

Integration by Parts:  
Let's Get Fancy

Rule  
 $\int u dv = uv - \int v du$

⊙  $\int x^3 \ln^2 x dx$

$u: \ln^2 x$

$du: 2 \ln x \cdot \frac{1}{x} dx$

$dv: x^3 dx$

$\rightarrow v: \frac{1}{4} x^4$

$= \frac{1}{4} x^4 \ln^2 x - \int \frac{1}{4} x^4 \cdot 2 \ln x \cdot \frac{1}{x} dx$   
 $= \frac{1}{4} x^4 \ln^2 x - \int \frac{1}{2} x^3 \ln x dx$

We don't know yet!  
how to antidiff. log  
So - can't choose it as  $du$

$u: \ln x$   
 $du: \frac{1}{x} dx$   
 $dv: \frac{1}{2} x^3 dx$   
 $\rightarrow v: \frac{1}{8} x^4$

$= \frac{1}{4} x^4 \ln^2 x - \left[ \frac{1}{8} x^4 \ln x - \int \frac{1}{8} x^4 \cdot \frac{1}{x} dx \right]$   
 $= \frac{1}{4} x^4 \ln^2 x - \frac{1}{8} x^4 \ln x + \int \frac{1}{8} x^3 dx$   
 $\boxed{\frac{1}{4} x^4 \ln^2 x - \frac{1}{8} x^4 \ln x + \frac{1}{8} \cdot \frac{1}{4} x^4 + C}$

ex

$$\int \ln x \, dx = \int 1 \cdot \ln x \, dx$$

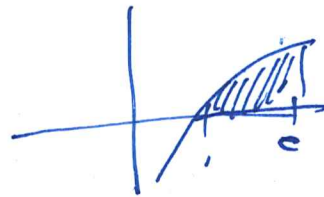
$$u: \ln x \quad du: \frac{1}{x} dx$$

$$dv: 1 \cdot dx \quad \rightarrow \quad v: x$$

$$= x \ln x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln x - \int 1 dx$$

$$= \boxed{x \ln x - x + C}$$



ex

$$\int_1^e \ln(x) \, dx =$$

$$[e \ln e - e + C] - [1 \ln 1 - 1 + C]$$

$$= [e - e - (-1)] = \boxed{1}$$

Rule:

$$\int u \, dv = uv - \int v \, du$$

(9x)

$$\int \arctan x \, dx = \int 1 \cdot \arctan x \, dx$$

$$u: \arctan x$$

$$dv: \frac{1}{1+x^2} \, dx$$

$$du: 1 \cdot dx$$

$$\rightarrow v: x$$

Rule:

$$\int u \, dv = uv - \int v \, du$$

$$= x \arctan x - \int \frac{x}{1+x^2} \, dx = x \arctan x - \int \frac{1}{2} \cdot \frac{1}{w} \cdot dw$$

$$w = 1+x^2$$

$$\frac{dw}{dx} = 2x$$

$$dw = 2x \, dx$$

$$\frac{1}{2} dw = x \, dx$$

$$= x \arctan x - \frac{1}{2} \ln|w| + C$$

$$= x \arctan x - \frac{1}{2} \ln|1+x^2| + C$$

$$= \boxed{x \arctan x - \frac{1}{2} \ln(1+x^2) + C}$$

$$\textcircled{ex} \int \arcsin x \, dx = \int 1 \cdot \arcsin x \, dx$$

$$u: \arcsin x \quad du: \frac{1}{\sqrt{1-x^2}} \, dx$$

$$dv: 1 \cdot dx \quad \rightarrow v: x$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$$w = 1-x^2$$

$$dw = -2x \, dx$$

$$\frac{-1}{2} dw = x \, dx$$

$$= x \arcsin x - \int \frac{1}{2} \frac{1}{\sqrt{w}} \, dw$$

$$= x \arcsin x + \frac{1}{2} \int w^{-1/2} \, dw$$

$$= x \arcsin x + \frac{1}{2} \cdot 2 w^{1/2} + C$$

$$= x \arcsin x + \sqrt{w} + C$$

$$= \boxed{x \arcsin x + \sqrt{1-x^2} + C}$$