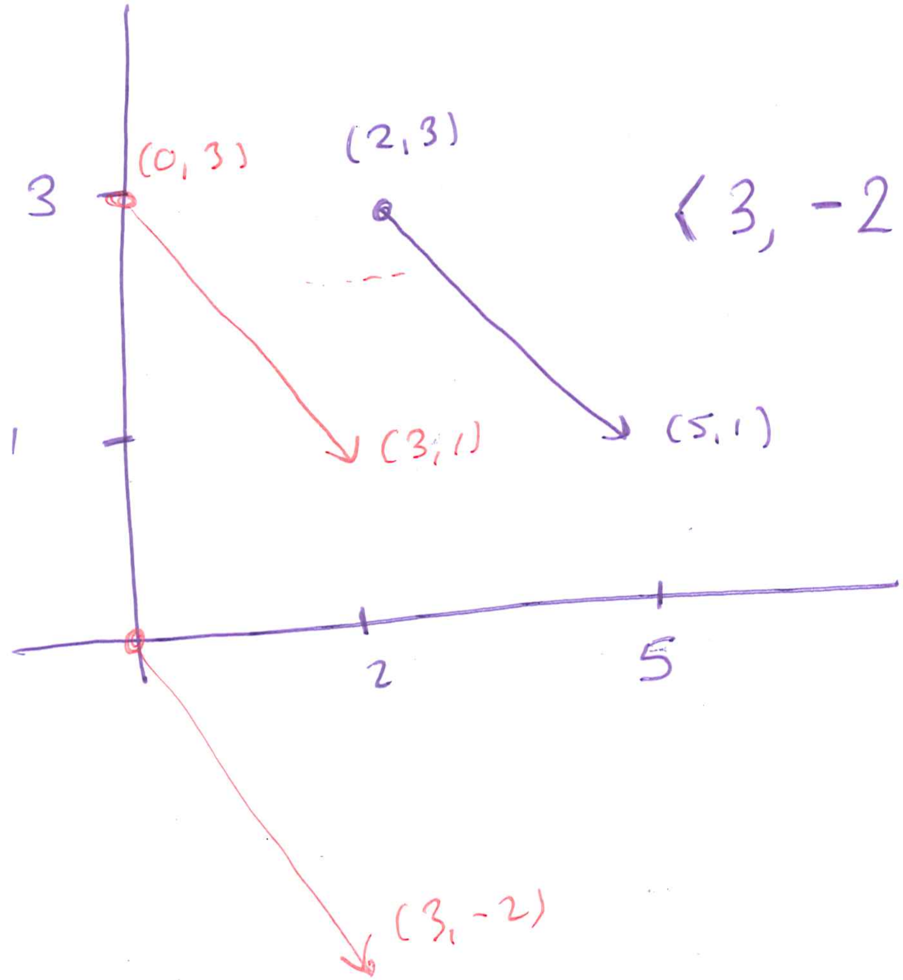
Last Time:
Vectors



$\langle 3, -2 \rangle$

$\langle 1, 2 \rangle + \langle 7, 8 \rangle =$

$\langle 8, 10 \rangle$

$-3 \langle 1, 2 \rangle = \langle -3, -6 \rangle$

Ch. 12.1 : Planes + Surfaces

Def: Given a fixed point P_0 and a nonzero vector $\vec{n}$ in $\underbrace{\mathbb{R}^3}$, → (xyz system / 3 dimensions)

the set of points P in $\mathbb{R}^3$ such that $\overrightarrow{P_0P}$ is perpendicular (orthogonal) to $\vec{n}$ is called a <u>plane</u>.

The vector $\vec{n}$ we call a <u>normal vector</u> to P.

Find equation for plane P that passes through point (p_1, p_2, p_3) and has normal vector $\langle n_1, n_2, n_3 \rangle$

Def $\Rightarrow$ P is all points (x, y, z) such that the vector from (p_1, p_2, p_3) to (x, y, z) $\Big\}$ $\langle x-p_1, y-p_2, z-p_3 \rangle$ is perp. to $\langle n_1, n_2, n_3 \rangle$.

That is: $\langle n_1, n_2, n_3 \rangle \cdot \langle x-p_1, y-p_2, z-p_3 \rangle = 0$

$n_1(x-p_1) + n_2(y-p_2) + n_3(z-p_3) = 0$

$n_1 x - n_1 p_1 + n_2 y - n_2 p_2 + n_3 z - n_3 p_3 = 0$

$$\boxed{n_1 x + n_2 y + n_3 z = n_1 p_1 + n_2 p_2 + n_3 p_3}$$

Suppose a plane P has normal vector

$\vec{n} = \langle 1, 2, 1 \rangle$.

Also: $\langle -1, -2, -1 \rangle$ is a normal vector to P

$\langle 3, 6, 3 \rangle$

$\langle -4, -8, -4 \rangle$

$\langle \pi, 2\pi, \pi \rangle$

ex What is the equation for a plane w/ normal vector $\langle 2, 7, -19 \rangle$ passing through point $(1, -1, 0)$?

$2x + 7y - 19z = 2 - 7 + 0 = -5$

$\boxed{2x + 7y - 19z = -5}$

(ex) P is a plane

$\vec{n} = \langle 5, 18, 0 \rangle$

has point $P_0 = (3, 0, 9)$

Equation of P:

$$5x + 18y = 15$$

Q: Is the point $(0, 1, 3)$ on plane P?

Q: Is the point $(0, 1, 3)$ on plane P? For what value of b is $(1, b, 7)$ on the plane P?

Point $(0, 1, 3)$: $5x + 18y = 5 \cdot 0 + 18 \cdot 1 = 18 \neq 15$

<u>not</u> on plane

Point $(1, b, 7)$: $5(1) + 18(b) = 15$

$18b = 10$

$b = \frac{10}{18}$ $\boxed{b = \frac{5}{9}}$

ex Plane has equation:

$$14x - 32y + \pi z = 521$$

1. give a normal vector to plane $\langle 14, -32, \pi \rangle$

2. give me a point on the plane

e.g. $\left(0, 0, \frac{521}{\pi}\right)$

$\left(\frac{521}{14}, 0, 0\right)$

Parallel & Orthogonal Planes

perpendicular
right angles

Parallel planes have parallel normal vectors
Perpendicular planes have perpendicular normal vectors

ex Give a plane <u>parallel</u> to

$$3x - 2y + z = 0$$

passing through the point $(1,1,1)$

use $\vec{n}$ parallel to $\langle 3,-2,1\rangle$

use $\langle 3,-2,1\rangle$

$$\boxed{3x - 2y + z = 2}$$

ex Find $\vec{v}$ that is perpendicular to $\langle 3,-2,1\rangle$ ($\vec{v}$ not all 0s)

$$\langle 2,3,0\rangle \cdot \langle 3,-2,1\rangle = 0$$

$$\overbrace{\langle 1,1,-1\rangle}^{\vec{v}} \cdot \langle 3,-2,1\rangle = 0$$

$$\underbrace{\langle 0,10,20\rangle}_{\vec{v}} \cdot \langle 3,-2,1\rangle = 0$$

e.g. $\vec{v} = \langle 2,3,0\rangle$

ex Find a plane perpendicular to $3x - 2y + z = 0$ that passes through $(2,0,1)$.

$2x + 3y = 4$] $\vec{n} = \langle 2,3,0\rangle$, $P_0 = (2,0,1)$

$\vec{n} = \langle 1,1,-1\rangle$, $P_0 = (2,0,1)$

$x + y - z = 1$] another plane

ex Describe all vectors of the form $\langle 1, y, z \rangle$ that are perpendicular to $\langle 1, 1, -1 \rangle$.

$$\langle 1, 1, -1 \rangle \cdot \langle 1, y, z \rangle = 0$$

$$1 + y - z = 0$$

$$y - z = -1$$

$$y = z - 1$$

$$\boxed{\langle 1, z-1, z \rangle \text{ for any } z}$$

e.g.:

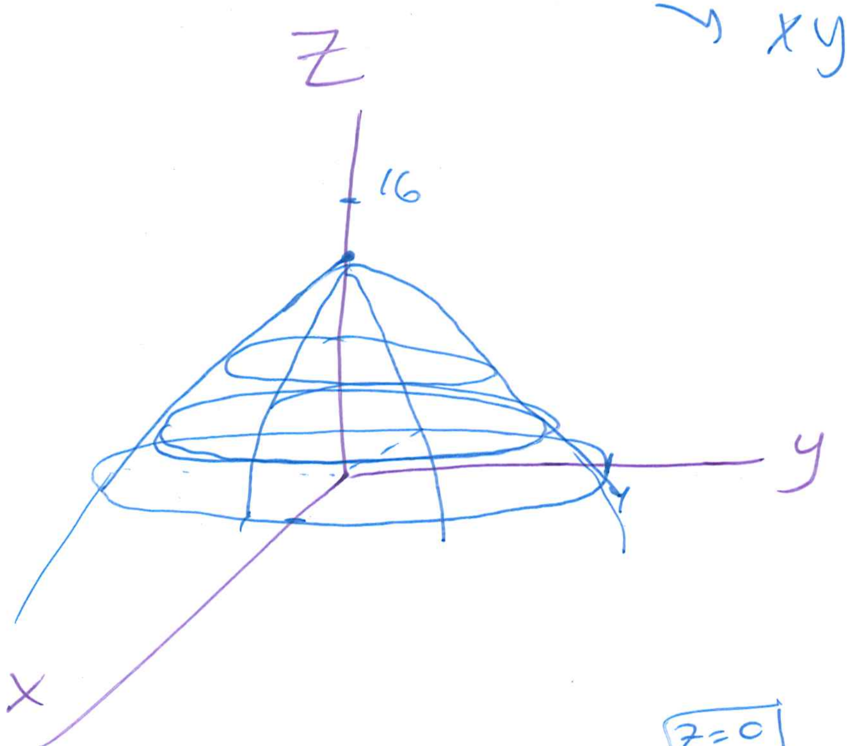
$\langle 1, -1, 0 \rangle$

$\langle 1, 0, 1 \rangle$

$\langle 1, 2, 3 \rangle$

Drawing Surfaces in $\underbrace{\mathbb{R}^3}$ using "traces"

$\to$ xyz system



ex $\quad z = 16 - 4x^2 - y^2$

$0 \le \underbrace{4x^2 + y^2} = 16 - z$

$0 \le 16 - z$

$z \le 16$

$\boxed{z=0}$ $\quad 0 = 16 - 4x^2 - y^2 \to 4x^2 + y^2 = 16$

$\boxed{z=1}$ $\quad 1 = 16 - 4x^2 - y^2 \to 4x^2 + y^2 = 15$

$\boxed{z=2}$ $\quad 2 = 16 - 4x^2 - y^2 \to 4x^2 + y^2 = 14$

$\boxed{z=-10}$ $\quad -10 = 16 - 4x^2 - y^2 \to 4x^2 - y^2 = 26$

what if x constant?

$$z = 16 - 4x^2 - y^2$$

$\boxed{x = 0}$ $\quad \boxed{z = 16 - y^2}$

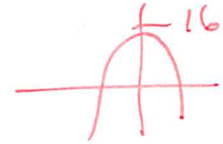
$\boxed{x = 1}$ $\quad z = 16 - 4 - y^2$

$\boxed{z = 12 - y^2}$

$\boxed{x = 2}$ $\quad z = 16 - 4(4) - y^2$

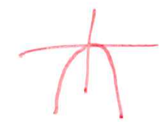
$\boxed{z = -y^2}$

Think: $y = 16 - x^2$



think: $y = 12 - x^2$

think: $y = -x^2$



SHAPES

$x^2 + y^2 = c$	circle
$ax^2 + by^2 = c$	ellipse
$ax^2 + by = c$	parabola
$ax + by = c$	line