Integration by parts

$$\int u\,dv = uv - \int v\,du$$

ex $\int (x+1)\sec^2 x\,dx = (x+1)\tan - \int \tan x \cdot dx$

$u : x+1$ $\quad$ $du : 1\,dx$

$dv : \sec^2 x\,dx$ $\quad$ $v : \tan x$

$$= (x+1)\tan x - \int \frac{\sin x}{\cos x}\,dx$$

$s = \cos x$

$ds = -\sin x\,dx$

$$= (x+1)\tan x - \int \frac{-ds}{s}$$

$$= (x+1)\tan x + \ln|s| + C$$

$$= \boxed{(x+1)\tan x + \ln|\cos x| + C}$$

ex) $\int \underbrace{x}\underbrace{e^{6x}}\,dx = \frac{x}{6}e^{6x} - \int \frac{1}{6}e^{6x}\,dx$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$u: x$ $\quad du: 1\cdot dx$

$dv: e^{6x}dx$ $\quad v: \frac{1}{6}e^{6x}$

$$= \frac{x}{6}e^{6x} - \frac{1}{6}\left(\frac{1}{6}e^{6x}\right) + C$$

$$= \boxed{\frac{1}{6}e^{6x}\left(x - \frac{1}{6}\right) + C}$$

ex) $\int (3t+5)\cos\left(\frac{t}{4}\right)dt =$

$(3t+5)(4\sin(\frac{t}{4}))$

$- \int 12\sin(\frac{t}{4})\,dt$

$u: 3t+5$ $\quad du: 3\,dt$

$dv: \cos(\frac{t}{4})dt$ $\quad v: 4\sin(\frac{t}{4})$

$$= (3t+5)(4\sin(\frac{t}{4})) - 12\cdot(4)\left[-\cos(\frac{t}{4})\right] + C$$

$$= \boxed{4(3t+5)\sin\frac{t}{4} + 48\cos(\frac{t}{4}) + C}$$

$v = \int dv = \int \cos(\frac{t}{4})dt = \int \cos(u)\cdot 4\,du$

$u = \frac{t}{4}$ $\quad = 4\sin u$

$du = \frac{1}{4}dt$ $\quad = \boxed{4\sin(\frac{t}{4})}$

$4\,du = dt$

(ex) $\int x^3 \ln x \, dx = \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$

$u: \ln x \qquad du: \frac{1}{x}dx$

$dv: x^3 dx \qquad v: \frac{1}{4}x^4$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\left(\frac{1}{4}x^4\right) + C$

$= \boxed{\frac{1}{4}x^4\left(\ln x - \frac{1}{4}\right) + C}$

(ex) $\int x^2 \ln^2 x \, dx$

$u: \ln^2 x \qquad du: 2\ln x \cdot \frac{1}{x}\,dx$

$dv: x^2 dx \qquad v: \frac{1}{3}x^3$

$= \frac{1}{3}x^3 \ln^2 x - \int \frac{1}{3}x^3 \cdot 2\ln x \cdot \frac{1}{x}\,dx$

$= \frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\int x^2 \ln x \, dx =$

$u: \ln x \qquad du = \frac{1}{x}dx$

$dv: x^2 dx \qquad v = \frac{1}{3}x^3$

$\frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left[\frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx\right]$

$= \frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left[\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx\right]$

$= \frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left[\frac{1}{3}x^3 \ln x - \frac{1}{3}\cdot\frac{1}{3}x^3\right] + C$

(ex) $\int \ln x \, dx = x\ln x - \int x\left(\frac{1}{x}\right)dx$

$u: \ln x \qquad du: \frac{1}{x}\,dx$

$dv: 1\,dx \qquad v: x$

$= x\ln x - \int 1\,dx$

$= \boxed{x\ln x - x + C}$

(ex) $\int \underbrace{\arctan x}\,\underbrace{dx} = x\arctan x - \int \frac{x}{1+x^2}\,dx =$

$\frac{1}{2}dw$

$u: \arctan x \qquad du: \frac{1}{1+x^2}\,dx$

$dv: 1\cdot dx \qquad v = x$

Substitution:

$w = 1+x^2$

$\frac{dw}{dx} = 2x$

$dw = 2x\,dx$

$\frac{1}{2}dw = x\,dx$

$x\arctan x - \int \frac{1}{2}\frac{1}{w}\,dw$

$= x\arctan x - \frac{1}{2}\ln|w| + C$

$= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

$= \boxed{x\arctan - \frac{1}{2}\ln(1+x^2) + C}$

ex) $\int \arcsin x \, dx$

Recall:

$$\frac{d}{dx}\{\arcsin x\} = \frac{1}{\sqrt{1-x^2}}$$

$$\int u\,dv = uv - \int v\,du$$

$u: \arcsin x \qquad du: \frac{1}{\sqrt{1-x^2}}\,dx$

$dv: dx \qquad v: x$

$$= x\arcsin x - \int \frac{\overbrace{x\,dx}^{-\frac{1}{2}dw}}{\underbrace{\sqrt{1-x^2}}}$$

$w = 1-x^2$

$dw = -2x\,dx$

$-\frac{1}{2}dw = x\,dx$

$$= x\arcsin x - \int -\frac{1}{2}\frac{1}{\sqrt{w}}\,dw = x\arcsin x + \frac{1}{2}\int w^{-1/2}\,dw$$

$$= x\arcsin x + \frac{1}{2}\cdot(2)\,w^{1/2} + C$$

$$= x\arcsin x + \sqrt{w} + C$$

$$= \boxed{x\arcsin x + \sqrt{1-x^2} + C}$$

"Integrating Around in a Circle"

$$\int u\,dv = uv - \int v\,du$$

(ex) $\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$

$u: e^x$ $\quad du: e^x\,dx$

$dv: \cos x\,dx$ $\quad v: \sin x$

$u: e^x$ $\quad du: e^x\,dx$

$dv: \sin x\,dx$ $\quad v: -\cos x$

$$= e^x \sin x + \left[+ e^x \cos x + \int - e^x \cos x\,dx \right]$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$+\int e^x \cos x\,dx$ to both sides

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$$

$$\boxed{\int e^x \cos x\,dx = \frac{1}{2}\left[e^x \sin x + e^x \cos x\right] + C}$$

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

(ex) $\boxed{\int e^x \sin x \, dx} =$

$u : e^x$ $\quad du = e^x dx$

$dv : \sin x \, dx$ $\quad v = -\cos x$

$$-e^x \cos x - \int -e^x \cos x \, dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

$u : e^x$ $\quad du = e^x dx$

$dv : \cos x \, dx$ $\quad v = \sin x$

$$= \boxed{-e^x \cos x + e^x \sin x - \int e^x \sin x \, dx}$$

$+\int e^x \sin x \, dx$ to both sides

$$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x + C$$

$$\boxed{\int e^x \sin x \, dx = \frac{1}{2}\left[-e^x \cos x + e^x \sin x\right] + C}$$

ex $\int_0^{10} xe^x\,dx = xe^x\Big|_0^{10} - \int_0^{10} e^x\,dx$

$u: x \qquad du: 1\,dx$

$dv: e^x dx \qquad v: e^x$

$= (10e^{10} - 0) - \int_0^{10} e^x\,dx$

$= 10e^{10} - [e^{10} - e^0]$

$= 10e^{10} - e^{10} + e^0$

$= \boxed{9e^{10} + 1}$ Area under curve

ex $\int_0^{2\pi} x^2 \sin x\,dx = -x^2\cos x\Big|_0^{2\pi} - \int_0^{2\pi} -2x\cos x\,dx$

$u: x^2 \qquad du: 2x\,dx$

$dv: \sin x\,dx \qquad v: -\cos x$

$= -(2\pi)^2\cos(2\pi) - (-0)\cos 0 + \int_0^{2\pi} 2x\cos x\,dx$

$= -4\pi^2 + \int_0^{2\pi} 2x\cos x\,dx$

$u: 2x \qquad du = 2\,dx$

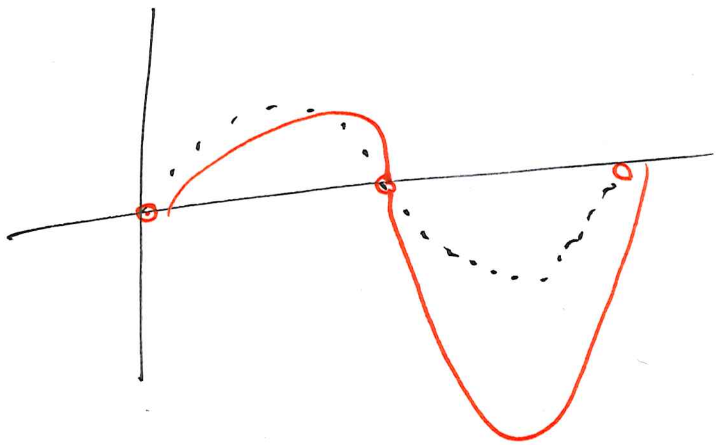
$dv: \cos x\,dx \qquad v: \sin x$

$= -4\pi^2 + 2x\sin x\Big|_0^{2\pi} - \int_0^{2\pi} 2\sin x\,dx$

$= -4\pi^2 + (4\pi\cancel{\sin(2\pi)}^{0} - 0) - \int_0^{2\pi} 2\sin x\,dx$

$= -4\pi^2 - 2\left[-\cos x\Big|_0^{2\pi}\right] = -4\pi^2 - 2(-1 - (-1)) = \boxed{-4\pi^2}$

$x^2 \sin x$:



net area should be negative

$\boxed{\text{Trig Identities}}$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$