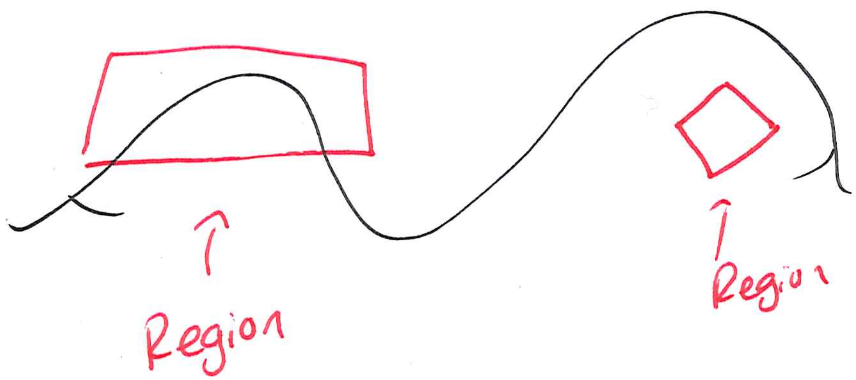
Absolute Maxima / Minima Over a closed Region



Check: - CPs
- Boundary

Last Semester (abs max/min) of $f(x)$ when $-2 \le x \le 5$:
- check CPs
- check endpoints

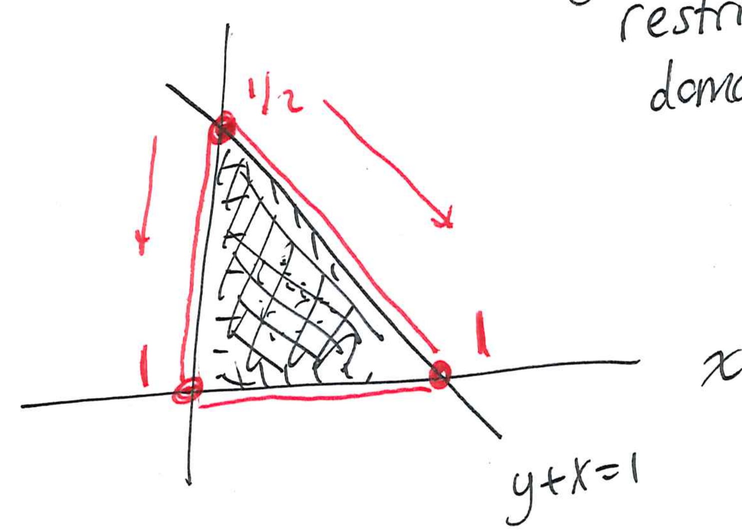
(ex) Want to find absolute extrema of

max/min

$$f(x,y) = \frac{x+1}{x+y+1}$$

in the region:

- $x+y \leq 1$
- $x \geq 0$
- $y \geq 0$



Region: restricted domain

- Check CPs

$$f(x,y) = \frac{x+1}{x+y+1}$$

$$f_x = \frac{(x+y+1)(1) - (x+1)(1)}{(x+y+1)^2} = \frac{y}{(x+y+1)^2}$$

$$f_y = \frac{(x+y+1)(0) - (x+1)(\cancel{1})}{(x+y+1)^2} = -\frac{(x+1)}{(x+y+1)^2}$$

Inside region, f_x & f_y exist everywhere

$$\begin{cases} 0 = \dfrac{y}{(x+y+1)^2} \longrightarrow y = 0 \\ 0 = -\dfrac{(x+1)}{(x+y+1)^2} \longrightarrow 0 = (x+1) \to x = -1 \end{cases}$$

$0 = -(x+1)$

CP: $(-1, 0)$. NOT IN REGION

$\boxed{\text{Boundaries}}$

$\boxed{x+y=1}$

$0 \le x \le 1$

$$f(x,y) = \frac{x+1}{\boxed{x+y}+1} = \frac{x+1}{1+1} = \frac{x+1}{2}$$

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$$f(0,1) = \frac{0+1}{2} = \frac{1}{2}$$

$$f(1,0) = \frac{1+1}{2} = \frac{2}{2} = 1$$

$\boxed{x=0}$

$0 \le y \le 1$

$$f(0,y) = \frac{0+1}{0+y+1} = \frac{1}{y+1}$$

$$f(0,0) = \frac{1}{0+1} = 1$$

$$f(0,1) = \frac{1}{1+1} = \frac{1}{2}$$

$\boxed{y=0}$

$0 \le x \le 1$

$$f(x,y) = f(x,0) = \frac{x+1}{x+0+1} = \frac{x+1}{x+1} = 1$$

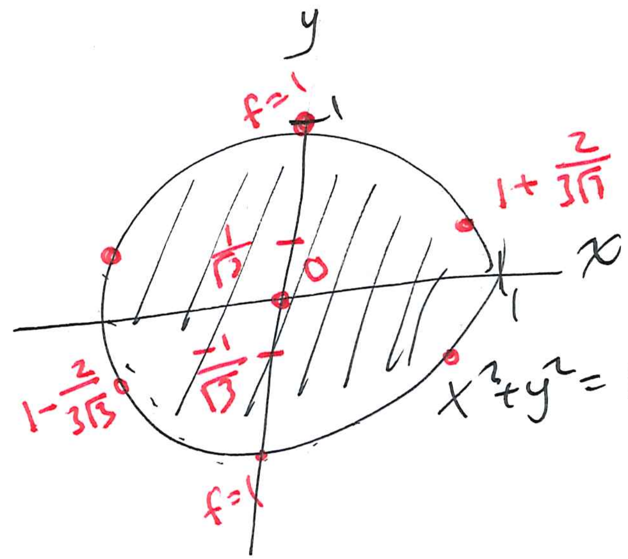
$\boxed{\text{COMPARE}}$ (no CPs)

ABS MIN: $f(0,1) = \underline{1/2}$

ABS MAX: $\underline{1}$, when $y=0$

ex $f(x,y) = x^2 + x^2 y + y^2$

Find abs. max/min when $x^2+y^2 \le 1$



$\boxed{\text{CPs}}$ $\begin{cases} f_x = 2x + 2xy = 0 \\ f_y = x^2 + 2y = 0 \end{cases}$

$\begin{cases} 2x(1+y) = 0 \longrightarrow x=0 \text{ or } y=-1 \\ x^2+2y=0 \end{cases}$

If $x=0$ and $x^2+2y=0$

$\Rightarrow 0+2y=0$

$\Rightarrow y=0$

CP: $(0,0)$

If $y=-1$ and $x^2+2y=0$

$\Rightarrow x^2-2=0$

$\Rightarrow x^2=2$

$\Rightarrow x=\pm\sqrt{2}$

$\cancel{(\sqrt{2},-1)}$
$\cancel{(-\sqrt{2},-1)}$ not in region

$f(0,0)=0$

$\boxed{\text{Boundary}}$

$\boxed{x^2+y^2=1}$

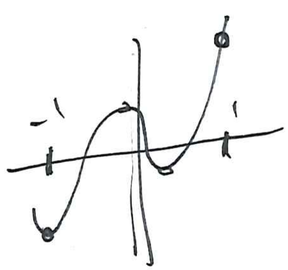
$x^2=1-y^2$

$-1 \le y \le 1$

$$f(x,y) = \underbrace{x^2}+x^2y+\underbrace{y^2}$$
$$= 1 + x^2y$$
$$= 1 + (1-y^2)y$$
$$= 1 + y - y^3$$

Question: When $-1 \le y \le 1$, what are max / min values of

$$g(y) = 1+y-y^3 \quad ?$$
$$= 1+y(1-y^2)$$



(last ~~semest~~ semester)

$$g'(y) = 1-3y^2 = 0$$
$$1 = 3y^2$$
$$\frac{1}{3} = y^2$$
$$y = \pm\frac{1}{\sqrt{3}} \rightarrow \text{CPs}$$

$$g(-1) = 1-1-(-1) = 1$$
$$g(1) = 1+1-1 = 1$$
$$g\left(\frac{1}{\sqrt{3}}\right) = 1+\frac{1}{\sqrt{3}}\left(1-\frac{1}{3}\right) = 1+\frac{2}{3}\cdot\frac{1}{\sqrt{3}} > 1$$
$$g\left(\frac{-1}{\sqrt{3}}\right) = 1-\frac{1}{\sqrt{3}}\left(1-\frac{1}{3}\right) = 1-\frac{2}{3}\cdot\frac{1}{\sqrt{3}} < 1$$

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Boundary: MAX: $1+\frac{2}{3\sqrt{3}}$ when $y = 1/\sqrt{3}$

MIN: $1-\frac{2}{3\sqrt{3}}$

$\boxed{\text{COMPARE}}$

(CP) $f(0,0) = 0$ } ABS MIN

(Boundary) $f\left(\pm\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right) = 1 + \frac{2}{3\sqrt{3}}$ } ABS MAX

$f\left(\pm\sqrt{\frac{2}{3}}, \frac{-1}{\sqrt{3}}\right) = 1 - \frac{2}{3\sqrt{3}}$

Bdy: $x^2 = 1 - y^2 = 1 - \frac{1}{3} = 2/3$