Last Time: Given a fixed point $P_0 = (p_1, p_2, p_3)$ in space, and a normal vector $\vec{n} = \langle n_1, n_2, n_3 \rangle$, the plane P that passes through P_0 and has normal vector $\vec{n}$ consist of:

All points $Q = (x y z)$ such that $\vec{PQ}$ is orthogonal to $\vec{n}$.

An equation of P is:

$$\boxed{n_1 x + n_2 y + n_3 z = n_1 p_1 + n_2 p_2 + n_3 p_3}$$

example: If P is a plane with normal vector $\langle -2, 4, 3 \rangle$ that passes through the point $(1, 2, 1)$, then an equation of P is:

$$-2x + 4y + 3z = -2 + 8 + 3$$

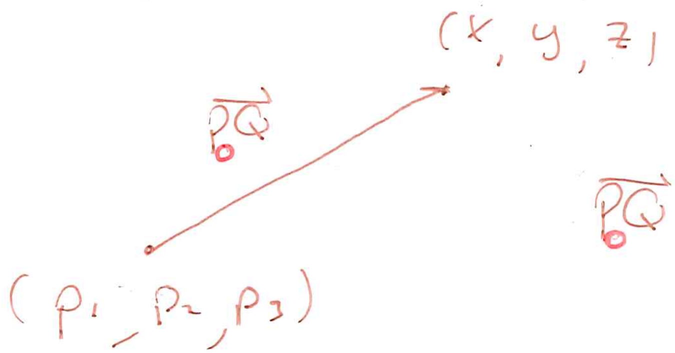
$$\boxed{-2x + 4y + 3z = 9}$$

Why is that the equation of a plane?

If $Q = (x, y, z)$ is a point on my plane

(plane has normal vector $\langle n_1, n_2, n_3 \rangle$
& has point $P_0 = (p_1, p_2, p_3)$)

Then: $\overrightarrow{P_0Q}$ perpendicular (orthogonal) to $\vec{n} = \langle n_1, n_2, n_3 \rangle$



$\overrightarrow{P_0Q} = \langle (x-p_1), (y-p_2), (z-p_3) \rangle$

So: If Q is on the plane:

$\overrightarrow{P_0Q} \cdot \vec{n} = 0$

$\langle (x-p_1), (y-p_2), (z-p_3) \rangle \cdot \langle n_1, n_2, n_3 \rangle = 0$

$$n_1(x-p_1) + n_2(y-p_2) + n_3(z-p_3) = 0$$

$$n_1 x \underbrace{-n_1 p_1} + n_2 y \underbrace{-n_2 p_2} + n_3 z \underbrace{-n_3 p_3} = 0$$

$$\boxed{n_1 x + n_2 y + n_3 z = n_1 p_1 + n_2 p_2 + n_3 p_3}$$

equation of plane

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ex Plane w/ $\vec{n} = \langle 1, 2, 3 \rangle$
$P_0 = (0, 1, -1)$

Equation: $x + 2y + 3z = 0 + 2(1) + 3(-1) = -1$

$$\boxed{x + 2y + 3z = -1}$$

ex Is $(-1, 1, 2)$ on the plane? NO

$-1 + 2(1) + 3(2) \stackrel{?}{=} -1$

$-1 + 2 + 6 = 7 \neq -1$

2 planes are orthogonal if their normal vectors are orthogonal

2 planes are parallel if their normal vectors are parallel.

ex Find a plane parallel to: $2x - 4y + 5z = 1$ $\Big\}\ \vec{n} = \langle 2, -4, 5 \rangle$ that passes through the point $(1,1,1)$.

ANS: $2x - 4y + 5z = 2 - 4 + 5 = 3$

$$\boxed{2x - 4y + 5z = 3}$$

ex) What vector of the form $\langle 1, 1, c \rangle$ is perpendicular to $\langle 2, -4, 5 \rangle$?

$$2 + (-4) + 5c = 0$$
$$-2 + 5c = 0$$
$$5c = 2$$
$$c = 2/5$$

So: $\langle 1, 1, 2/5 \rangle$ is perp. to $\langle 2, -4, 5 \rangle$.

ex Find a plane perp to $2x - 4y + 5z = 3$ that passes through $(1,1,1)$:

one ans: $x + y + \frac{2}{5}z = 2 + 2/5 = 12/5$