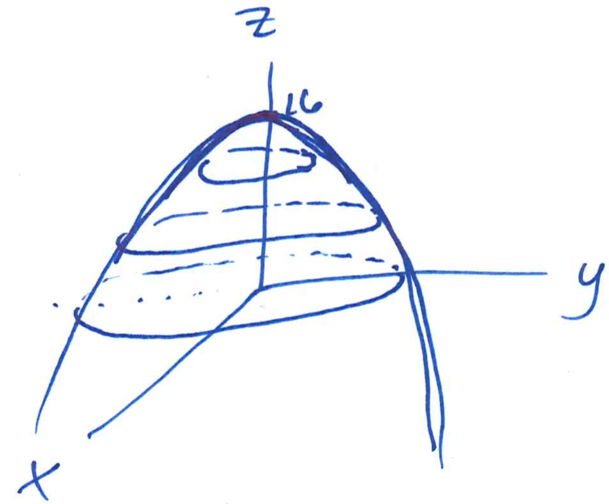
ex Sketch: $z = 16 - 4x^2 - y^2$



what if $z = 0$?

$0 = 16 - 4x^2 - y^2$

$16 = 4x^2 + y^2$

ellipse

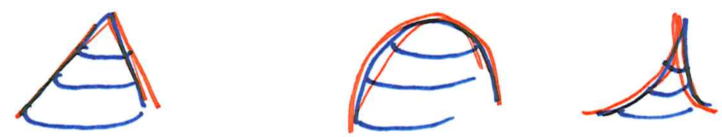
what if $z = 2$?

$2 = 16 - 4x^2 - y^2$

$14 = 4x^2 + y^2$

ellipse

MAYBE. MAYBE? MAYBE?



what if $z = 4$?

$4 = 16 - 4x^2 - y^2$

$12 = 4x^2 - y^2$

what if $x = 0$?

$z = 16 - y^2$ } parabola

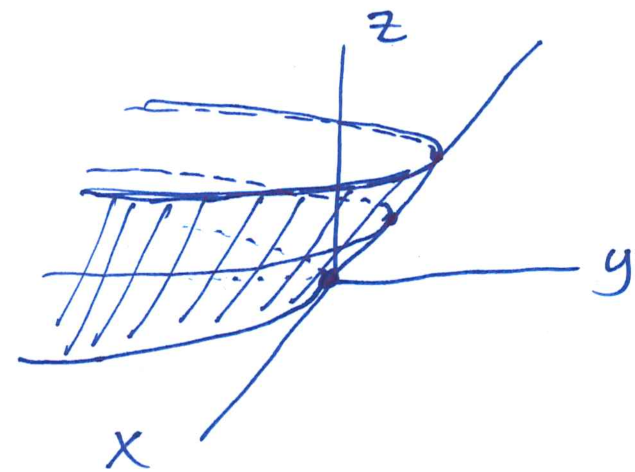
A *trace* of a surface is its intersection with a plane parallel to a coordinate plane

If z = constant: xy - trace $\}$ level curves

If y = const: xz - trace

If x = const: yz - trace

(ex) $y = z - x^2$



Level Curves:

$\underline{z=0}$ $y = -x^2$

$\underline{z=1}$ $y = 1 - x^2$

$\underline{z=2}$ $y = 2 - x^2$

yz-trace:

$\underline{x=0}$ $y = z$

Shapes to Remember

circle: $x^2 + y^2 = c$

ellipse: $ax^2 + by^2 = c$

parabola: $y = a \pm bx^2$

line: $c = x \pm y$

point: $0 = x^2 + y^2$

examples: p. 869

(ignore last one)

12.2 Graphs & Level Curves

$\underbrace{f(x,y)}_{\text{output}}$ function of $\underbrace{x \text{ \& } y}_{\text{inputs}}$

output "dependent variable"

inputs "independent variables"

any input (x,y) (in domain) should lead to only one output $f(x,y)$

e.g. : $z = x+y$ $\quad z = f(x,y)$: ✓

$z^2 = x+y$ ← NOT a fcn of x,y

$\left.\begin{matrix} x=1 \\ y=1 \end{matrix}\right\}$ $z^2 = 2$ → $z = \sqrt{2}$, $z = -\sqrt{2}$

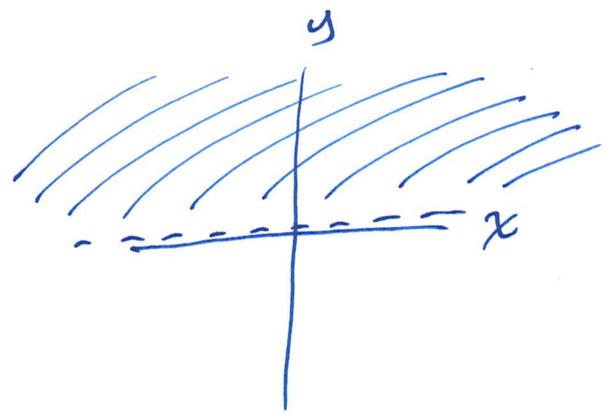
ex $f(x,y) = \sin\left(\frac{x}{\sqrt{y}}\right)$

eg: $f(1,2) = \underbrace{\sin\left(\frac{1}{\sqrt{2}}\right)}_{\text{some number}}$

DOMAIN: b/c $\sqrt{y}$: $y \geq 0$

b/c $\frac{1}{\sqrt{y}}$: $y \neq 0$

Domain: $\boxed{\begin{array}{l} x: \text{any number} \\ y > 0 \end{array}}$



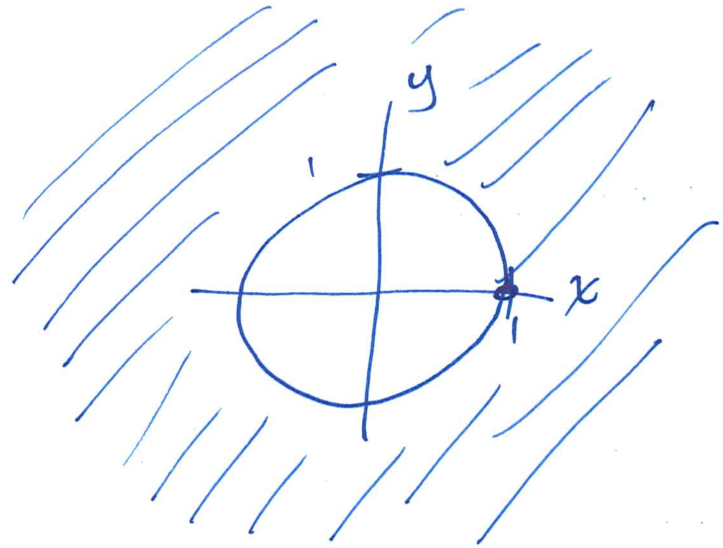
RANGE: $[-1, 1]$

(z values; $-1 \leq \underbrace{f(x,y)} \leq 1$)

ex $f(x,y) = \sqrt{y^2+x^2-1}$

DOMAIN: $y^2+x^2-1 \geq 0$

$y^2+x^2 \geq 1$



RANGE: $[0, \infty)$

z-values

$z = f(x,y)$

output values

Ch. 12.4 : Partial Derivatives

Remember : $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

} Δy, change in output

} Δx, change in input

Partial deriv with respect to x:

$$\underbrace{f_x(x,y)}_{\text{slope when we face in the } x\text{-direction}} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

} change in output

} change in x

Partial Deriv w. respect to y:

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

$\frac{\text{change in output}}{\text{change in } y}$

Leibnitz notation: $f_x(x,y) = \frac{\partial f}{\partial x}(x,y)$

"partial"

$$f_y = \frac{\partial f}{\partial y}$$

ex $f(x,y) = 3xy^2 - 15x + \ln y$

$f(x,y) = \underbrace{(3y^2)}_{\text{"const"}}x - 15x + (\ln y)$

$$\boxed{f_x = 3y^2 - 15}$$

$f_y = (3x)\cdot 2y + \frac{1}{y}$

$$\boxed{f_y = 6xy + 1/y}$$