Partial Derivatives

ex $f(x,y) = 2xy^2 - (5x+3y)^3$

$\underbrace{f_x(x,y)}_{\text{partial deriv. with respect to } x} = 2y^2 - 3(5x+3y)^2 \cdot 5$

$= 2y^2 - 15(5x+3y)^2$

$\frac{\partial f}{\partial x}$ x-variable
y-constant

$f_y(x,y) = (2x) \cdot 2y - 3(5x+3y)^2 \cdot 3$

$= 4xy - 9(5x+3y)^2$

ex $z\sin x + y\sin z = 0$ (Implicit diff)

const

x - var

y - const

z - fcn of x

Partial deriv with resp to x:

const

$z \cdot \cos x + (\sin x) z_x + y(\cos z) \cdot z_x = 0$

partial deriv of z w/ resp to x

$2\sin(3x+1)$ d/dx

$\underbrace{2}_{y}\cos(\underbrace{3x+1}_{z}) \cdot 3$

Recall (Math 104)

$x\sin y = 0$

differentiate: $x(\cos y) \cdot y' + \sin y \cdot (1) = 0$

Recall: deriv $\frac{\Delta \text{ output}}{\Delta \text{ input}}$

(ex) $G(W, C)$

G: cost of garment ($)

W: cost of wool ($/lb)

C: cost of cotton ($/lb)

G_W: partial deriv of G with respect to W

$$\lim_{h \to 0} \frac{G(W+h, C) - G(W, C)}{h}$$

$$\frac{\Delta G}{\Delta W}$$

If $\Delta W = 1$

then $G_W \approx \frac{\Delta G}{\Delta W} = \Delta G$

So: G_W is amount of change in final cost per $1/lb change in wool

Suppose $G_W = 2$

Suppose also: wool increases \$3 / lb.

$$2 = G_W \approx \frac{\Delta G}{\Delta W} = \frac{\Delta G}{3}$$

So: $\Delta G = 6$] Garment increased by \$6

Higher-Order Partial Derivs

$$f(x,y) = x \sin y$$

$f_x = \sin y$

$f_{xx} = 0$

$f_{xy} = \cos y$

$f_y = x \cos y$

$f_{yx} = \cos y$

$f_{yy} = -x \sin y$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} \quad \Bigg\| \quad \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

Newton: 1^{st} x, Then y:

f fcn

f_x 1^{st} deriv

$(f_x)_y$

we write: 2^{nd} deriv

f_{xy}

1^{st} 2^{nd}

Clairaut's Theorem

all (x,y) in domain

Assume f is defined on $\mathbb{R}^2$ and that f_{xy} & f_{yx} are continuous on $\mathbb{R}^2$.

Then $f_{xy} = f_{yx}$.

ex Is it possible to have a function f defined everywhere and:

$f_x = 3x$ and $f_y = 3x$?

$f_{xy} = 0$ $f_{yx} = 3$

NOT POSSIBLE

Recall: A fcn is continuous

if its limit equals its value at all pts

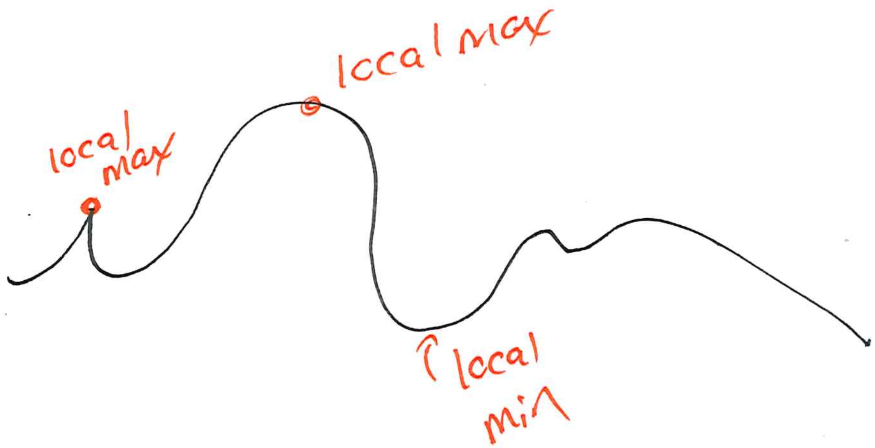
In $\mathbb{R}^2$ $(x y)$: $\lim\limits_{x \to a} f(x) = f(a)$

In $\mathbb{R}^3$ $(x y z)$: $\lim\limits_{(x,y) \to (a,b)} f(x,y) = f(a,b)$

Nice functions (polynomials, trig, exponential(s))
are continuous over their domains

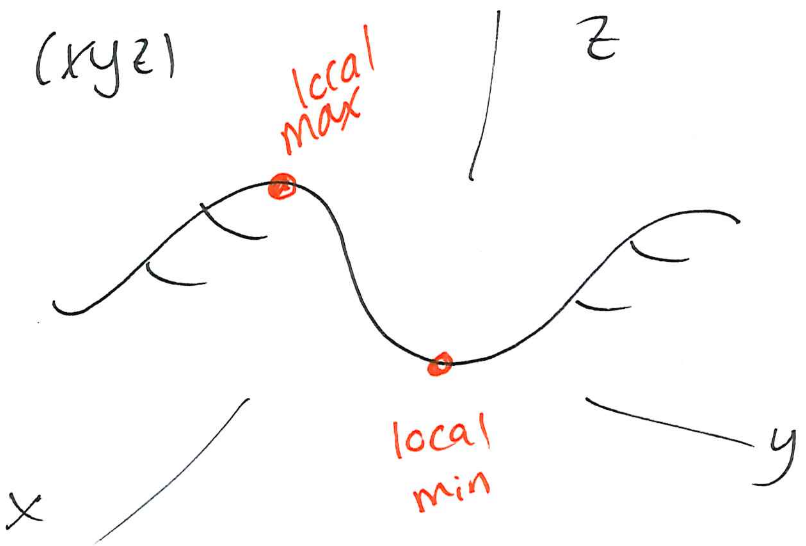
Ch 12.8: Max/Min Problems (local extrema)

Last Semester:



Local extrema occur when $f'(a) = 0$ or $f'(a)$ DNE

$\mathbb{R}^3$ (xyz)



Thm)

If a function f has a local max or min at (a,b) then:

- Maybe $f_x(a,b)$ DNE
- Maybe $f_y(a,b)$ DNE
- Otherwise, $f_x(a,b) = f_y(a,b) = 0$

If $f_x(a,b) = 0$ and $f_y(a,b) = 0$ but (a,b) is not a local max or min, we call (a,b) a saddle point.