GRADIENT

The gradient of a function $f(x,y)$ is the vector

$$\nabla f = \langle f_x , f_y \rangle$$

For example, if $f(x,y) = x^2+3y$, then

$$\nabla f = \langle 2x, 3 \rangle$$

∇f is a vector that points in the direction of steepest descent of the surface $z = f(x,y)$ at the position (x,y).

We can write the system of equations from the Lagrange Multiplier Method using gradients:

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) = c \end{cases} \quad \text{is equivalent to} \quad \begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g(x,y) = c \end{cases}$$

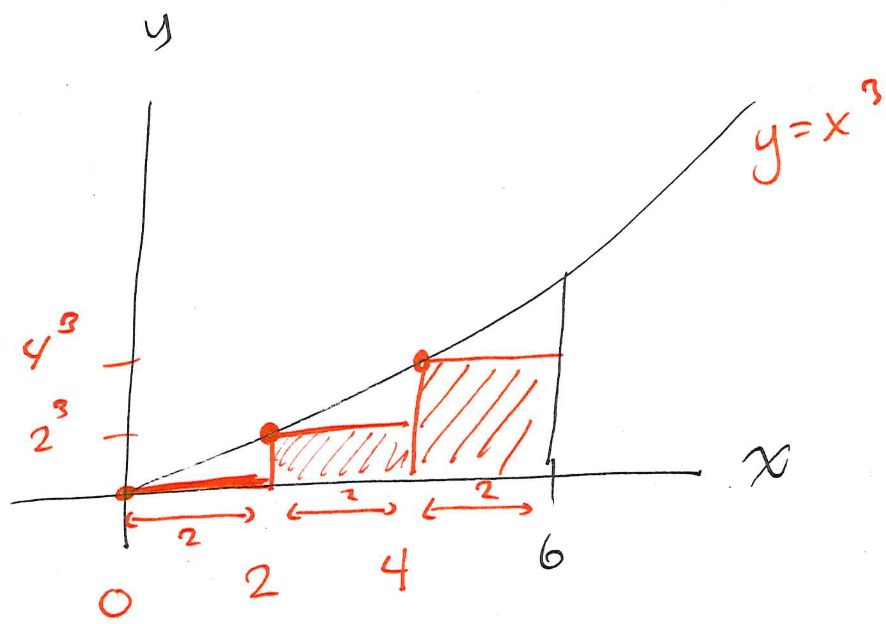
Riemann Sums

ex $y=x^3$

Approximate area under curve, $[0,6]$

using $n=3$ subintervals,

Riemann Sum

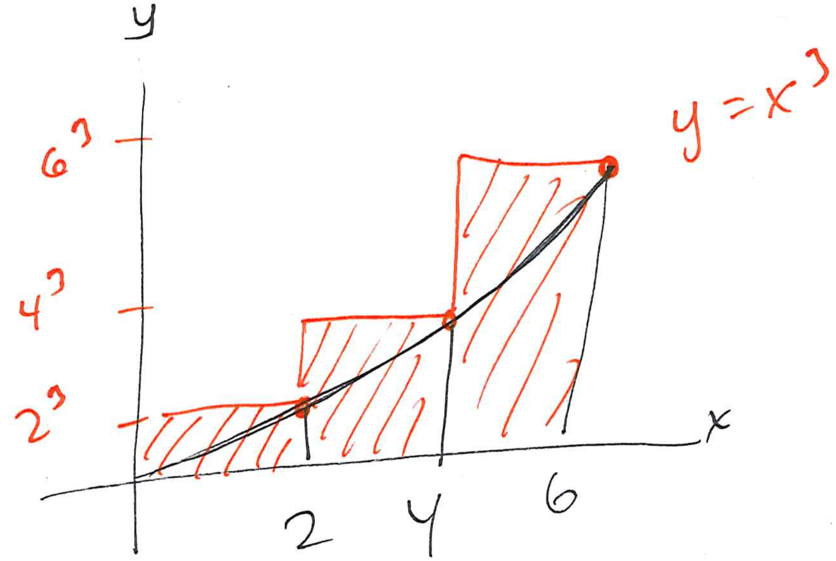


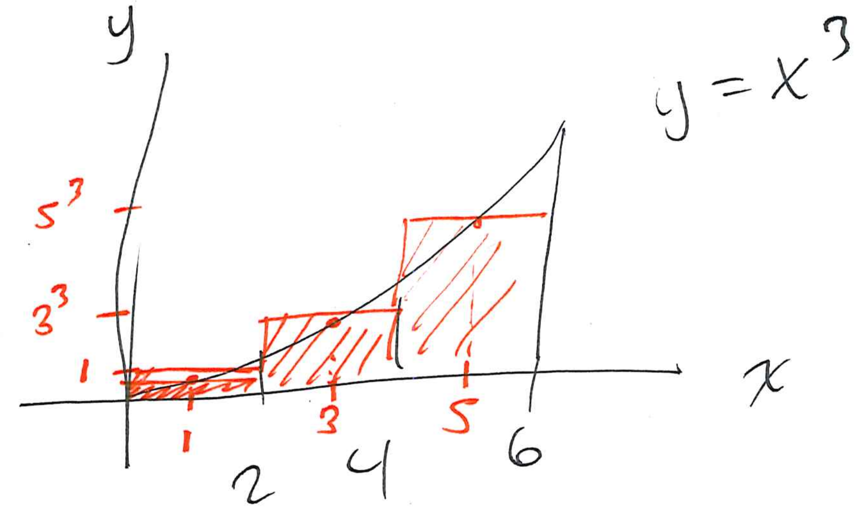
Left Riemann Sum: $0+2\cdot 2^3+2\cdot 4^3$

1^{st} ▯: $(2)(0)$

2^{nd} ▯: $(2)(2^3)$

3^{rd} ▯: $(2)(4^3)$





Right Riemann Sum:

1^{st} □: $(2)(2^3)$

2^{ND} □: $(2)(4^3)$

3^{RD} □: $(2)(6^3)$

Area = BASE × HEIGHT

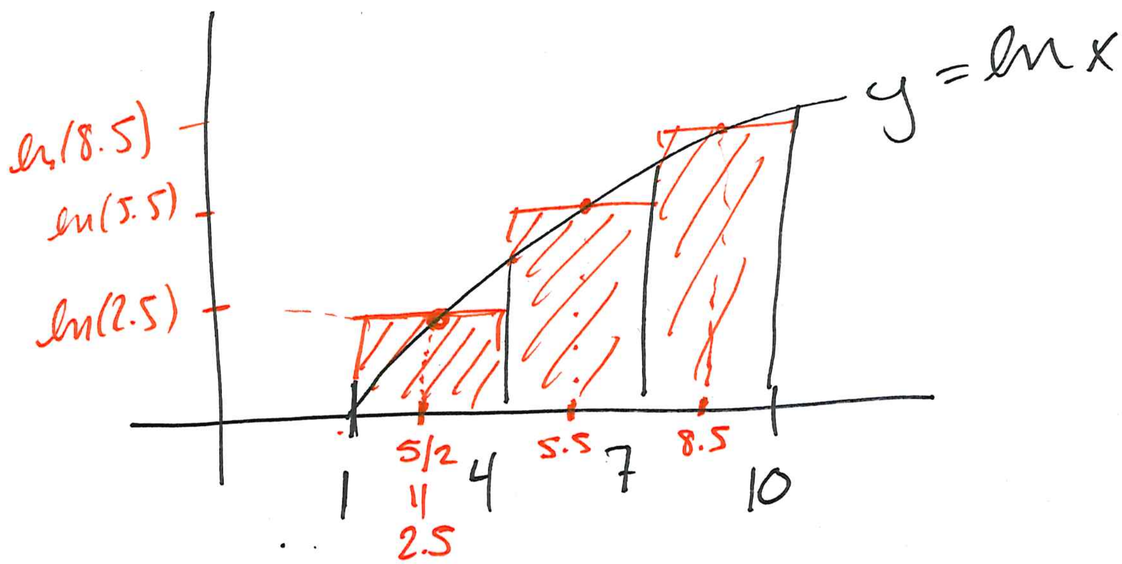
Sum: $2 \cdot 2^3 + 2 \cdot 4^3 + 2 \cdot 6^3$

Midpt Riemann Sum:

$(2)(1) + (2)(3^3) + (2)(5^3)$

(ex) Approx area under the curve $y = \ln x$ from $x=1$ to $x=10$, using $n=3$ intervals, Midpoint Riemann Sum

$$\Delta x = \frac{10-1}{3} = \frac{9}{3} = 3$$
(width)



Sum: $(3)\ln 2.5$
$+ (3)\ln(5.5)$
$+ (3)\ln(8.5)$

ex An object is observed travelling with the speeds below:

time	12:00	12:15	12:30	12:45	1:00
speed	60 kph	80 kph	100 kph	100 kph	40 kph

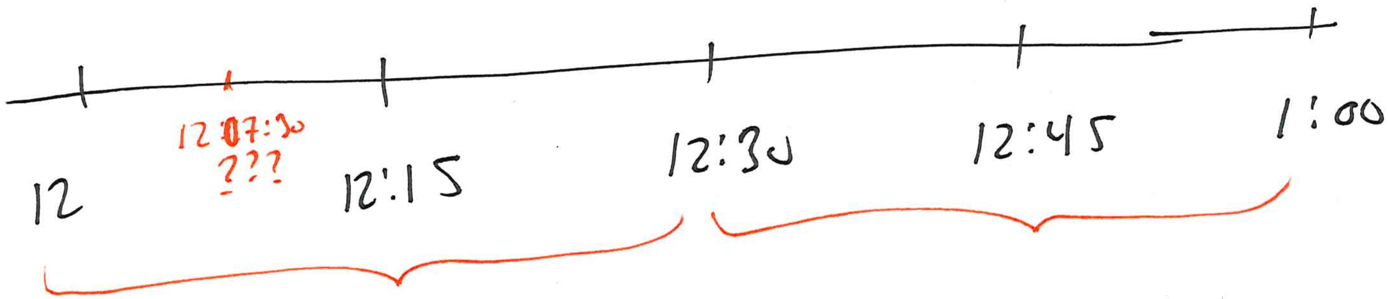
Use a Riemann Sum to approx. how far it travelled.

How many intervals did you use? 4

		left	right
$\boxed{12:00-12:15}$	$\frac{1}{4}$ hr,	60 kph	80 kph
$\boxed{12:15-12:30}$	$\frac{1}{4}$ hr,	80 kph	100 kph
$\boxed{12:30-12:45}$	$\frac{1}{4}$ hr,	100 kph	100 kph
$\boxed{12:45-1}$	$\frac{1}{4}$ hr,	100 kph	40 kph

Right:
$\frac{1}{4}(80)+\frac{1}{4}(100)+\frac{1}{4}(100)+\frac{1}{4}(40$

Left: $\frac{1}{4}(60)+\frac{1}{4}(80)+\frac{1}{4}(100)+(\frac{1}{4})(100)$



MIDPOINT RIEMANN SUM : 2 intervals

1ST Interval:
12:00–12:30 $\frac{1}{2}$ hr, 80 kph

2ND Interval:
12:30–1 $\frac{1}{2}$ hr, 100 kph

Sum: $\frac{1}{2}(80)+\frac{1}{2}(100) = 40+50 = 90$ km

Review of Σ-Notation

$$\sum_{k=a}^{b} f(k) = f(a) + f(a+1) + f(a+2) + \dots + f(b)$$

ex $$\sum_{k=3}^{7} k = 3+4+5+6+7$$

ex $$\sum_{k=-2}^{1} (2k+5) = \underset{(k=-2)}{(-4+5)} + \underset{(k=-1)}{(-2+5)} + \underset{(k=0)}{5} + \underset{(k=1)}{(2+5)}$$

ex $$\sum_{k=5}^{7} (k^2-k) = (25-5) + (36-6) + (49-7)$$

$$= 25+36+49-(5+6+7)$$

Which are true?

Addition: order doesn't matter

(A) $$\sum_{k=1}^{15} (k^2-k) = \left(\sum_{k=1}^{15} k^2\right) - \left(\sum_{k=1}^{15} k\right)$$

(B) $$\sum_{k=1}^{15} 3k = 3\sum_{k=1}^{15} k$$

FACTORING

$$3\cdot 1 + 3\cdot 2 + 3\cdot 3 + \dots + 3\cdot 15 =$$
$$3(1+2+3+\dots+15)$$

(C) $$\sum_{k=1}^{15} k(k-1) \neq k\sum_{k=1}^{15} (k-1)$$

$$1(0) + 2(1) + 3(2) + 4(3) + 5(4) + \dots + 15(14)$$

(D) $$\sum_{k=1}^{15} (k-1) = -15 + \sum_{k=1}^{15} k$$

$$= \sum_{k=1}^{15} k - \sum_{k=1}^{15} 1 = \left(\sum_{k=1}^{k} k\right) - 15$$

$$= (1-1) + (2-1) + (3-1) + (4-1) + \dots$$
$$(1+2+3+4+\dots) - (1+1+1+1\dots)$$