Quiz 2

Monday, 23 January

Ch. 12.1 – 12.8

(no Lagrange)

Midterm 1

Wednesday, 1 February

Up to + including Ch. 12.9

More info + practice exam on common course webpage

Absolute Extrema

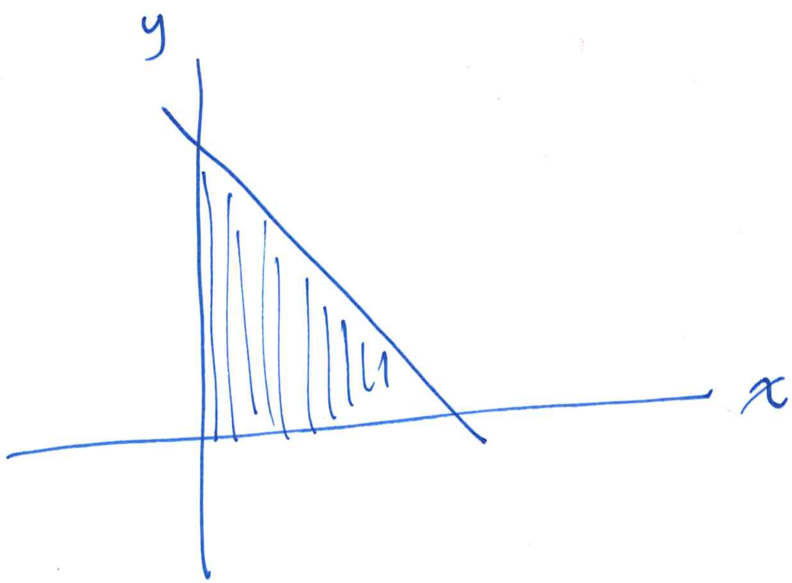
Find absolute max & min values of

$$f(x,y) = (xy)e^{-x-y}$$

over the region

$$R = \{ (x,y) : x \geq 0 , \; y \geq 0, \; x+y \leq 1 \}$$

collection of all thing — that look like this — and all these are true



Interior (CPs)

$$f_x = (xy)\cdot e^{-x-y}(-1) + (y)e^{-x-y} = ye^{-x-y}(1-x) = 0$$

$$f_y = (xy)e^{-x-y}(-1) + e^{-x-y}(x) = xe^{-x-y}(1-y) = 0$$

1st Equation: $y=0$ or $1-x=0$, $x=1$

2nd Equation: $x=0$ or $1-y=0$, $y=1$

if $y=0$, $x=0$ | **CP: (0,0)**

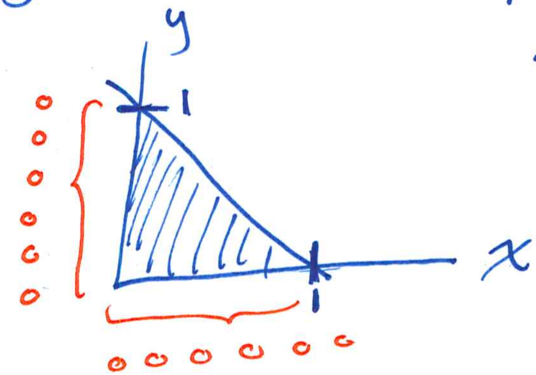
if $x=1$, $y=1$ | CP: (1,1) X NOT IN REGION

Boundary

$f(x,y) = (xy)e^{-x-y}$

$R = \{(x,y) : x \geq 0,\ y \geq 0,\ x+y \leq 1\}$

$x \leq 1-y$
$y \geq 0$
So:
$x \leq 1-0 = 1$



$x=0$

$f(0,y) = 0\,e^{-x-y} = 0$

$y=0$ $f(x,0) = 0$

$x+y=1$

$y = 1-x$

$0 \leq x \leq 1$

$f(x,y) = (xy)e^{-(x+y)}$ (with $x+y = 1$)

$= (xy)\cdot e^{-1}$

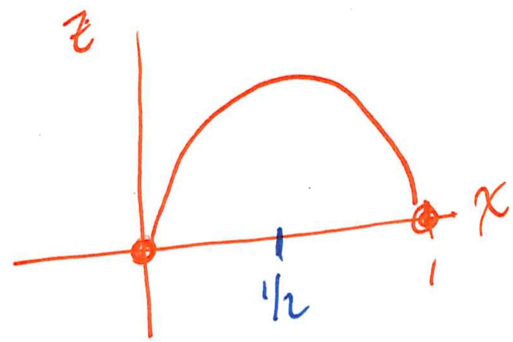
$= x(1-x)\cdot e^{-1}$

What are max/min values of

$g(x) = x(1-x)e^{-1}$

$= \frac{1}{e}x(1-x)$

when $0 \leq x \leq 1$?



MAX: $x = 1/2$

We're on bdry $x+y=1$, so if $x = 1/2$,

$y = 1/2$

$\frac{1}{e}\left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right) = \frac{1}{4e}$

$f(0,1) = 0$

$f(1,0) = 0$

$f\left(\frac{1}{2},\frac{1}{2}\right) = \frac{1}{4e}$

Compare

CP: $(0,0)$ $f(0,0) = 0$

Boundary: $f(x,y) = 0$ MIN when $x=0$ or $y=0$

$f(\frac{1}{2}, \frac{1}{2}) = \frac{1}{4e}$ MAX

Absolute max value: $\frac{1}{4e}$

Absolute min value: 0

Method of Lagrange Multipliers §12.9

Let the objective function f and the constraint function $g(x,y) = c$ (← constant) be differentiable, with $g_x(x,y)$ and $g_y(x,y)$ not both always zero.

To locate the maximum and minimum values of f subject to the constraint $g(x,y)=c$:

① Find the values of x, y, and λ that satisfy the equations:

$$\begin{cases} f_x(x,y) = \lambda\, g_x(x,y) \\ f_y(x,y) = \lambda\, g_y(x,y) \\ g(x,y) = c \end{cases}$$

② Among the values (x,y) from Step 1, select the largest & smallest corresponding function values. These are the maximum & minimum values of f subject to the constraint.

(ex) Find abs. max & min of

$$f(x,y) = xy + 14$$

given constraint

$$x^2 + y^2 = 18$$

$x^2 = 18 - y^2$

($x = \pm\sqrt{18 - y^2}$) to solve for x

... unpleasant

AVOID by using Lagrange

$f_x = y$ $\quad g_x = 2x$

$f_y = x$ $\quad g_y = 2y$

Solve:

$$\begin{cases} y = \lambda \cdot 2x \to & 2x = 0 \text{ OR } \lambda = \frac{y}{2x} \\ x = \lambda \cdot 2y \to & 2y = 0 \text{ OR } \lambda = \frac{x}{2y} \\ x^2 + y^2 = 18 \end{cases}$$

MAYBE: $2x = 0$ so $x = 0$ $\quad$ 1ST Eqn: $y = 0$ $\quad$ pt: $(0,0)$ ✗ 3rd Eqn FALSE

MAYBE: $2y = 0$, $y = 0$ $\quad$ 2nd Eqn: $x = 0$ $\quad$ pt $(0,0)$ ✗ " " "

SO: $\frac{y}{2x} = \lambda = \frac{x}{2y}$ (1ST) (2nd) $\to$ $2y^2 = 2x^2$; so $y^2 = x^2$

3rd Equation:
$x^2 + x^2 = 18$
$2x^2 = 18$

$x^2 = 9$
$x = \pm 3$

$y^2 = 9$
$y = \pm 3$

Points to Consider:

$f(3, 3) = 9+14 = 23$
$f(-3,-3) = 9+14 = 23$ } highest pts (abs maximal)

$f(-3, 3) = -9+14 = 5$
$f(3,-3) = -9+14 = 5$ } lowest pts (abs minimal)