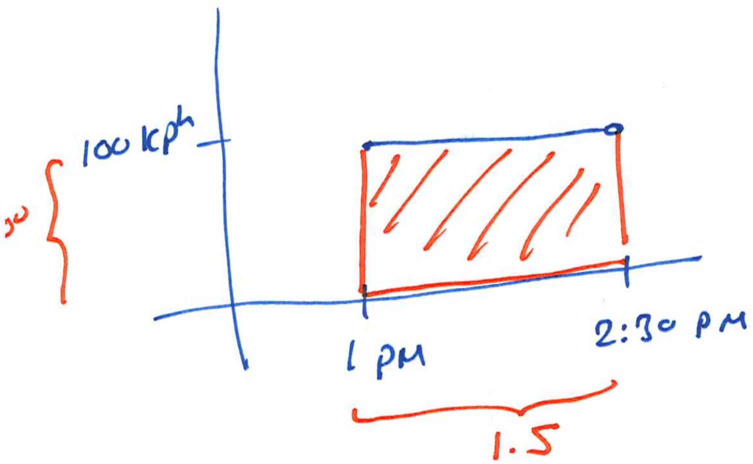
Ch. 5.1 : Approximating Area under Curves

DISTANCE = RATE × TIME

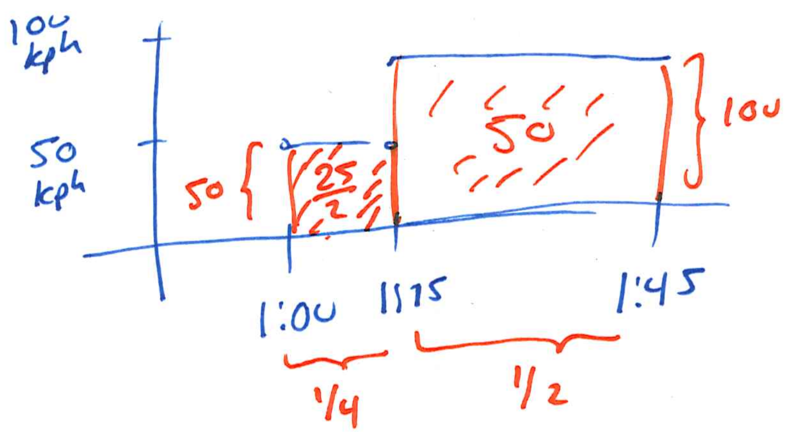


Dist travelled from 1 pm to 2:30 pm:

$(100 \frac{km}{hr})(1.5 \text{ hr})$

$= \boxed{150 \text{ km}}$

Area of rectangle under line



Dist travelled from 1 – 1:45 :

1–1:15 : $\frac{1}{4}$ hr, 50 kph

dist : $\frac{50}{4} = \frac{25}{2}$ km

1:15–1:45 : $\frac{1}{2}$ hr, 100 kph

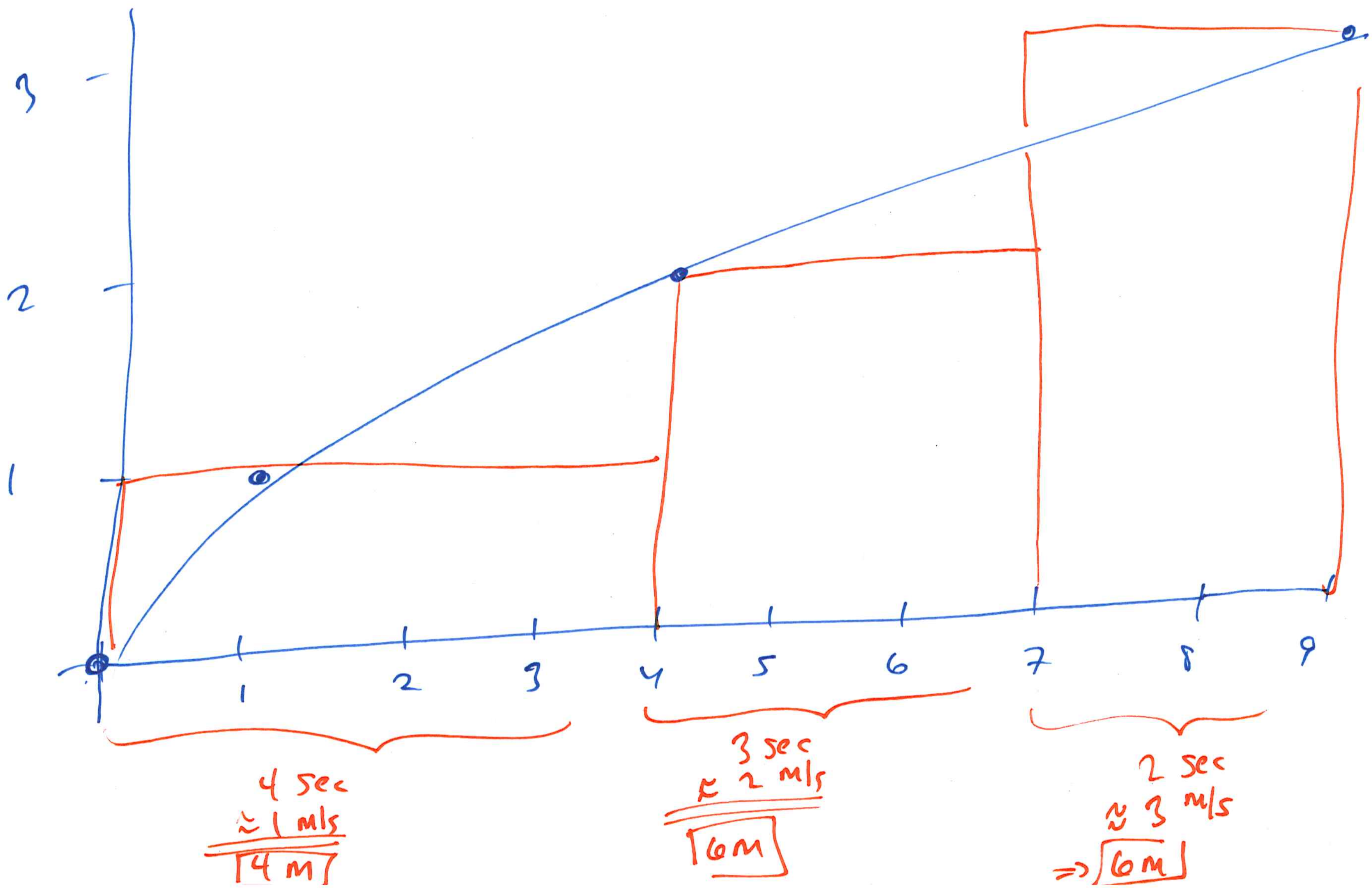
dist : $(\frac{1}{2})(100) = 50$ km

All together : $\boxed{50 + \frac{25}{2} \text{ km}}$

DISTANCE TRAVELLED: AREA UNDER LINE

At time t,
speed:
$s(t) = \sqrt{t}$ m/s

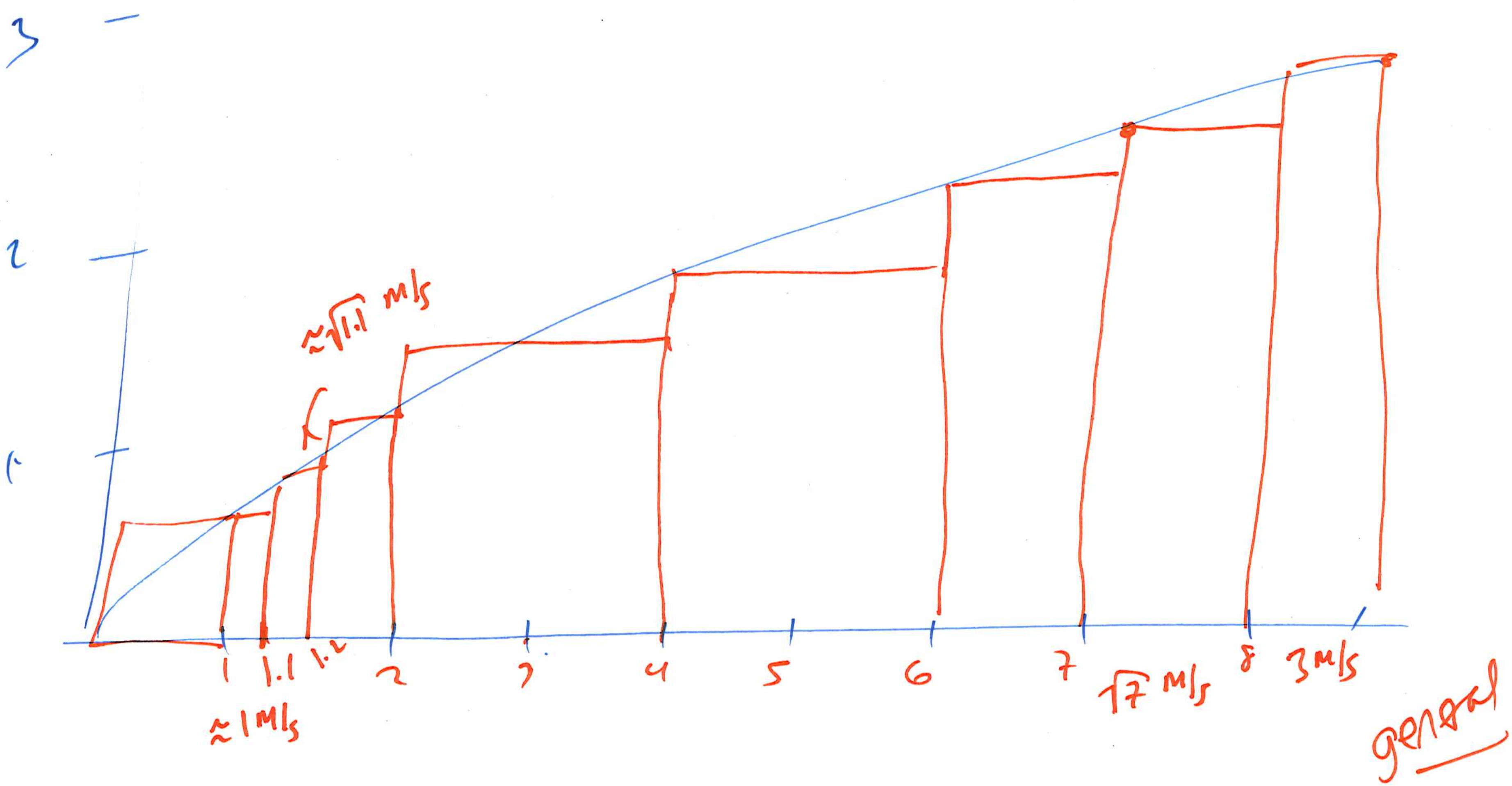
APPROX DIST TRAVELLED
≈ 16 m



Grid pointss 0, 1, 1.1, 1.2, 2, 4, 6, 7, 8, 9

Partition: Regular if cuts all same size
width of intervals: Δx

General if not same size

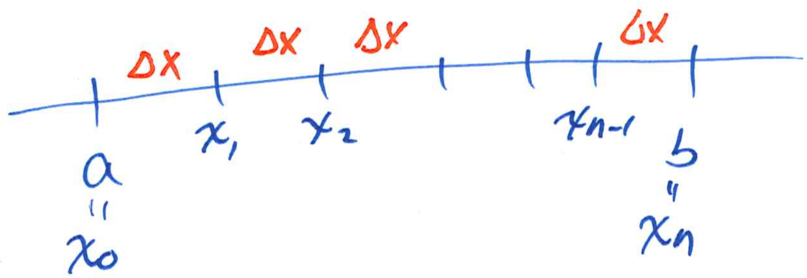


In a regular partition from a to b, with n intervals:

$$\Delta x = \frac{b-a}{n}$$

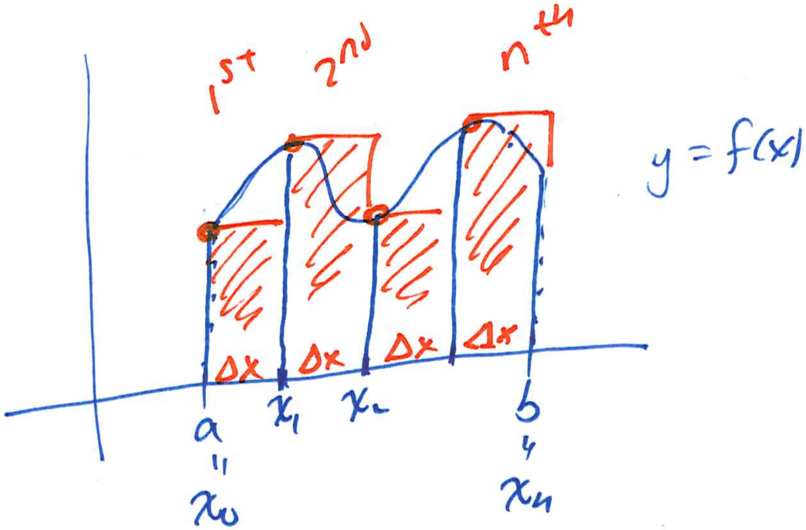
Grid points:

$$x_i = a + i\Delta x$$



Riemam Sums

Regular partitions



Left Riemann Sum

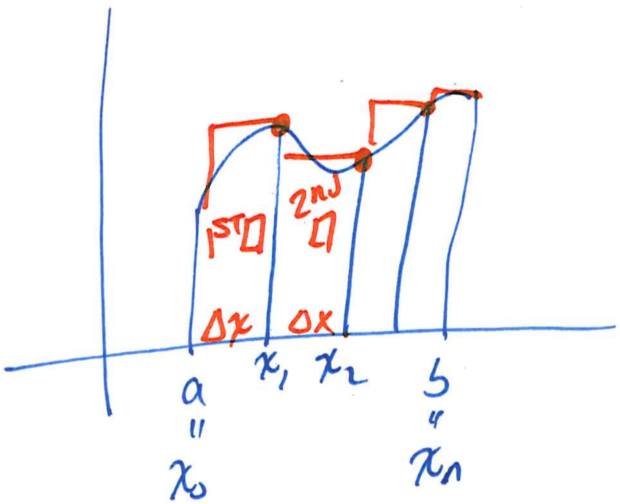
Area of i^{th} rectangle:

$= (\text{base})(\text{height})$

$= \Delta x \cdot f(x_{i-1})$

Approx of tt1 area:

$$\Delta x f(x_0) + \Delta x\, f(x_1) + \Delta x\, f(x_2) + \dots + \Delta x\, f(x_{n-1})$$



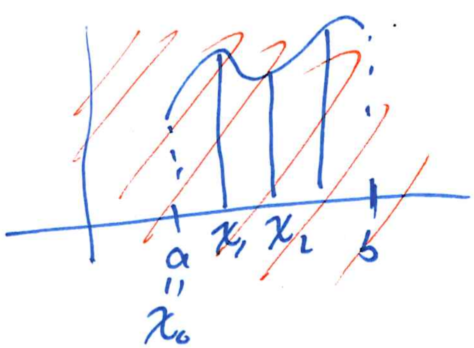
Right Rieman Sum

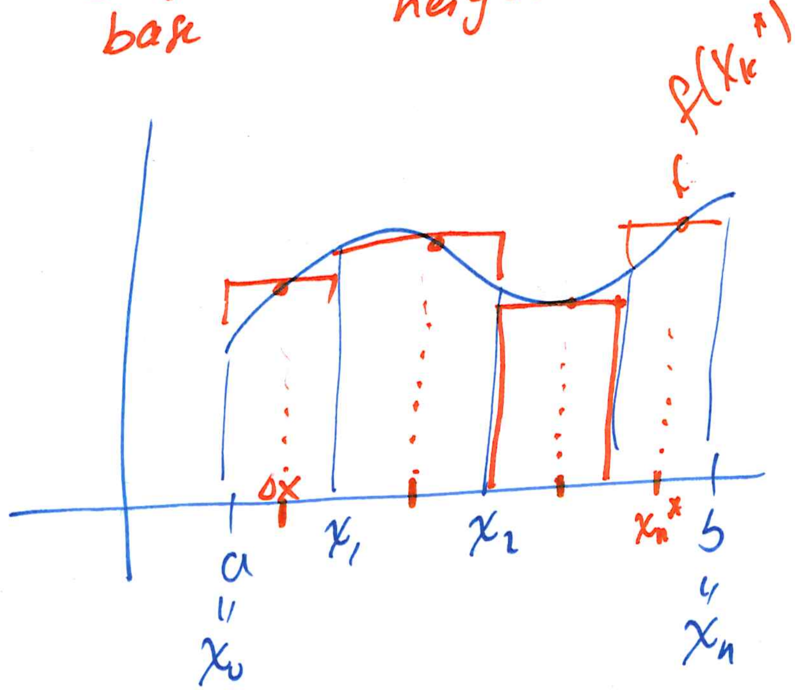
Area of i^{th} □ : $\underbrace{(\Delta x)}_{\text{base}} \cdot \big(\underbrace{f(x_i)}_{\text{height}}\big)$

Midpoint Riemann Sum

height of i^{th} rectangle: $f\left(\frac{x_{i-1}+x_i}{2}\right)$

base: ΔX





Riemann Sum

Suppose f is defined on a closed interval $[a,b]$, which is divided into n intervals of equal length, Δx.

If x_k^* is any point in the k^{th} interval $[x_{k-1}, x_k]$, for $k=1,\dots,n$, then

$$f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x$$

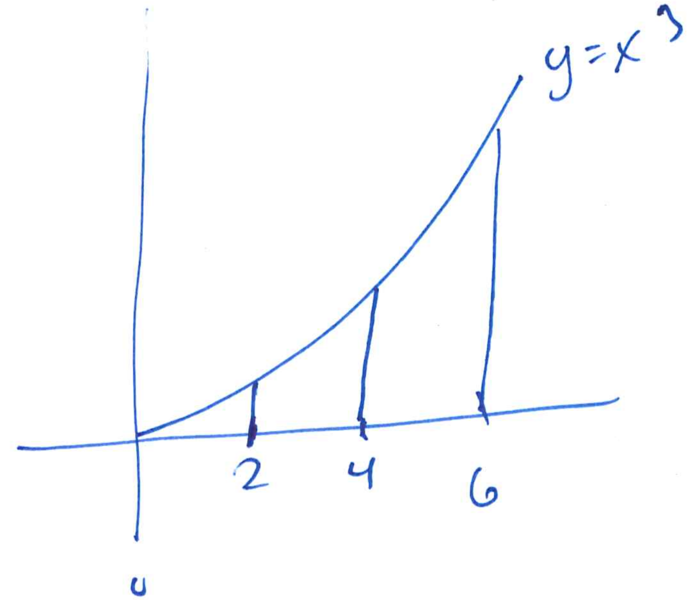
is called a Riemann Sum of f on $[a,b]$.

It's called a: $\left(\begin{array}{c}\text{left}\\ \text{right}\\ \text{midpoint}\end{array}\right)$ Riemann sum if

x_i^* is the (") of the interval $[x_{i-1}, x_i]$.

(ex) $y = x^3$

Approx area under curve, on $[0, 6]$, using 3 subintervals + Riemann Sum.



$n = 3$

$\Delta x = \frac{6-0}{3} = 2$

width of each partition

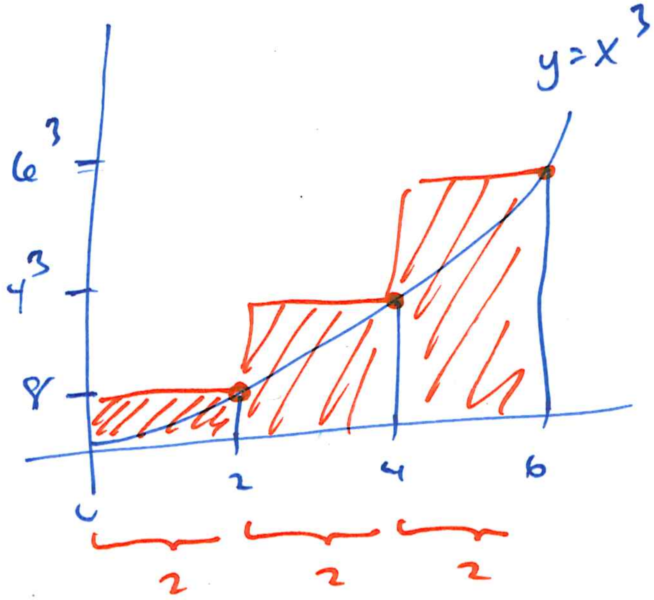
Grid points: 0, 2, 4, 6

Riemann Sum:

$\Delta x\, f(x_1^*) + \Delta x\, f(x_2^*) + \Delta x\, f(x_3^*)$

$= 2\, f(x_1^*) + 2\, f(x_2^*) + 2\, f(x_3^*)$

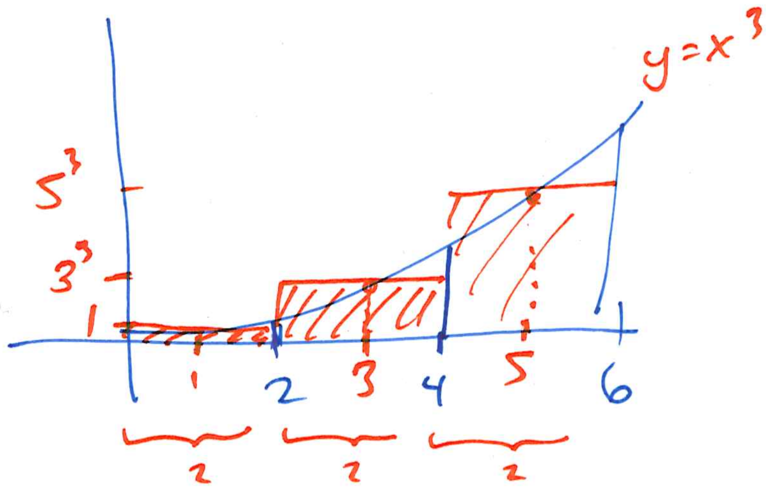
Right RS:



$$\boxed{2(8) + 2(4^3) + 2(6^3)}$$

Approximation of area under curve (Right RS)

Midpoint RS:



$$\boxed{(2)(1) + (2)(3^3) + (2)(5^3)}$$

Approx of area under curve: midpoint RS

ex Suppose a car's speed is :

time	12:00	12:15	12:30	12:45	1:00
speed	60 kph	80 kph	100 kph	100 kph	40 kph

How far did the car travel from 12:00 to 1:00 ?

$\Delta x = \frac{1}{4}$
$n=4$

Left RS:

$\left(\frac{1}{4}\right)(60) + \left(\frac{1}{4}\right)(80) + \left(\frac{1}{4}\right)(100) + \frac{1}{4}(100)$

Using Riemann Sum:
How many intervals? (n)
What is Δx ?
Write out sum
Left, Right, MP

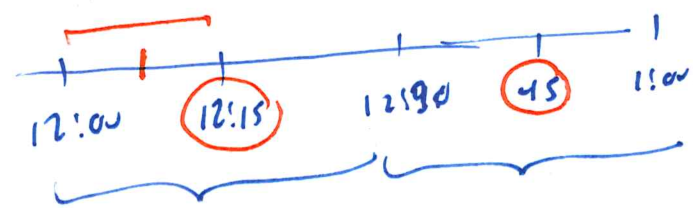
$\Delta x = \frac{1}{4}$
$n=4$

Right RS:

$\left(\frac{1}{4}\right)(80) + \frac{1}{4}(100) + \frac{1}{4}(100) + \frac{1}{4}(40)$

$n=2$
$\Delta x = \frac{1}{2}$

Midpoint RS:



$\frac{1}{2}(80) + \frac{1}{2}(100)$