$\boxed{\text{Substitution}}$

ex $\int x \sec(x^2)\tan(x^2)\,dx = \int \frac{1}{2} \sec u \cdot \tan u \, du$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$= \frac{1}{2}\sec u + C$

$= \boxed{\frac{1}{2}\sec(x^2) + C}$

ex $\int \frac{e^x}{e^x+15}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$

$u = e^x + 15$

$\frac{du}{dx} = e^x \quad du = e^x\,dx$

$= \ln|e^x+15| + C$

$= \boxed{\ln(e^x+15) + C}$

ex $\displaystyle \int \sin x \underbrace{\cos x\, dx}_{du} = \int u\, du = \frac{1}{2}u^2 + C$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x \rightarrow du = \cos x\, dx$$

$$= \boxed{\frac{1}{2}(\sin x)^2 + C}$$

Check: $\displaystyle \frac{d}{dx}\left\{ \frac{1}{2}(\sin x)^2 + C \right\}$

$$= \frac{1}{2} \cdot 2(\sin x) \cdot \cos x$$

$$= \sin x \cos x \quad ✓$$

(ex) $\int \frac{4x+2}{x^2+x+1}\,dx = \int 2(2x+1)\cdot\frac{1}{x^2+x+1}\,dx$

$u = x^2+x+1$

$\frac{du}{dx} = 2x+1$

$du = (2x+1)\,dx$

$= \int 2\cdot\frac{1}{u}\,du = 2\ln|u| + C$

$= \boxed{2\ln|x^2+x+1| + C}$

(ex) $\int x^4 (x^5+1)^8\,dx = \int \frac{1}{5}\cdot u^8\,du = \frac{1}{5}\cdot\frac{1}{9}u^9 + C$

$= \boxed{\frac{1}{45}(x^5+1)^9 + C}$

$u = x^5+1$

$\frac{du}{dx} = 5x^4$

$du = 5x^4\,dx$

$\frac{1}{5}du = x^4\,dx$

(ex) $\int x^2 \sin(x^3)\,dx = \int \frac{1}{3}\sin u \, du = \frac{1}{3}\cdot(-\cos u) + C$

$= \boxed{-\frac{1}{3}\cos(x^3) + C}$

$u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\boxed{\frac{1}{3}\,du} = \boxed{x^2\,dx}$

Check: $\frac{d}{dx}\left\{-\frac{1}{3}\cos(x^3) + C\right\}$

$= \frac{-1}{\cancel{3}}\cdot(-\sin(x^3))\cdot\cancel{3}x^2$

$= \sin(x^3)\cdot x^2$ ✓

ex $\int \frac{s}{\boxed{s-3}} ds = \int \frac{u+3}{\boxed{u}} du = \int \frac{u}{u} + \frac{3}{u} du$

$u = s-3 \qquad s = u+3$

$\frac{du}{ds} = 1$

$du = ds$

$= \int \left(1 + \frac{3}{u}\right) du =$

$u + 3\ln|u| + C$

$= \boxed{s-3+3\ln|s-3|+C}$

ex $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \int (-1)\frac{1}{u} du = -\ln|u| + C$

$\cos x = u$

$-\sin x \, dx = du$

$\sin x \, dx = -du$

$= -\ln|\cos x| + C$

$= \ln|(\cos x)^{-1}| + C$

$= \ln\left|\frac{1}{\cos x}\right| + C$

$= \boxed{\ln|\sec x| + C}$ *memorizing

Continuing ex:

$$\int \frac{\sin x}{\cos x}\,dx = \int \frac{u}{\cos x}\,dx$$

$$\frac{du}{\cos x}$$

difficult to sub in

what if $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x\,dx$$

(ex) $\int x^5\sqrt{x^3+1}\,dx = \int \underbrace{x^3}_{u-1}\underbrace{\sqrt{x^3+1}}_{\sqrt{u}}\underbrace{x^2\,dx}_{\frac{1}{3}du}$

$u = x^3+1 \rightarrow x^3 = u-1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$= \int (u-1)\sqrt{u}\left(\frac{1}{3}\right)du$

$= \frac{1}{3}\int (u-1)u^{1/2}\,du$

$= \frac{1}{3}\int u^{3/2} - u^{1/2}\,du = \frac{1}{3}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C$

$= \boxed{\frac{1}{3}\left[\frac{2}{5}(x^3+1)^{5/2} - \frac{2}{3}(x^3+1)^{3/2}\right] + C}$

$\int \sqrt{x^{13}+x^{10}}\,dx$ – not much improved ex $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^3 x}\, \overbrace{dx}^{du} = \int_{1/\sqrt{2}}^{1} \frac{1}{u^3}\, du$

$u = \sin x$

$du = \cos x\, dx$

$x = \pi/4 \to u = \sin(\pi/4) = 1/\sqrt{2}$

$x = \pi/2 \to u = \sin(\pi/2) = 1$

$$= \int_{1/\sqrt{2}}^{1} u^{-3}\, du = \left.\frac{u^{-2}}{-2}\right|_{1/\sqrt{2}}^{1}$$

$$= \left(\frac{1^{-2}}{-2}\right) + \left(\frac{\left(\frac{1}{\sqrt{2}}\right)^{-2}}{+2}\right) = \frac{-1}{2} + \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^{-2}$$

$$= \frac{-1}{2} + \frac{1}{2}\left(\sqrt{2}\right)^2 = \frac{-1}{2} + \frac{1}{2}(2) = \frac{-1}{2} + 1 = 1/2$$