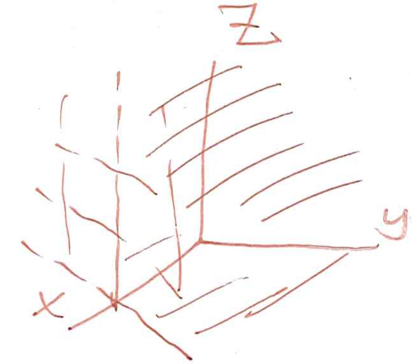
A <u>level curve</u> of a surface is the intersection of the surface with a plane parallel to thy xy-plane.

We find these by setting z equal to a constant

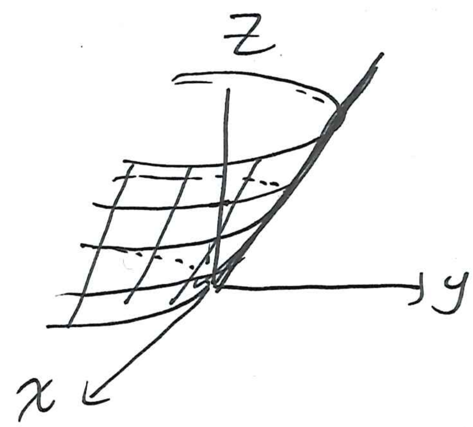
A <u>trace</u> of a surface is the intersection of the surface with a plane that is parallel to one of the coordinate planes.

$\boxed{z \to \text{constant}}$	xy-trace	(level curve)
$\boxed{y \to \text{constant}}$	xz-trace	
$\boxed{x \to \text{constant}}$	yz-trace	



Last time: $z = 16 - 4x^2 - y^2$

ex Graph $z = x^2 + y$



Level Curves (z = constant)

If $z = 0$: $0 = x^2 + y \rightarrow y = -x^2$
(parabola)

If $z = 2$: $2 = x^2 + y \rightarrow y = 2 - x^2$

If $z = 4$: $4 = x^2 + y \rightarrow y = 4 - x^2$

yz-trace
If $x = 0$: $z = y$

Equations To Know in $\mathbb{R}^2$ (xy-plane)

$x^2 + y^2 = c$ circle
$ax^2 + by^2 = c$ ellipse
$ax^2 + by = c$ parabola
$ax + by = c$ line

Another one
(not in syllabus)
$x^2 - y^2 = c$: hyperbola

p 869: lots of examples
(ignore last one)

A function $\overset{\text{dependent}}{z} = f(x,y)$ assigns a real value (output)

to each input $\underbrace{(x,y)}_{\text{independent variables}}$ from some domain.

ex: $z = x + y$ only one z-value for a pair (x,y)

function

ex: $z^2 = x + y$

NOT A FUNCTION

ex: $\left.\begin{matrix} x = 1 \\ y = 0 \end{matrix}\right\} \; x + y = 1 \; \left\} \begin{matrix} z = 1 \\ z = -1 \end{matrix}\right.$

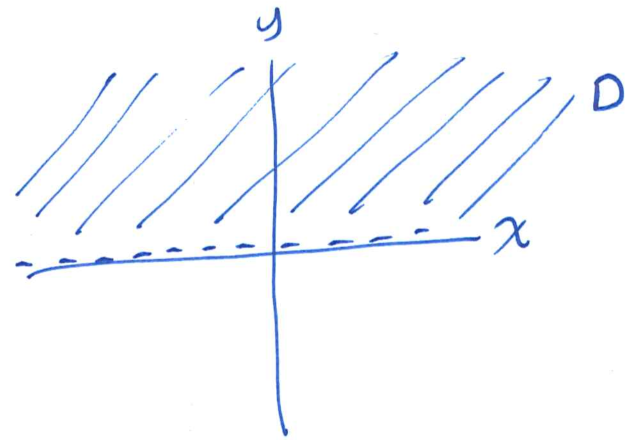
(ex) $f(x,y) = \sin\left(\frac{x}{\sqrt{y}}\right)$

DOMAIN:

Since we have $\sqrt{y}$: $y \geq 0$

Since $\frac{1}{\sqrt{y}} \leftarrow \sqrt{y} \neq y$ $\quad y \neq 0$

$$\begin{cases} y > 0 \\ x \text{ any real number} \end{cases}$$



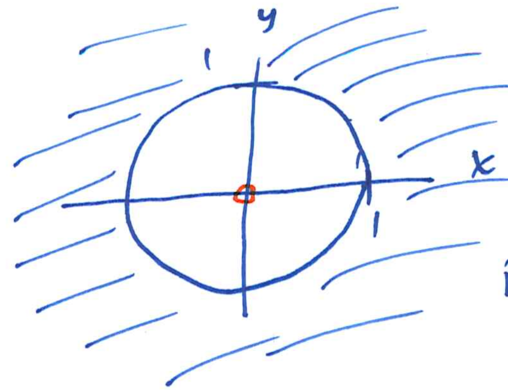
RANGE: $[-1, 1]$

Usually, range of sine is $[-1, 1]$

$f(x,y) = \sqrt{y^2+x^2-1}$

DOMAIN: $y^2+x^2-1 \geq 0$

$y^2+x^2 \geq 1$

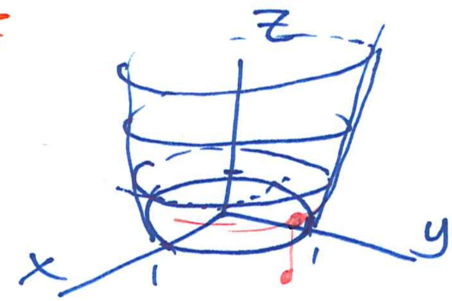


DOMAIN

example: $(0,0)$

$x=0 \quad y=0$

$\sqrt{0^2+0^2-1} = \sqrt{-1}$ DNE



RANGE: $[0, \infty)$

GRAPH: $z = \sqrt{y^2+x^2-1}$

Level Curves:

If $z=0$: $0 = \sqrt{y^2+x^2-1}$

$0 = y^2+x^2-1$

$1 = y^2+x^2$

If $z=1$: $1 = \sqrt{y^2+x^2-1}$

$1 = y^2+x^2-1$

$2 = y^2+x^2$

$z=2$: $2 = \sqrt{y^2+x^2-1}$

$4 = y^2+x^2-1$

$5 = y^2+x^2$

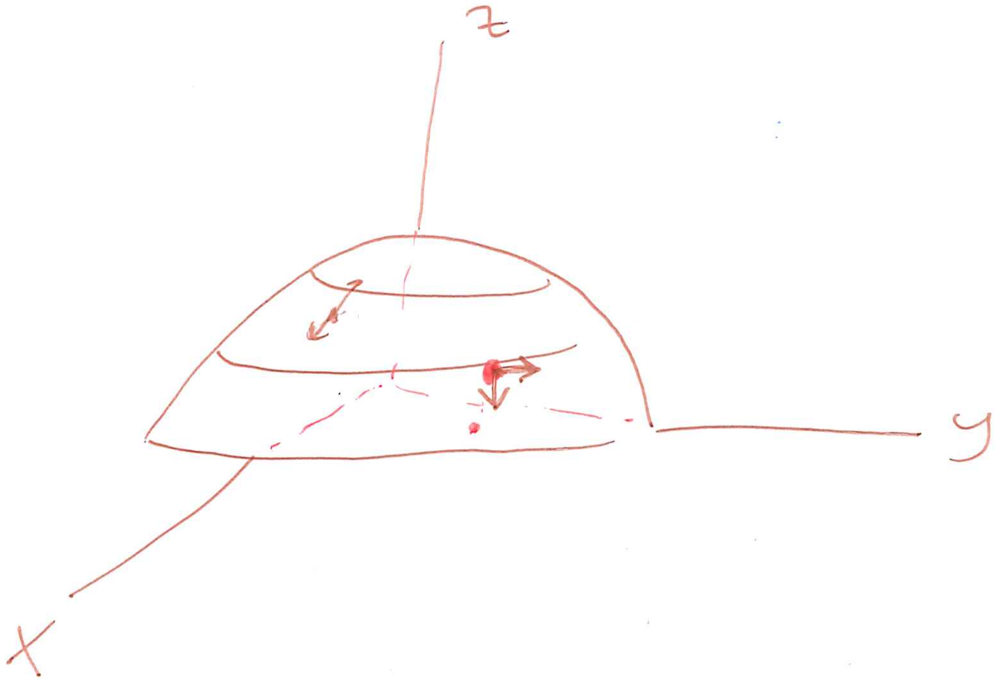
Recall:

$f(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$\frac{\text{change in output}}{\text{change in input}}$$



If we change x, but keep y same:

$$\frac{\text{change output}}{\text{change input}} : \frac{f(x+h, y) - f(x,y)}{h}$$

$$\boxed{\underbrace{f_x(x,y)}_{\text{partial derivative of } f \text{ with respect to } x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}}$$

Similarly, if we change y (not x):

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

Evaluate Partial Derivatives:

$$f(x,y) = 3xy^2 - 15x + \ln y$$

$f_x(x,y)$: treat y like constant

(also write: $\frac{\partial f}{\partial x}$)

$$f(x,y) = (3y^2)x - 15x + \ln y$$

$$f_x(x,y) = 3y^2 - 15$$

$$f(x,y) = (3x)y^2 - 15x + \ln y$$

$$f_y(x,y) = (3x)\cdot 2y + \frac{1}{y}$$

$$= 6xy + 1/y$$

(also write: $\frac{\partial f}{\partial y}$)

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ex $f(x,y) = 2xy^2 - (5x+3y)^3$

$$f_x(x,y) = 2y^2 - 3(5x+3y)^2 \cdot 5$$

$$= \boxed{2y^2 - 15(5x+3y)^2}$$