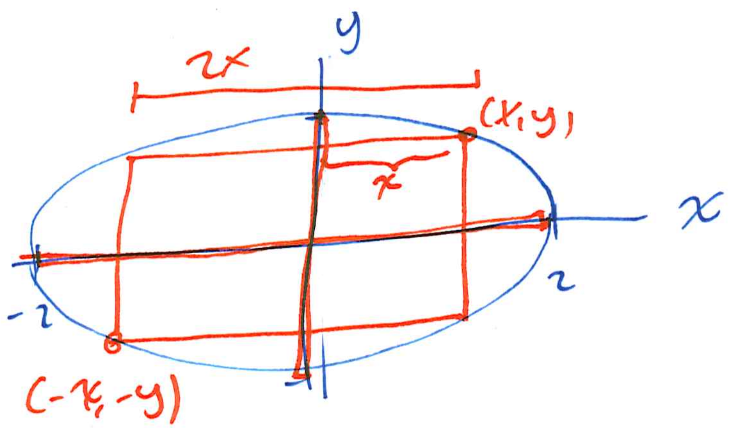
One final optimization example:

What is the maximum area of a rectangle (with sides parallel to the coordinate axes) inscribed in the ellipse $4x^2+16y^2=16$?



Area: $(2x)(2y) = 4xy$

$x, y > 0$

Constraint: $4x^2+16y^2=16$

$f(x,y) = 4xy \qquad g(x,y) = 4x^2+16y^2=16$

$f_x = 4y \qquad g_x = 8x$

$f_y = 4x \qquad g_y = 32y$

Solve

$$\begin{cases} 4y = \lambda \cdot \underbrace{8x} \\ 4x = \lambda \cdot \underbrace{32y} \\ 4x^2+16y^2 = 16 \end{cases}$$

$\cancel{x = 0 = y}$ or

$\cancel{y = 0 = x}$ or

$$\boxed{\begin{aligned} \lambda &= \frac{4y}{8x} = \frac{y}{2x} \\ \lambda &= \frac{4x}{32y} = \frac{x}{8y} \end{aligned}}$$

1st Eq: If $x=0$ then $y=0$
2nd eq FALSE - ignore

2nd Eq: If $y=0$ then $x=0$
3rd eq FALSE - ignore

So: $\lambda = \frac{y}{2x}$ and $\lambda = \frac{x}{8y}$

So: $\frac{y}{2x} = \frac{x}{8y}$ so $8y^2 = 2x^2$, so $4y^2 = x^2$

3rd Eq:

$$4x^2 + 16y^2 = 16$$
$$4x^2 + 4 \cdot \underbrace{4y^2}_{x^2} = 16$$
$$4x^2 + 4x^2 = 16$$
$$8x^2 = 16$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

$$4y^2 = 2$$
$$y^2 = \frac{1}{2}$$
$$y = \frac{\pm 1}{\sqrt{2}}$$

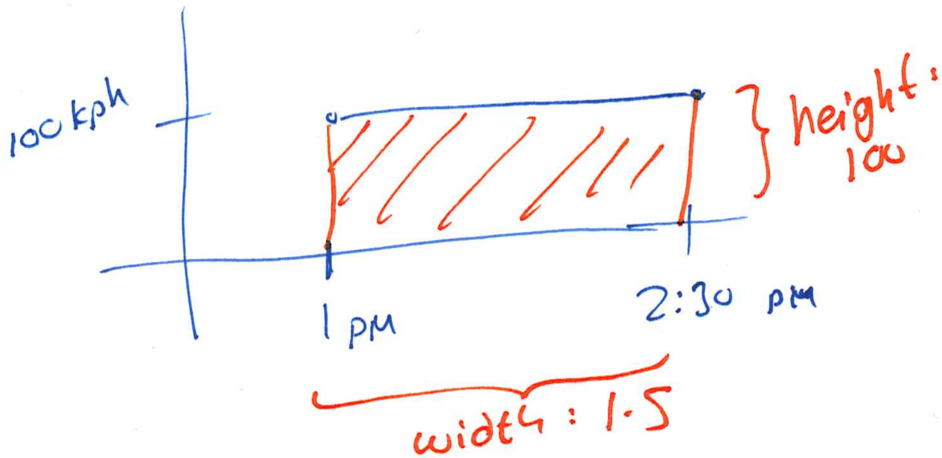
Consider:
$\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$, $\cancel{\left(-\sqrt{2}, \frac{1}{\sqrt{2}}\right)}$, $\cancel{\left(-\sqrt{2}, \frac{-1}{\sqrt{2}}\right)}$,
$\cancel{\left(\sqrt{2}, \frac{-1}{\sqrt{2}}\right)}$

Max area occurs when one corner of rectangle is at $(\sqrt{2}, \frac{1}{\sqrt{2}})$:

$$\text{Area} = 4(\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = \boxed{4}$$

Ch. 5.1 : Approximating Areas Under Curves

DISTANCE = RATE · TIME



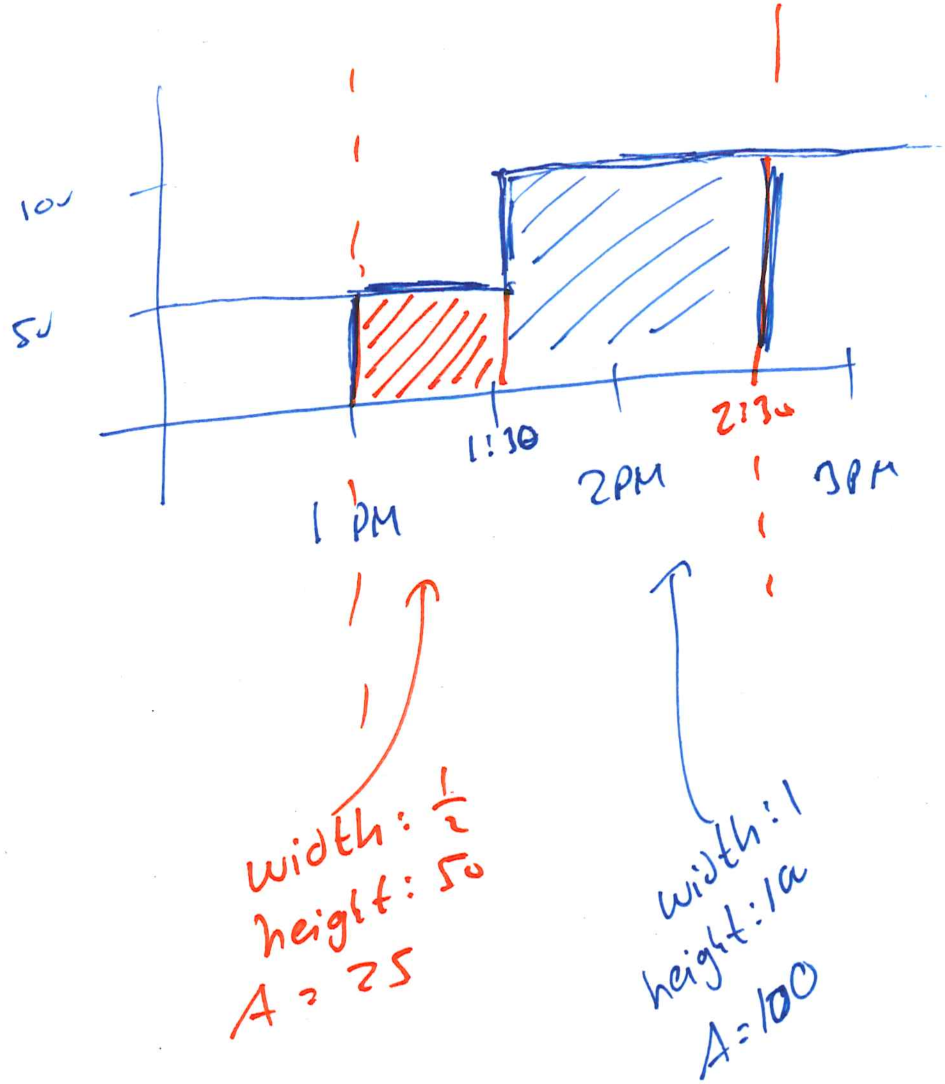
Area of : $(1.5)(100)$
$= 150$

Q: How far did car travel from 1 PM to 2:30 PM?

DIST = RATE · TIME

$= \left(100 \frac{km}{hr}\right)(1.5\ hr)$

$= 150\ km$



Area: 125

Q: How far did car travel from 1 PM to 2:30 PM?

From 1 to 1:30:

$(\frac{1}{2}\ hr)(50\ \frac{km}{hr}) = 25\ km$

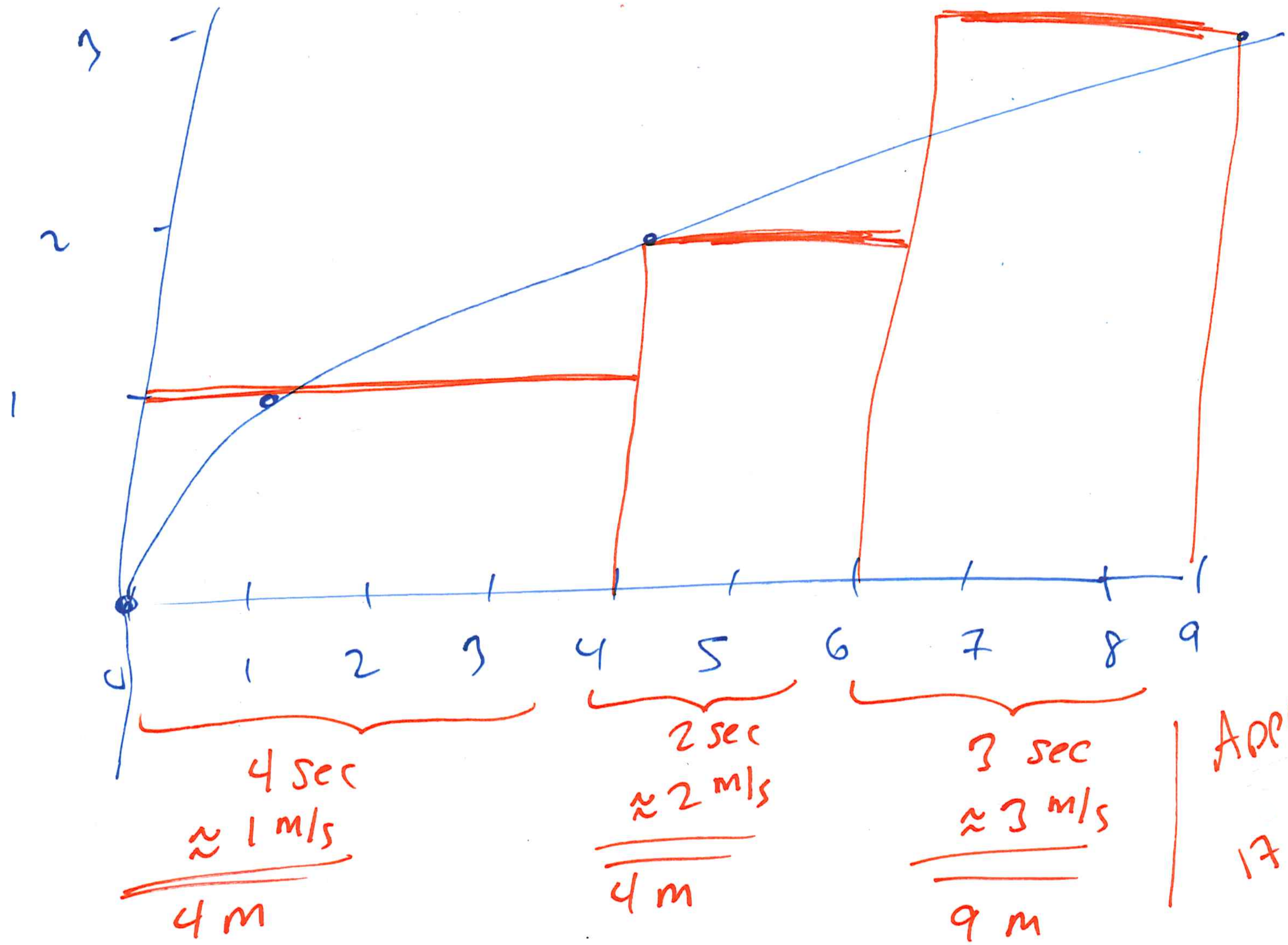
From 1:30 to 2:30:

$(1\ hr)(100\ \frac{km}{hr}) = 100\ km$

Total: 125 km

Speed @ time t :

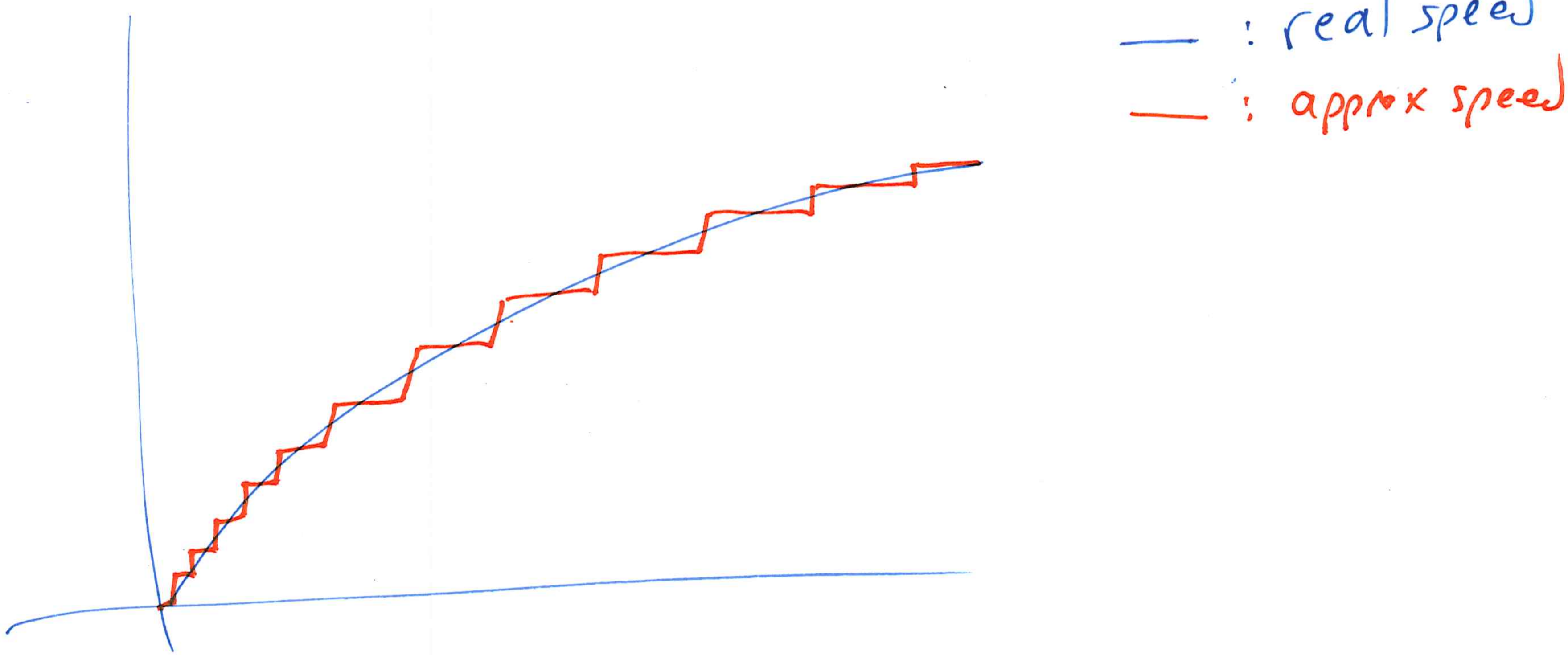
$s(t) = \sqrt{t} \ \frac{m}{s}$



Approx:

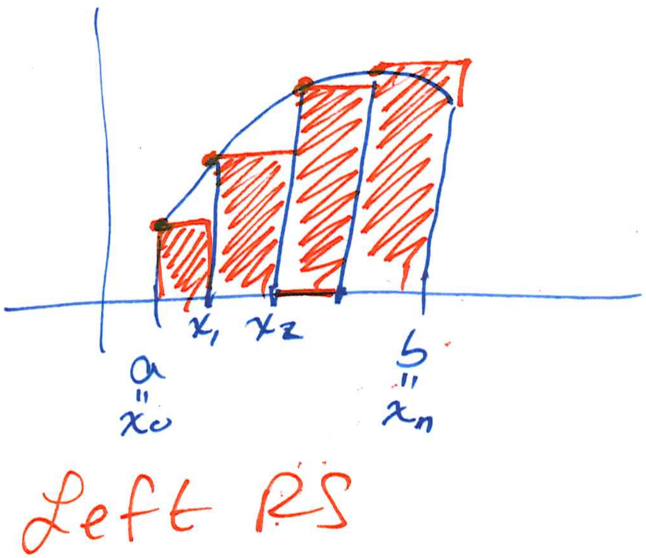
17 meters moved in first 9 sec

Better Approx:



Riemann Sum

Left / Right / Middle Riemann Sum:
how do you choose height?



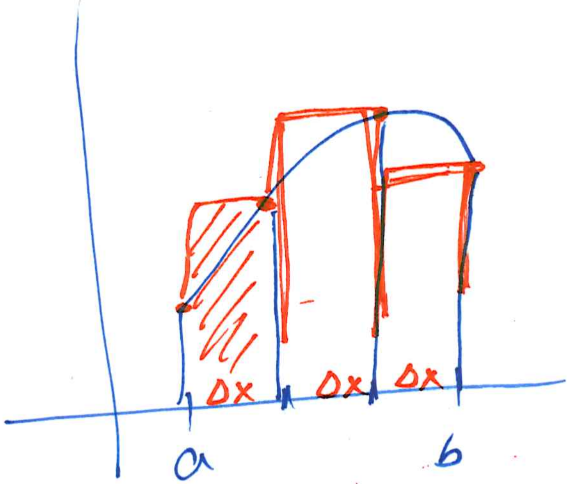
Each section:
Approx as rectangle

Grid points: $x_0, x_1, \dots, x_n$
(make cuts)

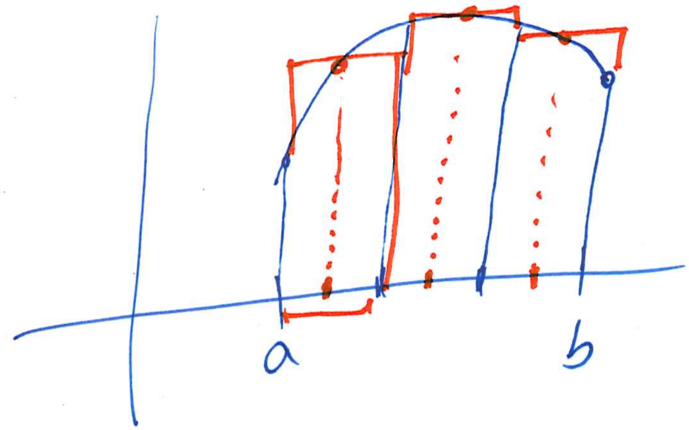
If grid points are all same distance from each other (same width □)

"regular partition"
width: Δx

If not same size: "general partition"



Right RS



Midpoint RS

Riemann Sum: Suppose f is defined on $[a, b]$, which is divided into n subintervals of equal length, Δx. If x_k^* is any point in the k^{th} interval, $[x_{k-1}, x_k]$ for $k = 1, \dots, n$, then

$$f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x$$

is called a Riemann Sum for f on $[a, b]$.

We call it a $\begin{pmatrix}\text{left} \\ \text{midpt} \\ \text{right}\end{pmatrix}$ R.S. if x_i^* is the $\begin{pmatrix}\text{left pt} \\ \text{midpt} \\ \text{right pt}\end{pmatrix}$ in $[x_{k-1}, x_k]$.