X$ .

The cumulative distribution function (CDF) has the following properties:

- $0 \leq F(x) \leq 1$

- $\lim_{x \rightarrow \infty} F(x) = 1$

- $\lim_{x \rightarrow -\infty} F(x) = 0$

- $F(x)$  is nondecreasing

eg  $F(1) \leq F(2) \leq F(3) \leq F(3.1)$



(ex) Half of days are less than  $20^\circ$   
 $\frac{1}{3}$  of days are more than  $30^\circ$

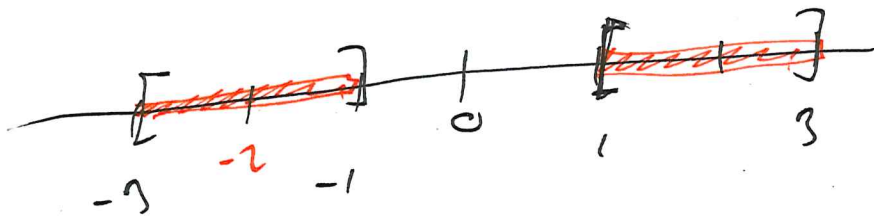
$F(x)$  is CDF  
 for the temp.

$$F(20) = P(T \leq 20) = \frac{1}{2}$$

$$F(30) = P(T \leq 30) = \frac{2}{3}$$

( $1 - \frac{1}{3}$ )

(ex)



$$[-3, -1] \cup [1, 3]$$

Choose any number  
 in intervals  
 "uniformly" (no preference)

$F(x)$ : cumulative distribution function

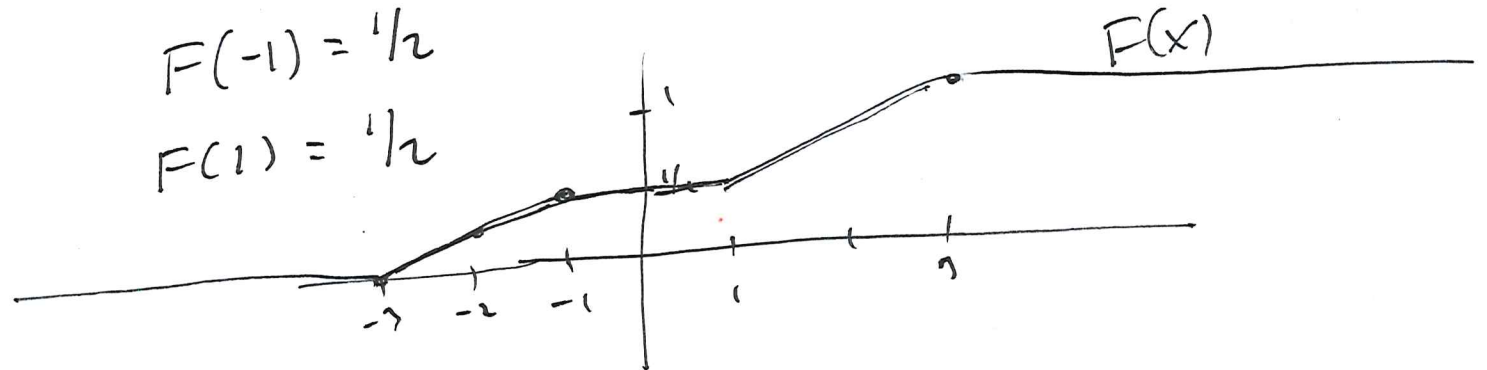
$$F(0) = \frac{1}{2}$$

$$F(3) = 1$$

$$F(-2) = \frac{1}{4}$$

$$F(-1) = \frac{1}{2}$$

$$F(1) = \frac{1}{2}$$



a Continuous random variable is a variable that has a continuous cumulative distribution function,

$$F(x) = P(X \leq x)$$