Differential Equations

Which of the following satisfies (↳ makes true)

$$\frac{dy}{dx} + x^2 - 1 = y$$

(A) $y = x^2 + 1$

$(2x) + x^2 - 1 = x^2 + 1$

$x^2 + 2x - 1 = x^2 + 1$ FALSE

not a solution

(B) $y = x^2 + 2x + 1$ ✓

$(2x+2) + x^2 - 1 = x^2 + 2x + 1$

$x^2 + 2x + 1 = x^2 + 2x + 1$ TRUE

a solution to our diff. eq.

(C) $y = -\frac{1}{3}x^3 + x$

$(-x^2 + 1) + x^2 - 1 = -\frac{1}{3}x^3 + x$

$0 = -\frac{1}{3}x^3 + x$ FALSE

not a solution

(ex) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x+2$

Which is a solution?

(A) $y = e^x$

$e^x + 2e^x + e^x = x+2$

$4e^x = x+2$

FALSE

(B) $y = x^2$

$y' = 2x$

$y'' = 2$

$2 + 2(2x) + x^2 = x+2$

$x^2 + 4x + 2 = x+2$

FALSE

✓ (C) $y = x$

$y' = 1$

$y'' = 0$

$0 + 2(1) + x = x+2$

$2 + x = x+2$

TRUE

(D) $y = x+1$

$y' = 1$

$y'' = 0$

$0 + 2\cdot(1) + (x+1) = x+2$

$x+3 = x+2$

FALSE

Separable Differential Equations

(ex) $\frac{dy}{dx} = y^2 x, \quad y(0) = 1$

long

$\frac{1}{y^2} \cdot \frac{dy}{dx} = x$

$\int \frac{1}{y^2} \frac{dy}{dx}\, dx = \int x\, dx$

$\frac{dy}{dx} = \frac{dy}{dx}$

$dy = \left(\frac{dy}{dx}\right) dx$

$\int \frac{1}{y^2}\, dy = \int x\, dx$

$\int \frac{1}{y^2} \left(y' dx\right) \rightarrow dy$

$y' = \frac{dy}{dx}$

$y' dx = dy$

short

$\frac{1}{y^2} \cdot \frac{dy}{dx} = x$

$\frac{1}{y^2}\, dy = x\, dx$

$\int \frac{1}{y^2}\, dy = \int x\, dx$

$\frac{-1}{y} = \frac{1}{2}x^2 + C$

find C

If $x=0$, $y=1$

$\frac{-1}{1} = 0 + C \rightarrow \boxed{C=-1}$

$\frac{-1}{y} = \frac{1}{2}x^2 - 1$

find y

$\frac{1}{y} = -\frac{1}{2}x^2 + 1$

$\boxed{y = \frac{1}{-\frac{1}{2}x^2 + 1}}$

(ex) $\frac{dy}{dx} = e^{x-y}$

$dy = e^{x-y}\,dx$

$dy = \frac{e^x}{e^y}\,dx$

$e^y\,dy = e^x\,dx$

$\int e^y\,dy = \int e^x\,dx$

$e^y = e^x + C$

$\boxed{y = \ln(e^x + C)}$

(ex) $\frac{dy}{dx} = y(4x^3 - 1)$, $\boxed{y(0) = -2}$

$\frac{1}{y}\,dy = (4x^3 - 1)\,dx$

$\int \frac{1}{y}\,dy = \int (4x^3 - 1)\,dx$

$\ln|y| = x^4 - x + C$

Find C:

If $x = 0$, $y = -2$

$\ln|-2| = 0^4 - 0 + C$

$\ln 2 = C$

$\ln|y| = x^4 - x + \ln 2$

$|y| = e^{x^4 - x + \ln 2}$

$\boxed{y = -e^{x^4 - x + \ln 2}}$

Find y

If $y > 0$, $|y| = y$

If $y < 0$, $|y| = -y$

(ex) $\frac{dy}{dx} \cdot \sqrt{y(9-x^2)} = -2x$ where $y > 0$ for all x

$$\frac{dy}{dx} \cdot \sqrt{y} \cdot \sqrt{9-x^2} = -2x$$

$$\int \sqrt{y}\, dy = \int \frac{-2x}{\sqrt{9-x^2}}\, dx$$

$$\int y^{1/2}\, dy = \int \frac{-2x}{\sqrt{9-x^2}}\, dx \quad du$$

$u = 9-x^2$

$du = -2x\, dx$

$$\frac{2}{3} y^{3/2} = \int \frac{1}{\sqrt{u}}\, du$$

$$\frac{2}{3} y^{3/2} = \int u^{-1/2}\, du$$

$$\frac{2}{3} y^{3/2} = 2u^{1/2} + C$$

$$\frac{2}{3} y^{3/2} = 2\sqrt{9-x^2} + C$$

$$y^{3/2} = 3\sqrt{9-x^2} + C$$

$$\boxed{y = \left[3\sqrt{9-x^2} + C\right]^{2/3}}$$

Note:

$$y = \sqrt[3]{3\sqrt{9-x^2} + C}^{\,2}$$

(ex) $\sec x \frac{dy}{dx} = y^3$

$\frac{1}{\cos x} \cdot \frac{dy}{dx} = y^3$

$\int y^{-3} dy = \int \cos x \, dx$

$-\frac{1}{2} y^{-2} = \sin x + C$

$\frac{-1}{2y^2} = \frac{\sin x + C}{1}$

$\frac{2y^2}{-1} = \frac{1}{\sin x + C}$

$2y^2 = \frac{-1}{\sin x + C}$

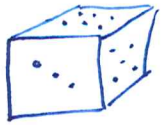
$y^2 = \frac{-1}{2\sin x + C}$

① $y = \sqrt{\frac{-1}{2\sin x + C}}$

or

② $y = -\sqrt{\frac{-1}{2\sin x + C}}$

Probability



- Probability: Number from 0 to 1
  Interpret: liklihood an event will happen

0: no matter how many tries, never happens

1: (100%) no many tries, always happens

1/3: if try lots & lots of times, event happens in ~1/3

(limit over #tries→∞ [# tries where event happened / # tries total])

Notation: (conventions)

Event: capital letter, X

(trial)

Value event might take: lower-case letter, x

X: dice roll

x: 4

* $\boxed{Pr(X=x)}$: Probability that trial X gives a value of x

X: rolling a dice

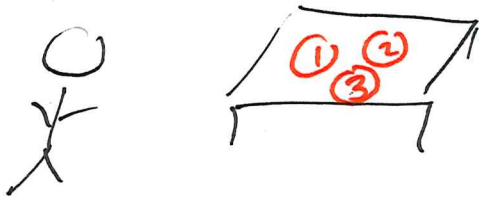
$Pr(X=6)$

Prob. that I roll a 6

$Pr(X=1)$

(ex)

X: selection of participant



① : your product
② : competitors' products
③ :

What is $Pr(X=1)$

probability that selection of participant is ①

$$Pr(X = x \text{ or } X \neq x) = 1$$

$$Pr(X = x) = 1 - Pr(X \neq x)$$

(ex) If an unfair coin flips Heads 70% of the time

$$Pr(X = H) = 0.7$$

Then: $Pr(X = T) = 0.3$

discrete: "listable"
possible outcomes of an event

- Roll 3 dice, add values

Outcomes:
3, 4, 5, ..., 18

DISCRETE

- Choose a whole # from 1 to 10

Outcomes:
1, 2, 3, ..., 10

DISCRETE

- Choose any real # from 1 to 10

Outcomes:
[1, 10]
exist along a continuum

NOT DISCRETE

CONTINUOUS

- The exact age of a person at noon today

Outcome:
[0, 200]

NOT DISCRETE

- Amount of oil spilled in an oil spill

Ambiguous →
molecules
whole # (discrete)

weight could be any #, [0, N]
N: weight of earth
(not discrete)