Power Rule

Only for your understanding - you won’t be assessed on it.

\[ \frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} \]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h}
\]

We want to expand \((x + h)^n\).
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Appendix E: Proofs

E.1: Proof of the power rule

Pascal’s Triangle
Pascal’s Triangle
Appendix E: Proofs

E.1: Proof of the power rule

Pascal’s Triangle

[Diagram of Pascal's Triangle showing the binomial coefficients]

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Pascal’s Triangle
Appendix E: Proofs

E.1: Proof of the power rule

Pascal’s Triangle

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
```

1 1 2 1
1 2 3 2 1
1 3 6 7 3 1
1 4 10 16 14 4 1
1 5 15 28 36 28 5 1

1 1 1 1 1 1 1
Pascal’s Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
Pascal’s Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
\[(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3\]
Pascal’s Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
\[(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3\]
\[(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\]
Pascal’s Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
\[(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3\]
\[(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\]
\[(x + h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\]
Pascal’s Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
\[(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3\]
\[(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\]
\[(x + h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\]
Pascal’s Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
\[(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3\]
\[(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\]
\[(x + h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\]
Pascal's Triangle

\[(x + h)^2 = x^2 + 2xh + h^2\]
\[(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3\]
\[(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\]
\[(x + h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\]
Pascal’s Triangle

\[ (x + h)^2 = x^2 + 2xh + h^2 \]
\[ (x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 \]
\[ (x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 \]
\[ (x + h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 \]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h}
\]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h} \\
= \lim_{h \to 0} \frac{(x^n + nx^{n-1}h + \cancel{x^{n-2}h^2} + \cancel{x^{n-3}h^3} + \cdots) - x^n}{h}
\]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h} \\
= \lim_{h \to 0} \frac{(x^n + nx^{n-1}h + □x^{n-2}h^2 + □x^{n-3}h^3 + \cdots) - x^n}{h} \\
= \lim_{h \to 0} \frac{(nx^{n-1}h + □x^{n-2}h^2 + □x^{n-3}h^3 + \cdots)}{h}
\]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h}
\]

\[
= \lim_{h \to 0} \frac{(x^n + nx^{n-1}h + \square x^{n-2}h^2 + \square x^{n-3}h^3 + \cdots) - x^n}{h}
\]

\[
= \lim_{h \to 0} \frac{(nx^{n-1}h + \square x^{n-2}h^2 + \square x^{n-3}h^3 + \cdots)}{h}
\]

\[
= \lim_{h \to 0} nx^{n-1} + \square x^{n-2}h + \square x^{n-3}h^2 + \cdots
\]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h}
= \lim_{h \to 0} \frac{(x^n + nx^{n-1}h + bx^{n-2}h^2 + \ldots) - x^n}{h}
= \lim_{h \to 0} \frac{nx^{n-1}h + bx^{n-2}h^2 + \ldots}{h}
= \lim_{h \to 0} nx^{n-1} + bx^{n-2}(0) + bx^{n-3}(0)^2 + \ldots
= nx^{n-1}
\]
Power Rule

Only for your understanding - you won’t be assessed on it.

\[
\frac{d}{dx} \{x^n\} = \lim_{h \to 0} \frac{(x + h)^n - x^n}{h} \\
= \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \square x^{n-2}h^2 + \square x^{n-3}h^3 + \cdots - x^n}{h} \\
= \lim_{h \to 0} \frac{nx^{n-1}h + \square x^{n-2}h^2 + \square x^{n-3}h^3 + \cdots}{h} \\
= \lim_{h \to 0} nx^{n-1} + \square x^{n-2}(0) + \square x^{n-3}(0)^2 + \cdots \\
= nx^{n-1}
\]

That is, \( \frac{d}{dx} \{x^n\} = nx^{n-1} \).