# Integration by Parts: Let's Get Fancy

**Rule**

$$\int u\,dv = uv - \int v\,du$$

(ex) $\int x^3 \ln^2 x\,dx$

$u: \ln^2 x \qquad du: 2\ln x \cdot \frac{1}{x}\,dx$

$dv: x^3\,dx \quad \rightarrow \quad v: \frac{1}{4}x^4$

$$= \frac{1}{4}x^4 \ln^2 x - \int \frac{1}{4}x^4 \cdot 2\ln x \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{4}x^4 \ln^2 x - \int \frac{1}{2}x^3 \ln x\,dx$$

We don't know (yet!) how to antidiff. log
So - can't choose it as $du$

$u: \ln x \qquad du: \frac{1}{x}\,dx$

$dv: \frac{1}{2}x^3\,dx \quad \rightarrow \quad v: \frac{1}{8}x^4$

$$= \frac{1}{4}x^4 \ln^2 x - \left[\frac{1}{8}x^4 \ln x - \int \frac{1}{8}x^4 \cdot \frac{1}{x}\,dx\right]$$

$$= \frac{1}{4}x^4 \ln^2 x - \frac{1}{8}x^4 \ln x + \int \frac{1}{8}x^3\,dx$$

$$= \boxed{\frac{1}{4}x^4 \ln^2 x - \frac{1}{8}x^4 \ln x + \frac{1}{8}\cdot\frac{1}{4}x^4 + C}$$

Rule:
$\int u\,dv = uv - \int v\,du$

(ex) $\int \ln x\,dx = \int 1 \cdot \ln x\,dx$

$u: \ln x \qquad du: \frac{1}{x}dx$

$dv: 1 \cdot dx \rightarrow v: x$

$= x\ln x - \int x \cdot \frac{1}{x}dx$

$= x\ln x - \int 1\,dx$

$= \boxed{x\ln x - x + C}$


(ex) $\int_1^e \ln(x)\,dx =$

$[e\ln e - e + C] - [1\underbrace{\ln 1}_{0} - 1 + C]$

$= [e - e] - [-1] = \boxed{1}$

(9x) $\int \arctan x \, dx = \int 1 \cdot \arctan x \, dx$

$u : \arctan x \qquad dv : \frac{1}{1+x^2} dx$

$du : 1 \cdot dx \qquad \rightarrow v : x$

Rule:

$\int u\,dv = uv - \int v\,du$

$= x\arctan x - \int \frac{x}{1+x^2} dx = x\arctan x - \int \frac{1}{2} \cdot \frac{1}{w} \cdot dw$

$w = 1+x^2$

$\frac{dw}{dx} = 2x$

$dw = 2x\,dx$

$\frac{1}{2} dw = x\,dx$

$= x\arctan x - \frac{1}{2} \ln|w| + C$

$= x\arctan x - \frac{1}{2} \ln|1+x^2| + C$

$= \boxed{x\arctan x - \frac{1}{2} \ln(1+x^2) + C}$

(ex) $\int \arcsin x \, dx = \int 1 \cdot \arcsin x \, dx$

$u : \arcsin x \qquad du : \frac{1}{\sqrt{1-x^2}} dx$

$dv : 1 \cdot dx \quad \rightarrow v : x$

$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$ ($\frac{-1}{2} dw$)

$= x \arcsin x - \int \frac{-1}{2} \frac{1}{\sqrt{w}} \, dw$

$w = 1-x^2$

$dw = -2x \, dx$

$\frac{-1}{2} dw = x \, dx$

$= x \arcsin x + \frac{1}{2} \int w^{-1/2} \, dw$

$= x \arcsin x + \frac{1}{2} \cdot 2 \, w^{1/2} + C$

$= x \arcsin x + \sqrt{w} + C$

$= \boxed{x \arcsin x + \sqrt{1-x^2} + C}$