Welcome to Math 105

My name is Elyse Yeager.

Course webpage: www.math.ubc.ca/~kliu/common105.html

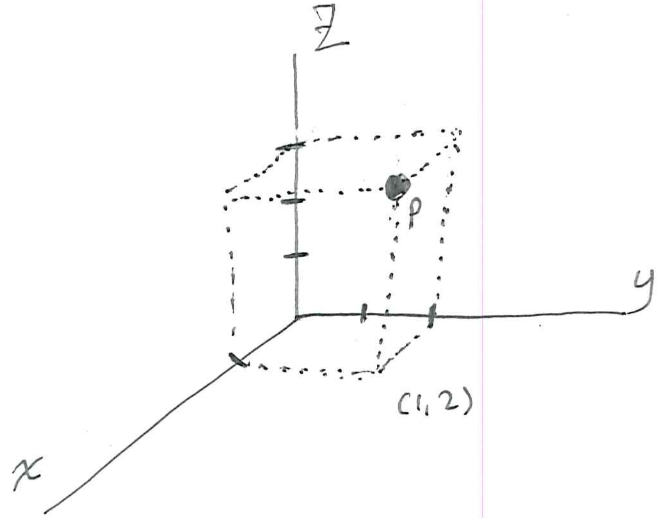
Section webpage: www.math.ubc.ca/~elyse/2017Math105.html

Book: Briggs, Cochran, Gillet
Calculus: Early Transcendentals
Volume 2
Fourth custom edition for UBC

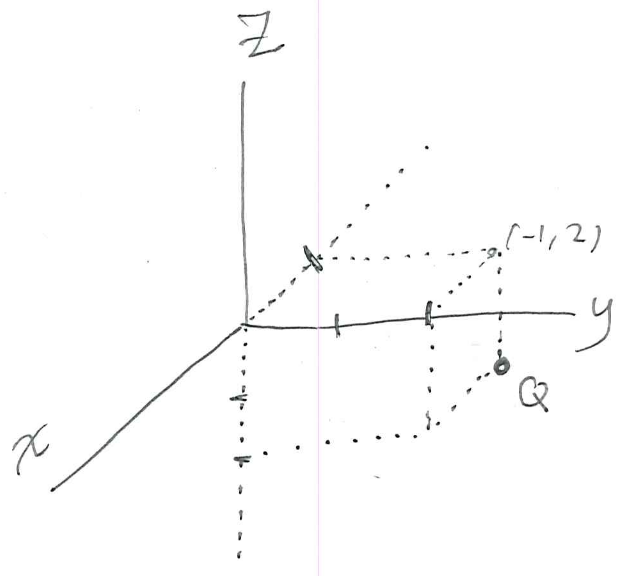
webwork.elearning.ubc.ca

Ch. 11.1–11.3 Vectors & Dot Products

Drawing in $\mathbb{R}^3$ ← xyz coordinates



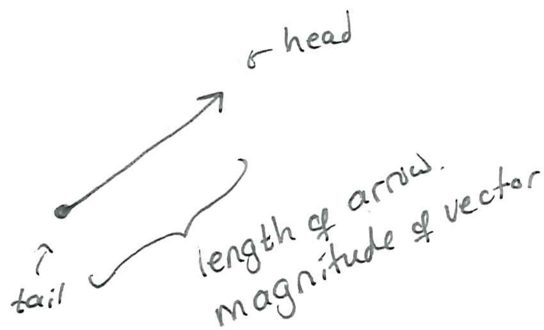
$P = (1, 2, 3)$

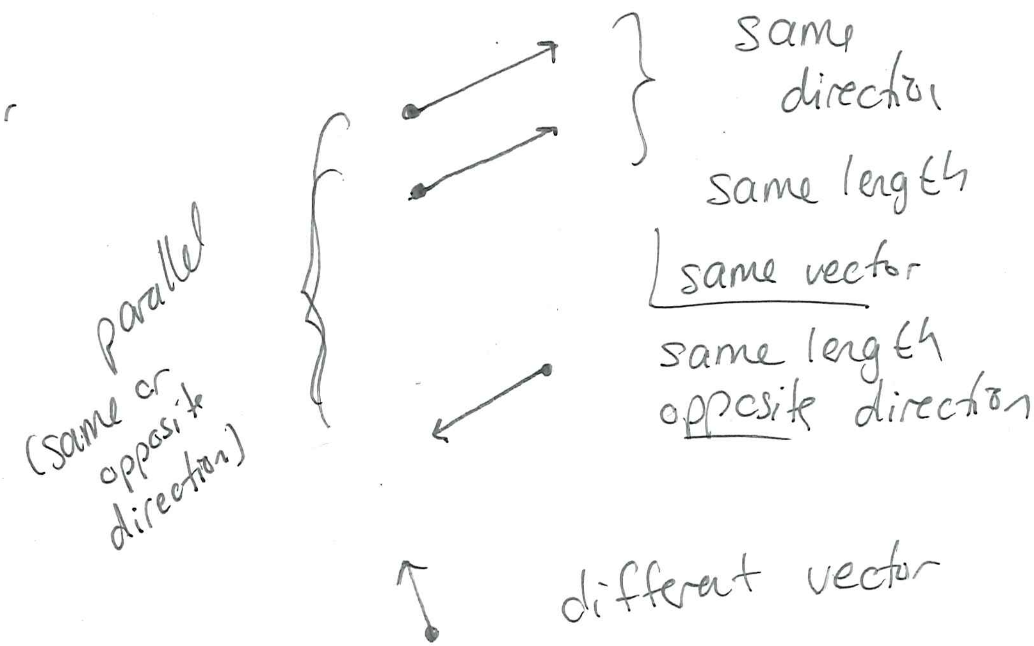


$Q = (-1, 2, -2)$

Vectors

A vector is a mathematical object with
a magnitude (length, size)
and a direction.

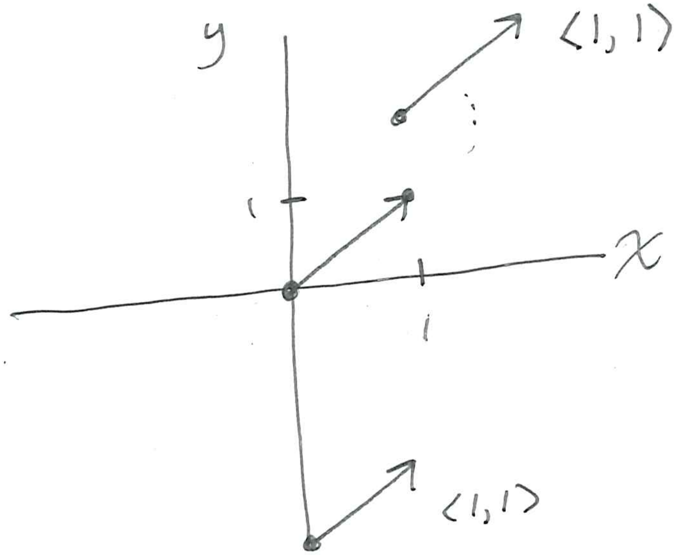


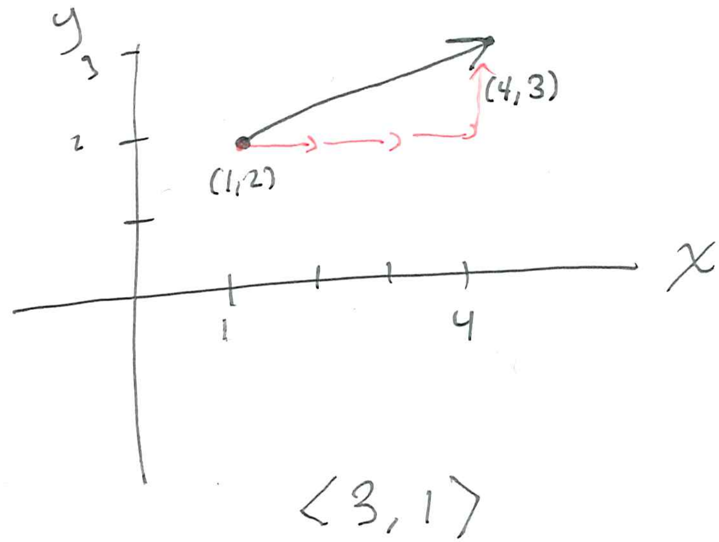


Naming Vectors

In a coordinate system (e.g. $\underbrace{xy\text{-plane}}_{\mathbb{R}^2}$, $\underbrace{xyz}_{\mathbb{R}^3}$)

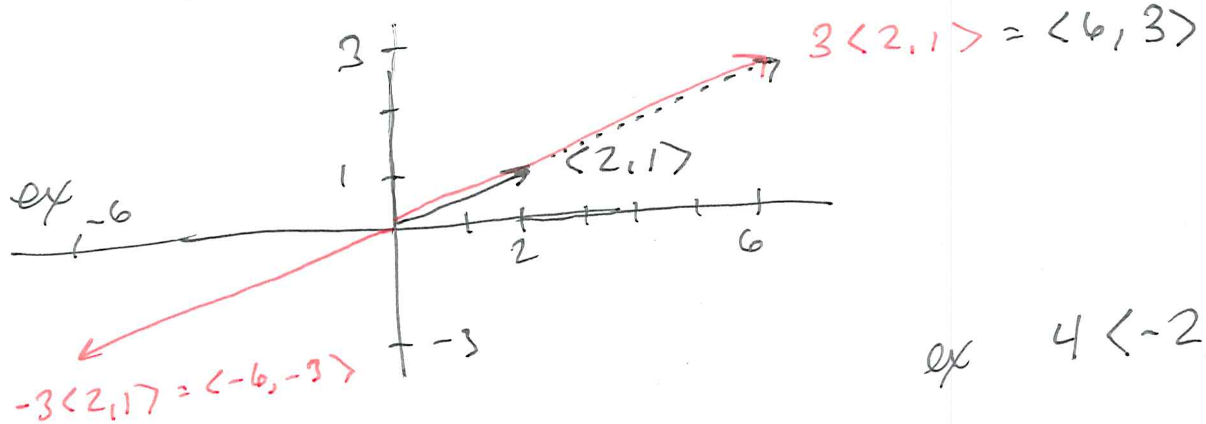
we name a vector by the position of its head when its tail is at origin.





Multiplying a Vector & a $\underbrace{\text{Scalar}}_{\text{number}}$

When we multiply a vector by a positive scalar,
we leave the direction alone, and multiply the length



ex $4\langle -2,1,0\rangle =$

$\langle -8, 4, 0\rangle$

When we multiply a vector by a negative scalar,
we get a vector in the opposite direction (swap head & tail)
& multiply length

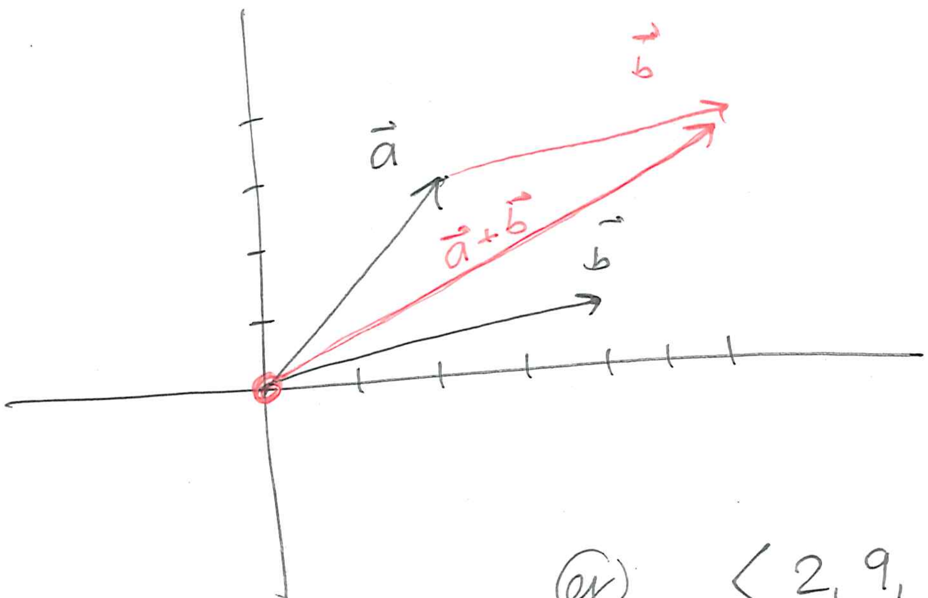
- Vectors that are parallel are scalar multiples of one another

Adding Vectors

If we add vectors $\vec{a}$ and $\vec{b}$:

we put them head-to-tail

$\vec{a}+\vec{b}$ is the vector w/ tail @ 1st tail
head @ 2nd head



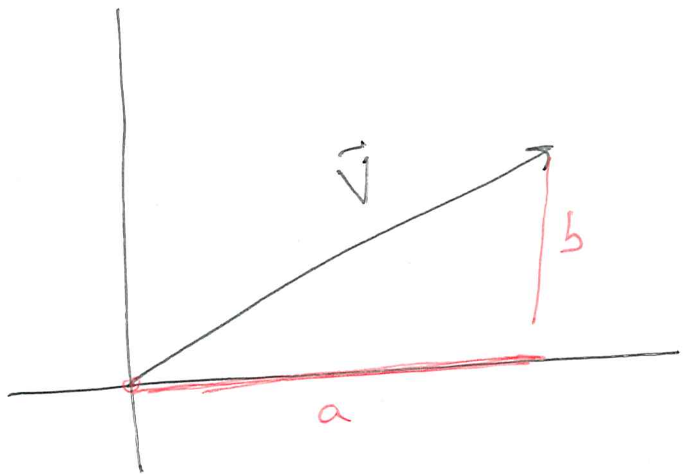
$\vec{a} = \langle 2, 3 \rangle$

$\vec{b} = \langle 4, 1 \rangle$

$\vec{a}+\vec{b} = \langle 6, 4 \rangle$

ex $\langle 2, 9, 20 \rangle + \langle -4, 8, 6 \rangle = \langle -2, 17, 26 \rangle$

Length + Direction of Vectors



$$\vec{v} = \langle a, b \rangle$$

$$\underbrace{\| \vec{v} \|}_{\text{"length" or "norm" or "magnitude" of } \vec{v}} = \sqrt{a^2+b^2} \quad \text{(Pythagorean Thm)}$$

$$\vec{w} = \langle a, b, c \rangle$$

Then $\underbrace{\| \vec{w} \|}_{\text{length}} = \sqrt{a^2+b^2+c^2}$

ex What is the length of $\vec{w} = \langle 2, 5, -1 \rangle$?

$$\sqrt{4+25+1} = \boxed{\sqrt{30}}$$

unit vector : any vector of length one

we use these to describe direction.

Compute : $\vec{u}$ unit vector in same direction as $\vec{w}$.

$$\vec{u} = \frac{1}{\sqrt{30}} \langle 2, 5, -1 \rangle = \boxed{\left\langle \frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{-1}{\sqrt{30}} \right\rangle}$$

Check that $\|\vec{u}\| = 1$:

$$\|\vec{u}\| = \sqrt{\frac{4}{30} + \frac{25}{30} + \frac{1}{30}} = \sqrt{\frac{30}{30}} = \sqrt{1} = 1 \checkmark$$

ex Find a vector of length l $(l > 0)$ in the same direction as $\langle a, b, c \rangle$.

($\langle a, b, c \rangle$ not all 0s)

$$\| \langle a, b, c \rangle \| = \sqrt{a^2+b^2+c^2}$$

vector: $$\frac{l}{\sqrt{a^2+b^2+c^2}} \langle a, b, c \rangle = \boxed{\left\langle \frac{la}{\sqrt{a^2+b^2+c^2}}, \frac{lb}{\sqrt{a^2+b^2+c^2}}, \frac{lc}{\sqrt{a^2+b^2+c^2}} \right\rangle}$$

has length l;
same direction as $\langle a, b, c \rangle$

Dot Product

The dot product is calculated like this:

(in $\mathbb{R}^2$) $$\underbrace{\langle a, b\rangle}_{\text{vector}} \cdot \underbrace{\langle x, y\rangle}_{\text{vector}} = \underbrace{ax + by}_{\substack{\text{number}\\ \text{(scalar)}}}$$

(in $\mathbb{R}^3$) $$\langle a, b, c\rangle \cdot \langle x, y, z\rangle = ax + by + cz$$

ex $$\langle 2, 5, -1\rangle \cdot \langle 3, -2, 0\rangle = 6 - 10 + 0 = -4$$

$\boxed{\text{FACT}}$ If $\vec{u}$ and $\vec{v}$ are perpendicular (orthogonal), then
$\vec{u} \cdot \vec{v} = 0$.

ex: $\langle 2, 5, -1\rangle$, $\langle 3, -2, 0\rangle$ <u>not</u> perpendicular ex

$\vec{a} = \langle 1, 0, 3 \rangle$

$\vec{b} = \langle 3, 0, -1 \rangle$

$\vec{c} = \langle -2, 0, -6 \rangle$

Which pairs are parallel? $(-2)\vec{a} = \vec{c}$ so $\boxed{\vec{a}, \vec{c} \text{ parallel}}$

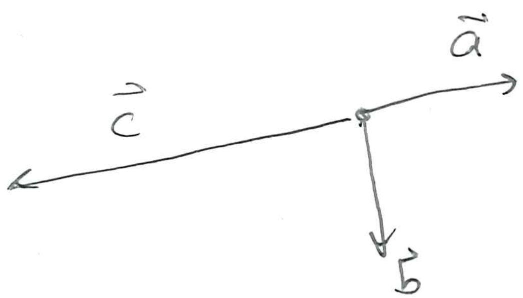
Which are perpendicular?

$\vec{a} \cdot \vec{b} = 3 + 0 - 3 = 0$ so $\boxed{\vec{a}, \vec{b} \text{ perpendicular}}$

$\vec{b} \cdot \vec{c} = -6 + 0 + 6 = 0$ so also $\boxed{\vec{b}, \vec{c} \text{ perpendicula}}$

$\perp$

$\parallel$



$\Big\}$ rough idea

Properties of Dot Product

$\vec{a}, \vec{b}, \vec{c}$ vectors

s scalar

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $s(\vec{a} \cdot \vec{b}) = (s\vec{a}) \cdot \vec{b}$
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

Suggested Problems

section webpage

$\boxed{11.1 - 11.3}$