Improper Fractions

$$\frac{2x^3-5x^2+8x-7}{x^2-3x+2}$$

We can do long division.
(video on website)

Hard to $\int$

Want to use partial fractions

problem: degree of num $\geq$ deg den

Analogy: $\frac{13}{3} = \frac{12+1}{3} = \frac{12}{3}+\frac{1}{3} = \frac{4\cdot\cancel{3}}{\cancel{3}}+\frac{1}{3} = 4+\frac{1}{3}$

identify largest multiple of denominator that fits in numerator

separate

cancel

$$\frac{2x^3-5x^2+8x-7}{x^2-3x+2} = \frac{2x^3-6x^2+4x \quad +x^2+4x-7}{x^2-3x+2}$$

(separate)

$2x(x^2-3x+2)$

$= 2x^3-6x^2+4x$

$$= \frac{2x^3-6x^2+4x}{x^2-3x+2} + \frac{x^2+4x-7}{x^2-3x+2}$$

$$= \frac{2x\cancel{(x^2-3x+2)}}{\cancel{x^2-3x+2}} + \frac{x^2+4x-7}{x^2-3x+2}$$

$$= 2x + \frac{x^2+4x-7}{x^2-3x+2}$$

still can't do partial fractions: divide more

x^2-3x+2

$$= 2x + \frac{x^2-3x+2 \ +7x-9}{x^2-3x+2}$$

$$= 2x + \frac{\cancel{x^2-3x+2}}{\cancel{x^2-3x+2}} + \frac{7x-9}{x^2-3x+2}$$

$$= 2x+1+\frac{7x-9}{x^2-3x+2}$$

now can do partial fractions: deg of num < deg of denom

$$\frac{7x-9}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$ find A, B

$$\int \frac{2x^3-5x^2+8x-7}{x^2-3x+2}\,dx = \int 2x+1+\frac{A}{x-1}+\frac{B}{x-2}\,dx$$ etc...

ex $$\frac{3x^2-2x-3}{x^2-3x+2} = \frac{3x^2-9x+6+7x-9}{x^2-3x+2}$$

$$= \cancel{\frac{3(x^2-3x+2)}{x^2-3x+2}} + \frac{7x-9}{x^2-3x+2}$$

$3(x^2-3x+2)$

$= 3x^2-9x+6$

$$= 3 + \frac{7x-9}{x^2-3x+2}$$ } deg of numerator < deg of denominator

$$= 3 + \frac{A}{(x-1)} + \frac{B}{(x-2)}$$ find A, B etc.

§7.5 : all suggested

Quiz Monday

short video on section webpage : long division

Ch 7.7 : Numerical Integration

Motivation: $\int_a^b f(x)\,dx$ } don't want to (can't) antidifferentiate $f(x)$

eg $\int_1^2 \frac{1}{1+x^2}\,dx = \arctan(2) - \arctan(1)$

what is $\arctan(2)$?

eg $\int_0^1 e^{x^2}\,dx$

$\int e^{x^2}\,dx$: non-elementary function

we don't have the tools to write it

Absolute vs Relative Error

ex 1: A bag of flour weighs exactly 500g
it is mislabeled as 495g.

absolute error: $|500-495| = 5g$

relative error: $\frac{5}{500} = \frac{1}{100} = 1\%$

ex 2: A bottle of medicine contains exactly 5g
it is mislabeled as 10g

abs error: $|5-10| = 5g$

relative error: $\frac{5}{5} = 1 = 100\%$

absolute error: $|\text{exact} - \text{approx}|$

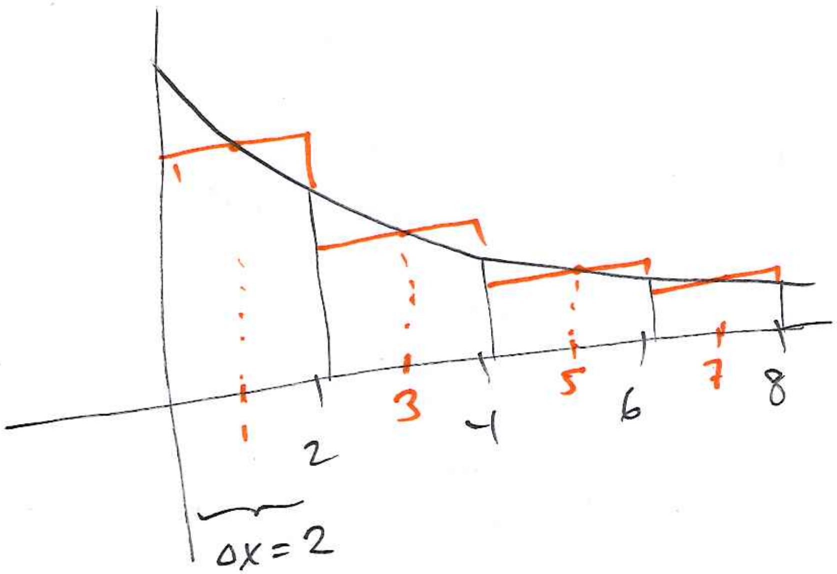
relative error: $\left|\frac{\text{absolute error}}{\text{exact}}\right|$

Reall Midpt Rieman Sum

Approx $\int_0^8 \frac{1}{1+x^2}dx$, $n=4$ intervals

Area $\approx 2f(1)+2f(3)+2f(5)+2f(7)$

$= 2\left(\frac{1}{2}\right)+2\left(\frac{1}{10}\right)+2\left(\frac{1}{26}\right)+2\left(\frac{1}{50}\right)$



"Midpoint Rule"

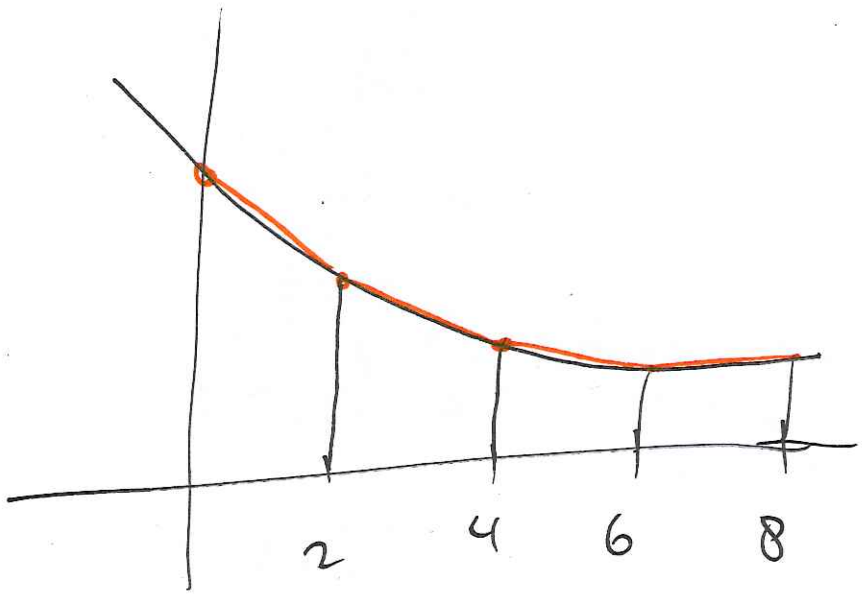
$$\int_a^b f(x)dx = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right)\Delta x$$

where $\Delta x = \frac{b-a}{n}$

$x_k = a+k\Delta x$

Approx f(x) by a constant over each interval

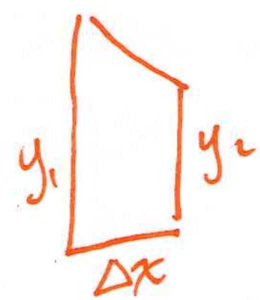
What if we approx $f(x)$ using lines



$$\int_0^8 f(x)\,dx \approx$$

$n=4$

Trapezoids!

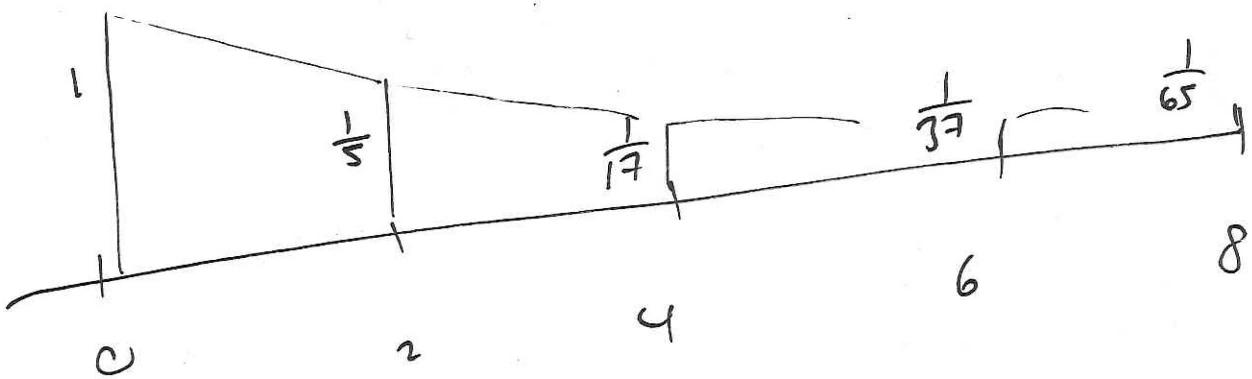


area: $\frac{1}{2}\Delta x(y_1+y_2)$

Using Trapezoid Rule:

$$\int_0^8 \frac{1}{1+x^2}\,dx \approx \frac{1}{\cancel{2}}(\cancel{2})\left(1+\frac{1}{5}\right)+\frac{1}{\cancel{2}}(\cancel{2})\left(\frac{1}{5}+\frac{1}{17}\right)+\frac{1}{\cancel{2}}(\cancel{2})\left(\frac{1}{17}+\frac{1}{37}\right)+\frac{1}{\cancel{2}}(\cancel{2})\left(\frac{1}{37}+\frac{1}{65}\right)$$

$n=4$



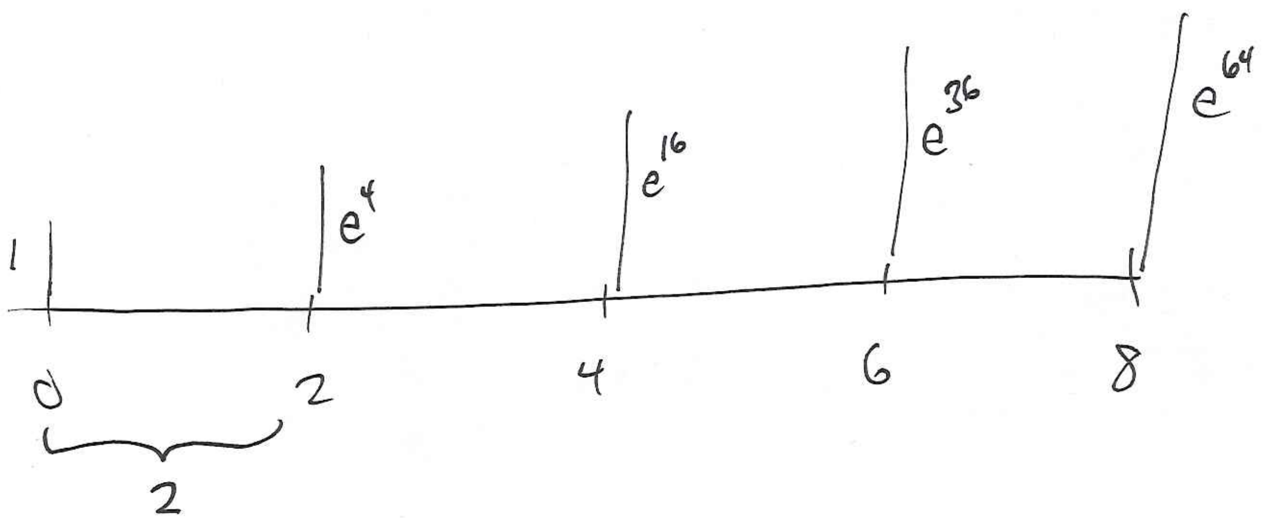
$\Delta x = 2$

Trapezoid Rule:

$$\int_a^b f(x)\,dx \approx \Delta x\left(\frac{1}{2}\underset{\overset{\shortparallel}{a}}{f(x_0)} + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2}\underset{\overset{\shortparallel}{b}}{f(x_n)}\right)$$

Use Trapezoid rule, $n=4$

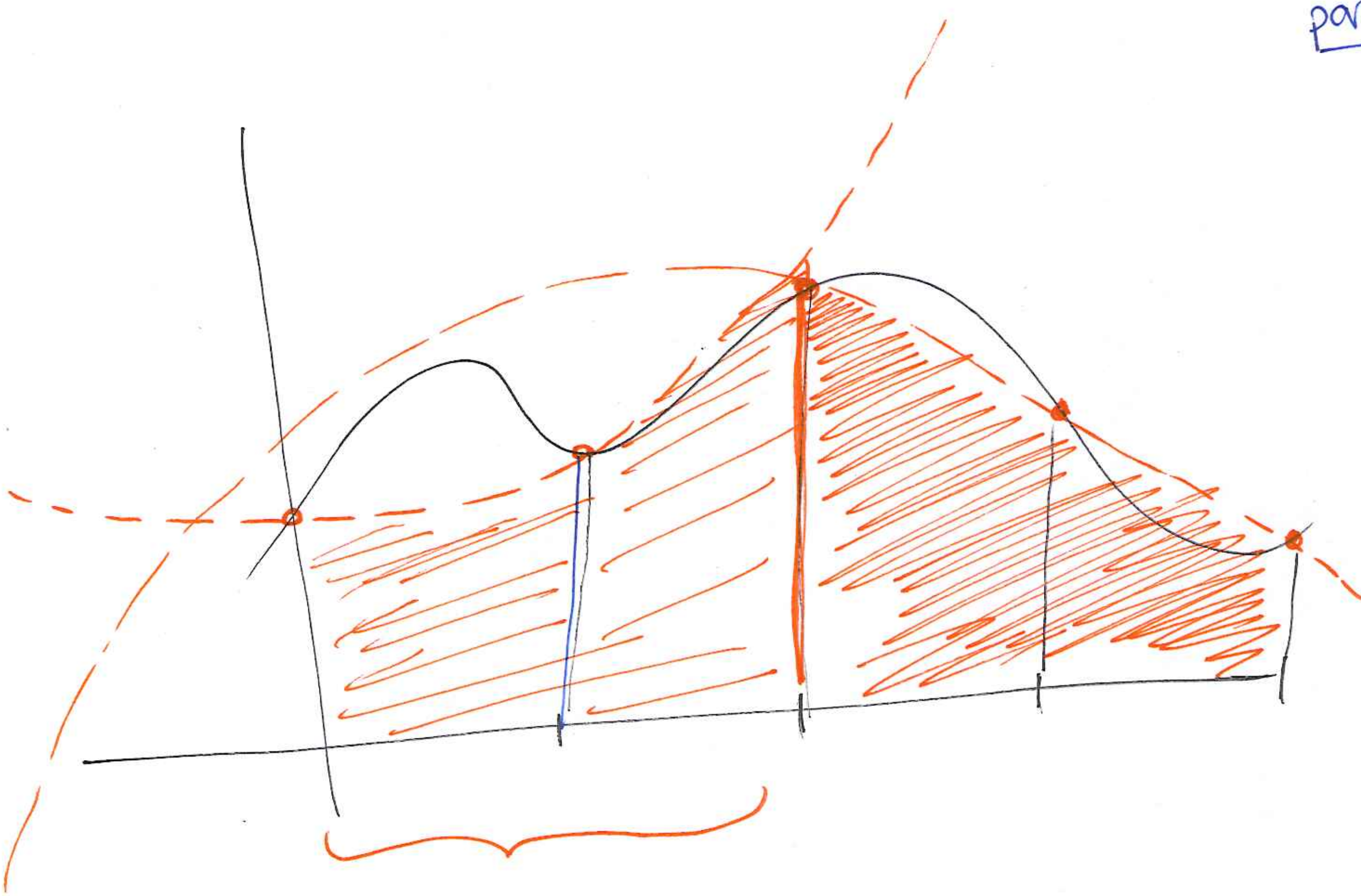
$$\int_0^8 e^{x^2}\,dx \approx 2\left[\frac{1}{2}(1) + e^{4} + e^{16} + e^{36} + \frac{1}{2}e^{64}\right]$$



$\Delta x = \frac{8-0}{n} = 2$

Simpson's Rule

Approx f(x) using <u>parabola</u> (over a pair of intervals)



$n=4$

(n-even for Simpson)