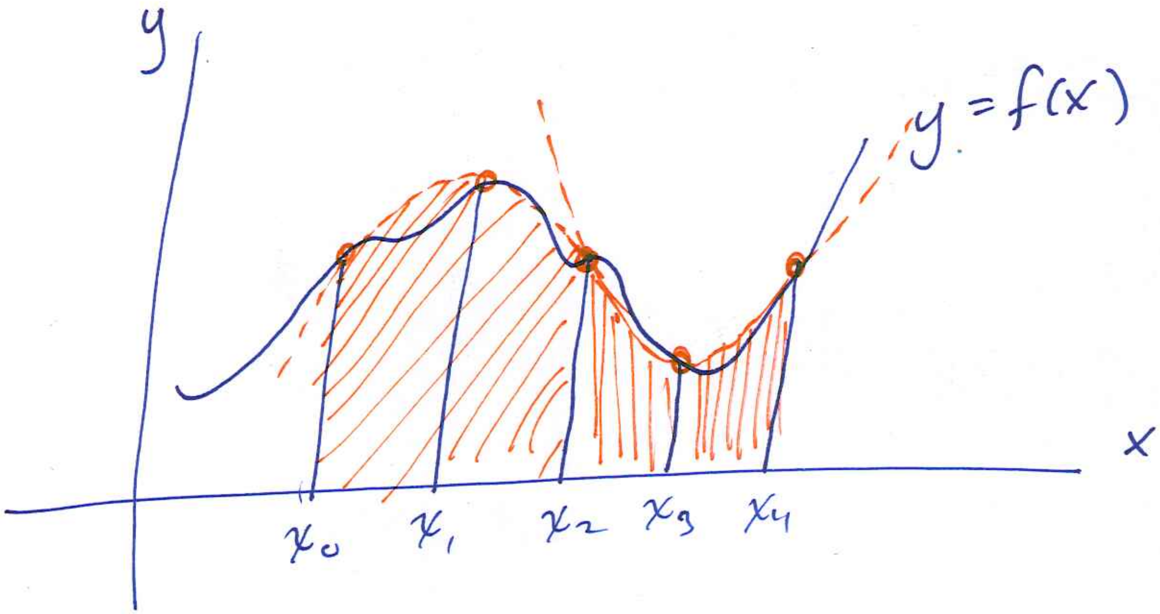
Simpson's Rule



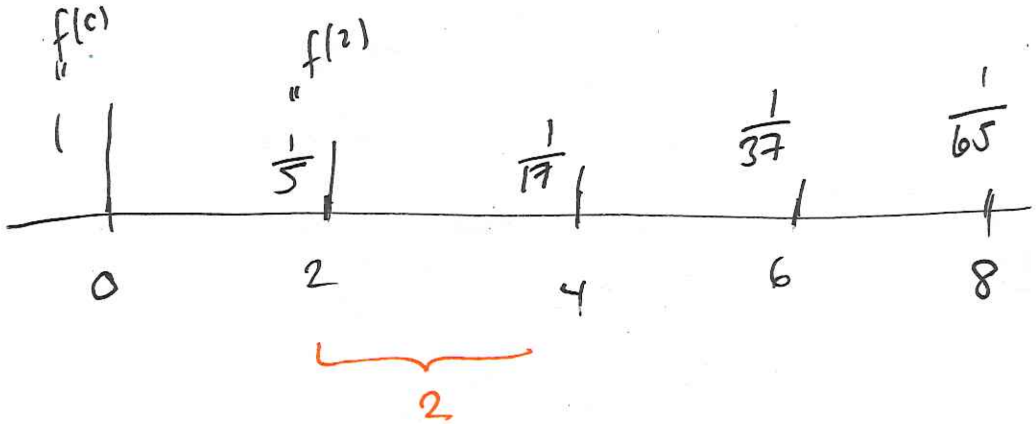
Approximate area under a pair of intervals using a parabola

even # intervals

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)\right]$$

(ex) Approx $\int_0^8 \frac{1}{1+x^2}\,dx$, Simpson's Rule, $n=4$

$\Delta x = \frac{b-a}{n} = \frac{8}{4} = 2$

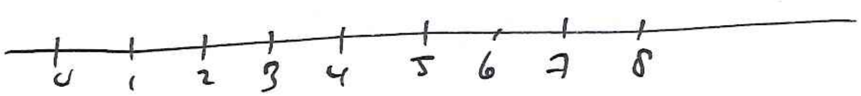


$$\frac{2}{3}\left[1 + 4\cdot\frac{1}{5} + 2\cdot\frac{1}{17} + 4\cdot\frac{1}{37} + \frac{1}{65}\right] \approx \arctan(8) = \int_0^8 \frac{1}{1+x^2}\,dx$$

(ex) Give Simpson's Rule approx of $\int_0^8 e^{x^2}\,dx$, $n=8$

$$\int_0^8 e^{x^2}\,dx \approx \frac{1}{3}\left[e^0 + 4e^1 + 2e^4 + 4e^9 + 2e^{16} + 4e^{25} + 2e^{36} + 4e^{49} + e^{64}\right]$$

$\Delta x = \frac{8-0}{8} = 1$



Error in numerical Integration

Thm 7.2 p 565 + formula sheet

Using approx of $\int_0^1 \sin(2x)\,dx$, $n=10$

$f(x) = \sin(2x)$

$f'(x) = 2\cos(2x)$

$\boxed{f''(x) = -4\sin 2x} \longrightarrow |f''(x)| = |-4\sin(2x)| \le 4$ $\quad k=4$

Formula:

using midpt approx:

$|\text{error}| = E_M \le \dfrac{k(b-a)^3}{24n^2} = \dfrac{4(1-0)^3}{24\cdot 10^2} = \dfrac{1}{600}$

using trapezoid approx:

$|\text{error}| = E_T \le \dfrac{k(b-a)^3}{12n^2} = \dfrac{4(1-0)^3}{12\cdot 10^2} = \dfrac{1}{300}$

$f'''(x) = -8\cos(2x)$

$\boxed{f^{(4)}(x) = 16\sin 2x} \rightarrow |f^{(4)}(x)| = |16\sin 2x| \le 16$ $\quad K=16$

using Simpson: $E_S \le \dfrac{K(b-a)^5}{180\,n^4} = \dfrac{16(1-0)^5}{180\cdot 10^4} = \dfrac{16}{1800000}$