ex Write in Σ-notation

$$2+3+4+5+6+7 = \sum_{k=2}^{7} k = \sum_{k=1}^{6}(k+1)$$

IDEA: $\sum_{k=2}^{10}(k+2) = \underset{(k=2)}{4} + \underset{(k=3)}{\cancel{5}} + \cdots$ (doesn't work)

$$4+6+8+10+12 = \sum_{k=2}^{6} 2k = \underset{(k=2)}{4} + \underset{(k=3)}{6} + \underset{(k=4)}{8} + \underset{(k=5)}{10} + \underset{(k=6)}{12}$$

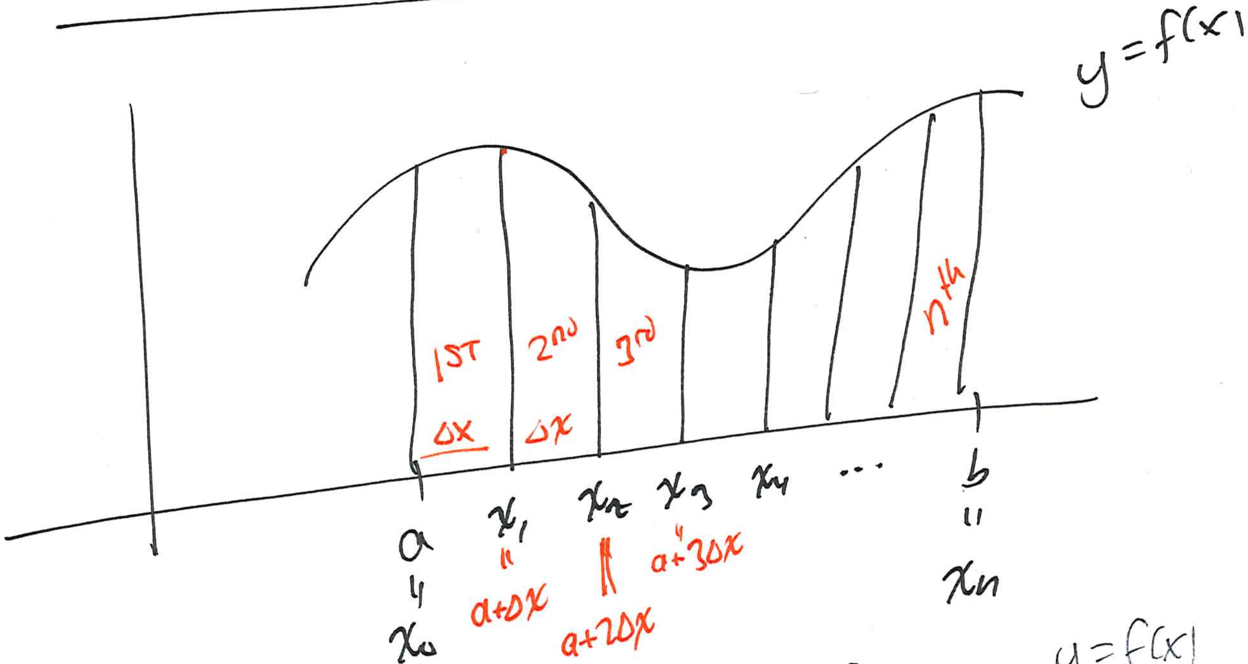
$$5+7+9+11+13 = \sum_{k=2}^{6}(2k+1)$$

$$3.5+6.5+9.5+12.5+15.5 = \sum_{k=1}^{5}\left(3k+\tfrac{1}{2}\right)$$

$$\underset{2^{-1}}{\underset{\parallel}{\tfrac{1}{2}}} + \underset{2^{0}}{\underset{\parallel}{1}} + \underset{2^{1}}{\underset{\parallel}{2}} + \underset{2^{2}}{\underset{\parallel}{4}} + \underset{2^{3}}{\underset{\parallel}{8}} + \underset{2^{4}}{\underset{\parallel}{16}} + \underset{2^{5}}{\underset{\parallel}{32}} = \sum_{k=-1}^{5} 2^k$$

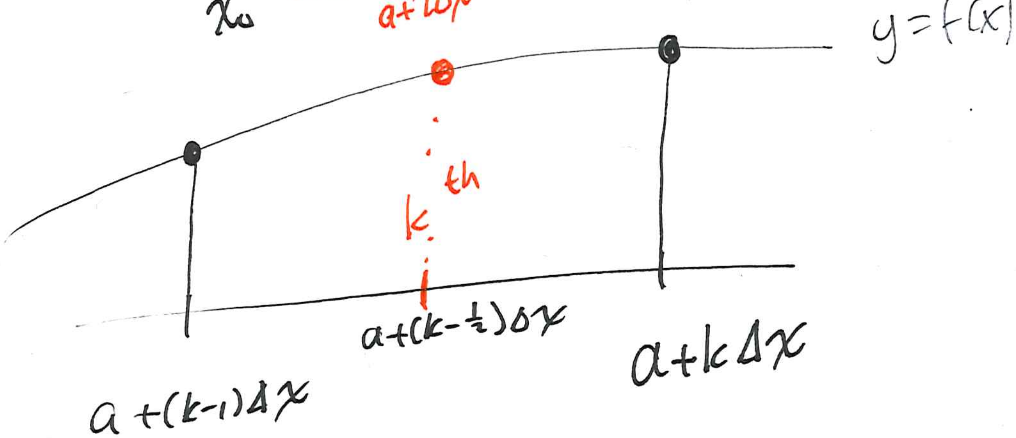
$$\underset{(-2)^{-1}}{\underset{\parallel}{-\tfrac{1}{2}}} + \underset{(-2)^{0}}{\underset{\parallel}{1}} \underset{(-2)^{1}}{\underset{\parallel}{-2}} + \underset{(-2)^{2}}{\underset{\parallel}{4}} \underset{(-2)^{3}}{\underset{\parallel}{-8}} + \underset{(-2)^{4}}{\underset{\parallel}{16}} \underset{(-2)^{5}}{\underset{\parallel}{-32}} = \sum_{k=-1}^{5}(-2)^k$$

Σ - formula for Riemann Sums



Area under $y = f(x)$, n slices

$$\underbrace{\Delta x}_{\text{width of slice}} = \frac{b-a}{n}$$



Left RS:

$f(a+(k-1)\Delta x)$ height of ▯

Right RS:

$f(a+k\Delta x)$ height of ▯

Midpt RS:

$f(a+(k-\frac{1}{2})\Delta x)$

$f(x)$; $[a,b]$,
n intervals (slices)
$\Delta x = \frac{b-a}{n}$

Left RS:

$$\underbrace{\sum_{k=1}^{n} \underbrace{\Delta x}_{\text{width of } \square} \cdot \underbrace{f(a+(k-1)\Delta x)}_{\text{height of } k^{th} \square}}_{\text{Area of } k^{th} \square}$$

Right RS:

$$\sum_{k=1}^{n} \Delta x \cdot f(a+k\Delta x)$$

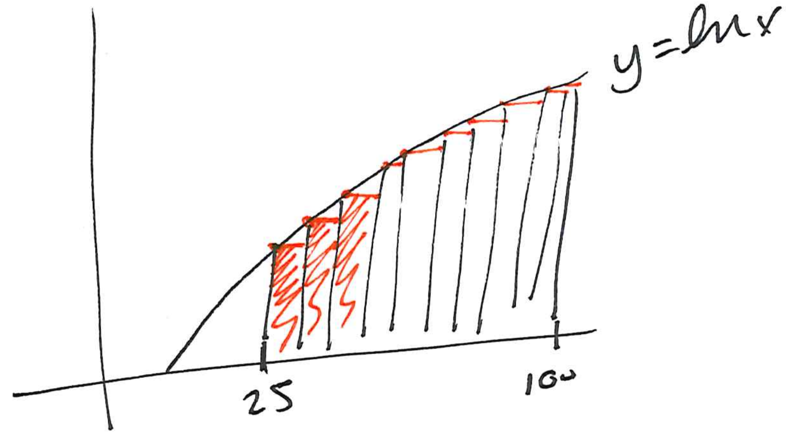
Midpoint RS:

$$\sum_{k=1}^{n} \Delta x \cdot f\left(a+\left(k-\tfrac{1}{2}\right)\Delta x\right)$$

ex $f(x) = \ln x$

Approx area under curve from $x = 25$ to $x = 100$, using 10 intervals

(Left RS)



Using Formula:

$n = 10$

$a = 25, \quad b = 100$

$$\underbrace{\Delta x}_{\text{width of } \square} = \frac{100-25}{10} = \frac{75}{10} = 7.5$$

$$\text{Sum: } \sum_{k=1}^{10} \Delta x \cdot f(25 + (k-1)\Delta x) = \boxed{\sum_{k=1}^{10} (7.5) \ln(25 + (k-1)7.5)}$$

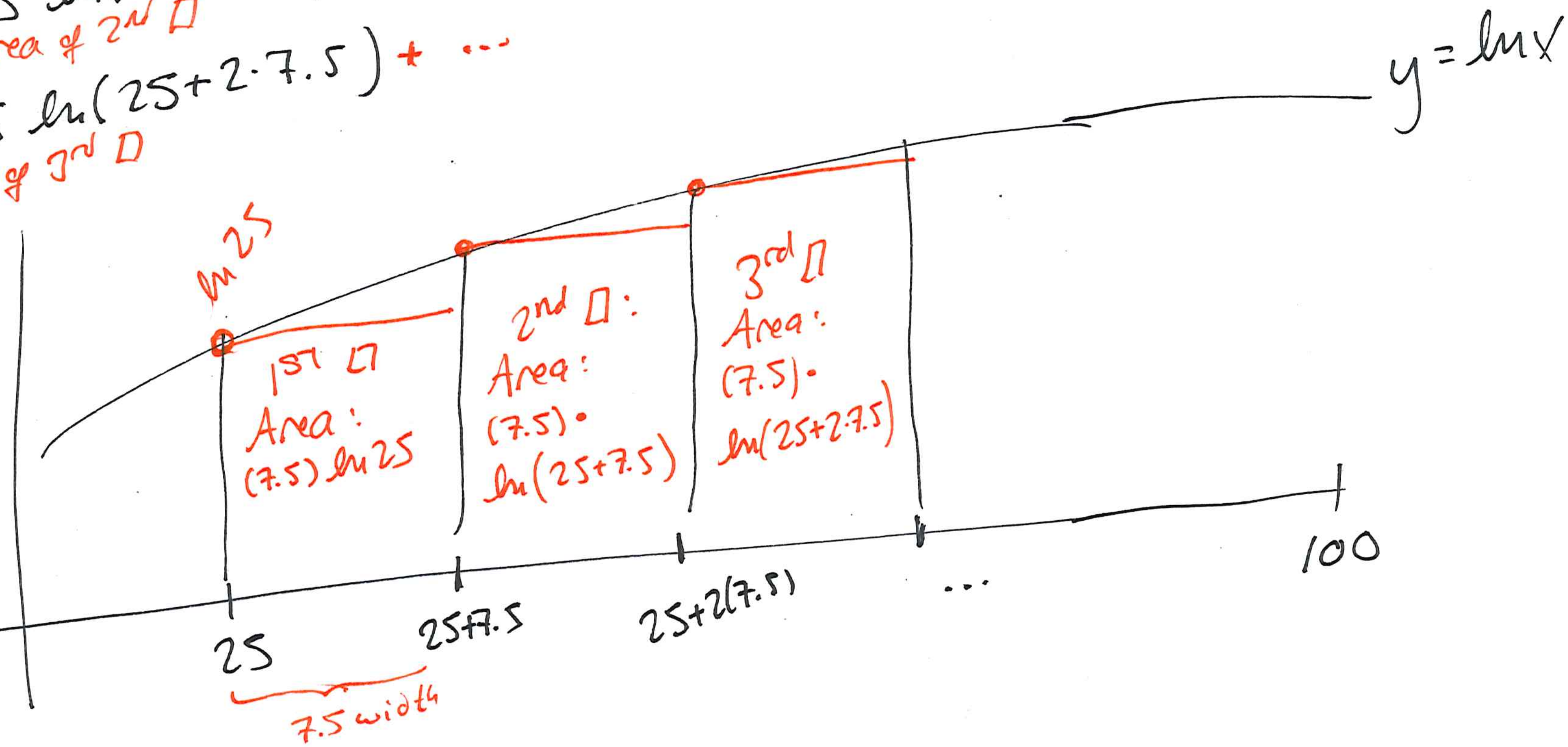
Let's see what's being added

$$\sum_{k=1}^{10} 7.5 \ln\big(25 + (k-1)\cdot 7.5\big)$$

$(k=1)$: $7.5 \ln(25 + 0\cdot 7.5) = 7.5 \ln(25)$ +
Area of 1st ▯

$(k=2)$: $7.5 \ln(25 + 7.5)$ +
Area of 2nd ▯

$(k=3)$: $7.5 \ln(25 + 2\cdot 7.5)$ + ...
Area of 3rd ▯



How to sum lots of #s ?

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(Add small powers of k)

- $\sum_{k=1}^{n} 7 = \underbrace{7+7+7+\cdots+7}_{n} = 7n$

$(7 = 7\cdot 1 = 7\cdot k^0)$

- $\sum_{k=1}^{n} k = 1+2+3+4+\cdots+n = \frac{(n+1)\cdot n}{2}$

- $\sum_{k=1}^{n} k^2 = 1+4+9+16+25+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

- $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

Don't need to memorize k^2, k^3

should know how to use formula

$1+2+3+\cdots+\overbrace{50+51}^{101}+\cdots+98+99+100$

101

101

101

101

$= (100+1)\cdot\frac{100}{2}$

$1+2+3+\cdots+\overbrace{998+999+1000}^{1001}$

1001

1001

$(1000+1)\left(\frac{1000}{2}\right)$

ex Evaluate:

$$\sum_{k=1}^{500}(2k^2-k+15) = \sum_{k=1}^{500} 2k^2 - \underbrace{\sum_{k=1}^{500} k}_{\substack{\text{FORMULA:}\\ 501\cdot 250}} + \underbrace{\sum_{k=1}^{500} 15}_{15\cdot 500}$$

$$= 2\underbrace{\sum_{k=1}^{500} k^2}_{\substack{\text{FORMULA}\\ n=500\\ \frac{(500)(501)(1001)}{6}}} - (250)(501) + (15)(500)$$

$$= \boxed{2\left(\frac{500\cdot 501\cdot 1001}{6}\right) - (250)(501) + (15)(500)}$$