Spreading Dough

You are rolling out 1 litre (1000 cm$^3$) of dough for a delicious pastry. The dough is in the shape of a thin cylinder.

If the radius is increasing by 2 cm per minute, how fast is the flat area changing when the height of the dough is 5 mm?
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Analyze the problem:

- **Given:** \( \frac{dr}{dt} = 2 \text{ cm/min} \)
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- We know the relationship between $A$ and $r$: $A = \pi r^2$. 
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Plan: \( \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} \): need to find \( \frac{dA}{dr} \)
Chapter 9: Chain rule – related rates and implicit differentiation

9.1 Applications of the chain rule to “related rates”

Analyze the problem:

- Given: $\frac{dr}{dt} = 2$ cm/min
- Want to find: $\frac{dA}{dt}$, when $h = 0.5$ cm
- We know the relationship between $A$ and $r$: $A = \pi r^2$.

Plan: $\frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dt}$: need to find $\frac{dA}{dr}$

Implement:
**Chapter 9: Chain rule – related rates and implicit differentiation**

**9.1 Applications of the chain rule to “related rates”**

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- **Given:** \( \frac{dr}{dt} = 2 \text{ cm/min} \)
- **Want to find:** \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)
- **We know the relationship between \( A \) and \( r \): \( A = \pi r^2 \).**

**Plan:** \( \frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dt} \): need to find \( \frac{dA}{dr} \)

**Implement:**

\[
A = \pi r^2 \\
\frac{dA}{dr} = 2\pi r
\]
Chapter 9: Chain rule – related rates and implicit differentiation

9.1 Applications of the chain rule to “related rates”

### Analyze the problem:

- **Given:** \( \frac{dr}{dt} = 2 \text{ cm/min} \)
- **Want to find:** \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)
- **We know the relationship between** \( A \) **and** \( r \): \( A = \pi r^2 \).

### Plan:

\[
\frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dt} : \text{need to find} \quad \frac{dA}{dr}
\]

### Implement:

\[
A = \pi r^2
\]
\[
\frac{dA}{dr} = 2\pi r
\]
\[
\frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = (2\pi r)(2)
\]
Analyze the problem:

- **Given:** \( \frac{dr}{dt} = 2 \text{ cm/min} \)
- **Want to find:** \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)
- **We know the relationship between** \( A \) **and** \( r \): \( A = \pi r^2 \).

Plan: \( \frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dt} \): need to find \( \frac{dA}{dr} \)

Implement:

\[
A = \pi r^2 \\
\frac{dA}{dr} = 2\pi r \\
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\]

Plan: Need to find \( r \) when \( h = 0.5 \text{ cm} \).
Analyze the problem:

- Given: \( \frac{dr}{dt} = 2 \text{ cm/min} \)
- Want to find: \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)
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**Plan:** \( \frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dt} \): need to find \( \frac{dA}{dr} \)

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\[
A = \pi r^2 \\
\frac{dA}{dr} = 2\pi r \\
\frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = (2\pi r)(2)
\]

**Plan:** Need to find \( r \) when \( h = 0.5 \text{ cm} \).
Know: \( \pi r^2 h = V = 1000 \text{ cm}^3 \)
Analyze the problem:

- Want to find: \( \frac{dA}{dt} \), when \( h = 0.5 \) cm

Implement: \( \frac{dA}{dt} = 4\pi r \)

Plan:

- Need to find \( r \) when \( h = 0.5 \) cm.
- Know: \( \pi r^2 h = V = 1000 \) cm\(^3\)
Analyze the problem:

- Want to find: \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)

Implement: \( \frac{dA}{dt} = 4\pi r \)

Plan:
- Need to find \( r \) when \( h = 0.5 \text{ cm} \).
- Know: \( \pi r^2 h = V = 1000 \text{ cm}^3 \)
- Solve for \( r \) when \( h = 0.5 \text{ cm} \)
Analyze the problem:

- Want to find: \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)

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Plan:

- Need to find \( r \) when \( h = 0.5 \text{ cm} \).
- Know: \( \pi r^2 h = V = 1000 \text{ cm}^3 \)
- Solve for \( r \) when \( h = 0.5 \text{ cm} \)

Implement:

\[
r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{1000}{0.5\pi}} = \sqrt{\frac{2000}{\pi}}
\]
Analyze the problem:

- Want to find: \( \frac{dA}{dt} \), when \( h = 0.5 \text{ cm} \)

Implement: \( \frac{dA}{dt} = 4\pi r \)

Plan:

- Need to find \( r \) when \( h = 0.5 \text{ cm} \).
- Know: \( \pi r^2 h = V = 1000 \text{ cm}^3 \)
- Solve for \( r \) when \( h = 0.5 \text{ cm} \)

Implement:

\[
\begin{align*}
  r &= \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{1000}{0.5\pi}} = \sqrt{2000} \\
  \frac{dA}{dt} &= 4\pi r = 4\pi \sqrt{\frac{2000}{\pi}} = 80\sqrt{5\pi}
\end{align*}
\]
A spider attaches one end of its silk to a stack of neglected books at a height of $H$ centimetres off a desk, and starts to walk along the desk with a constant rate of $k$ centimetres per second. How fast is the length of silk increasing?
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Stretching Silk

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Stretching Silk

\[
\frac{dS}{dt} = k \text{ (constant)} \quad S^2 + H^2 = L^2 \quad \text{Find } \frac{dL}{dt}
\]
Stretching Silk

\[ \frac{dS}{dt} = k \text{ (constant)} \quad S^2 + H^2 = L^2 \]

Find \( \frac{dL}{dt} \)


Chapter 9: Chain rule – related rates and implicit differentiation

9.1 Applications of the chain rule to “related rates”

Stretching Silk

\[ \frac{dS}{dt} = k \text{ (constant)} \quad S^2 + H^2 = L^2 \quad \text{Find } \frac{dL}{dt} \]

\[ S^2 + H^2 = L^2 \]

\[ (S(t))^2 + H^2 = (L(t))^2 \]
9.1 Applications of the chain rule to “related rates”

**Stretching Silk**

\[ \frac{dS}{dt} = k \text{ (constant)} \quad S^2 + H^2 = L^2 \]

Find \( \frac{dL}{dt} \)

\[
S^2 + H^2 = L^2
\]

\[
\left( S(t) \right)^2 + H^2 = \left( L(t) \right)^2
\]

If the expressions on either side of the equals sign are the same, then they must have the same derivative (with respect to \( t \)). We use the chain rule.

\[
2\left( S(t) \right) \frac{dS}{dt} + 0 = 2\left( L(t) \right) \frac{dL}{dt}
\]

\[
2\left( S(t) \right) (k) + 0 = 2\left( \sqrt{S^2 + H^2} \right) \frac{dL}{dt}
\]

\[
2\left( kt + S_0 \right) (k) + 0 = 2\left( \sqrt{(kt + S_0)^2 + H^2} \right) \frac{dL}{dt}
\]

\[
\frac{dL}{dt} = \frac{k^2 t + kS_0}{\sqrt{(kt + S_0)^2 + H^2}}
\]
Creeping Cylinder

A right circular cylinder has a fixed height of 1 metre, and a changing radius \( r \). At time \( t \), its volume is given by \( V = \sqrt{t} \).

How fast is the surface area of the cylinder increasing when \( r = 2 \)?
Creeping Cylinder

A right circular cylinder has a fixed height of 1 metre, and a changing radius $r$. At time $t$, its volume is given by $V = \sqrt{t}$.

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Creeping Cylinder

A right circular cylinder has a fixed height of 1 metre, and a changing radius $r$. At time $t$, its volume is given by $V = \sqrt{t}$.

How fast is the surface area of the cylinder increasing when $r = 2$?

Recall a right circular cylinder has volume $V = \pi r^2 h$ and surface area $S = 2\pi r^2 + 2\pi rh$. 
Creeping Cylinder

(1) \[ \sqrt{t} = V = \pi r^2 \]

\[ S = 2\pi r^2 + 2\pi r \]

\[ = 2V + 2\pi r \]

\[ = 2\sqrt{t} + 2\pi r \]

recall \( h = 1 \)

(2) \[ \frac{dS}{dt} = \frac{1}{\sqrt{t}} + 2\pi \frac{dr}{dt} \]

\[ \frac{1}{2\sqrt{t}} = \frac{dV}{dt} = 2\pi r \frac{dr}{dt} \]

differentiate

use (1) to find \( \frac{dr}{dt} \)

(3) \[ \frac{1}{4\pi r\sqrt{t}} = \frac{dr}{dt} \]

\[ \frac{dS}{dt} = \frac{1}{\sqrt{t}} + 2\pi \left( \frac{1}{4\pi r\sqrt{t}} \right) \]

plug (3) into (2)

\[ = \frac{1}{\sqrt{t}} + \frac{1}{2r\sqrt{t}} \]

\[ = \frac{1}{\sqrt{t}} + \frac{5}{4\sqrt{t}} \]

if \( r = 2 \)
So, we need to know what $\sqrt{t}$ is when $r = 2$. Since $\sqrt{t} = V = \pi r^2$, that tells us $\sqrt{t} = 4\pi$. So:

$$\frac{dS}{dt} = \frac{5}{4\sqrt{t}} = \frac{5}{16\pi}$$
Creeping Cylinder - Solution Method 2

We want to know \( \frac{dS}{dt} \). Using the chain rule:

\[
\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}
\]

We can find \( \frac{dS}{dr} \) pretty easily from the equation given for \( S \). (Remember \( h = 1 \).)

\[
S = 2\pi r^2 + 2\pi r
\]

\[
\frac{dS}{dr} = 4\pi r + 2\pi
\]

So,

\[
\frac{dS}{dt} = (4\pi r + 2\pi) \cdot \frac{dr}{dt}
\]

Now, we should find \( \frac{dr}{dt} \). We know how to relate \( r \) to \( V \), and \( V \) to \( t \).
Now, we should find $\frac{dr}{dt}$. We know how to relate $r$ to $V$, and $V$ to $t$.

\[ \sqrt{t} = V = \pi r^2 \]

Differentiating both sides with respect to $t$:

\[ \frac{1}{2\sqrt{t}} = 2\pi r \frac{dr}{dt} \]

When $r = 2$,

\[ \frac{1}{2\sqrt{t}} = 4\pi \frac{dr}{dt} \]

\[ \frac{dr}{dt} = \frac{1}{8\pi \sqrt{t}} \]

If $r = 2$, then $V = \pi r^2 = 4\pi$, so $\sqrt{t} = 4\pi$

\[ \frac{dr}{dt} = \frac{1}{32\pi^2} \]
Now, can figure out \( \frac{dS}{dt} \)

\[
\frac{dS}{dt} = (4\pi r + 2\pi) \frac{dr}{dt}
\]

\[
= (4\pi \cdot 2 + 2\pi) \cdot \frac{1}{32\pi^2}
\]

\[
= \frac{5}{16\pi}
\]
Suppose $A$ and $B$ satisfy the relationship

$$A^3 + B^3 + 3A^2B = 15$$

and $\frac{dA}{dt} = 3$. What is $\frac{dB}{dt}$ when $A = 1$ and $B = 2$? What is $\frac{dA}{dB}$ at the same point?
Chapter 9: Chain rule – related rates and implicit differentiation

9.2 Implicit Differentiation

\[ A^3 + B^3 + 3A^2B = 15 \]

\[ 3A^2 \frac{dA}{dt} + 3B^2 \frac{dB}{dt} + 3A^2 \frac{dB}{dt} + 6A \frac{dA}{dt} B = 0 \]

\[ 3(1)^2(3) + 3(2)^2 \frac{dB}{dt} + 3(1)^2 \frac{dB}{dt} + 6(1)(3)(2) = 0 \]

\[ \frac{dB}{dt} (12 + 3) + 45 = 0 \]

\[ \frac{dB}{dt} = -3 \]

For the second part, we use the chain rule.

\[ \frac{dA}{dB} \frac{dB}{dt} = \frac{dA}{dt} \]

\[ \frac{dA}{dB} (3) = -3 \]

\[ \frac{dA}{dB} = -1 \]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

Still has a slope: \( \frac{\Delta y}{\Delta x} \)
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

Still has a slope: \[ \frac{\Delta y}{\Delta x} \]  
Locally, \( y \) is still a function of \( x \).
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

Still has a slope: \( \frac{\Delta y}{\Delta x} \)

**Locally**, \( y \) is still a function of \( x \).
Consider $y$ as a function of $x$.

- \[ \frac{d}{dx}[y] = \]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

Consider \( y \) as a function of \( x \).

- \( \frac{d}{dx}[y] = \frac{dy}{dx} = y' \)
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

Consider \( y \) as a function of \( x \).

- \[ \frac{d}{dx}[y] = \frac{dy}{dx} = y' \]
- \[ \frac{d}{dx}[x] = \]
Implicitly Defined Functions

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Consider \( y \) as a function of \( x \).

- \( \frac{d}{dx}[y] = \frac{dy}{dx} = y' \)
- \( \frac{d}{dx}[x] = 1 \)
- \( \frac{d}{dx}[1] = 0 \)
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

Consider \( y \) as a function of \( x \).

\[ \frac{d}{dx}[y] = \frac{dy}{dx} = y' \quad \frac{d}{dx}[x] = 1 \quad \frac{d}{dx}[1] = 0 \]
**Implicitly Defined Functions**

\[ y^2 + x^2 + xy + x^2y = 1 \]

Consider \( y \) as a function of \( x \).

\[ \frac{d}{dx}[y] = \frac{dy}{dx} = y' \]

\[ \frac{d}{dx}[x] = 1 \]

\[ \frac{d}{dx}[1] = 0 \]

\[ 1 = y^2 + x^2 + xy + x^2y \]

Differentiate both sides with respect to \( x \).

\[ 0 = 2y \frac{dy}{dx} + 2x + \left( x \frac{dy}{dx} + y(1) \right) + \left( x^2 \frac{dy}{dx} + 2xy \right) \]

\[ 0 = \frac{dy}{dx} (2y + x + x^2) + (2x + y + 2xy) \]

\[ - (2x + y + 2xy) = \frac{dy}{dx} (2y + x + x^2) \]

\[ \frac{2x + y + 2xy}{2y + x + x^2} = \frac{dy}{dx} \]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

\[
\frac{dy}{dx} = - \frac{2x + y + 2xy}{2y + x + x^2}
\]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

\[ \frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2} \]

Necessarily, \( \frac{dy}{dx} \) depends on both \( y \) and \( x \). Why?
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

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Necessarily, \( \frac{dy}{dx} \) depends on both \( y \) and \( x \). Why?

\[ \left. \frac{dy}{dx} \right|_{(1,0)} = \quad \left. \frac{dy}{dx} \right|_{(1,-2)} = \]
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

\[
\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}
\]

Necessarily, \( \frac{dy}{dx} \) depends on both \( y \) and \( x \). Why?

\[
\left. \frac{dy}{dx} \right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1
\]

\[
\left. \frac{dy}{dx} \right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2
\]

Points with the same \( x \)-value may have different slopes. We need both the \( x \)-value and the \( y \)-value to figure out which point we're talking about.
Implicitly Defined Functions

\[ y^2 + x^2 + xy + x^2y = 1 \]

\[ \frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2} \]

Necessarily, \( \frac{dy}{dx} \) depends on both \( y \) and \( x \). Why?

\[ \left. \frac{dy}{dx} \right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1 \]

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Points with the same \( x \)-value may have different slopes. We need both the \( x \)-value and the \( y \)-value to figure out which point we’re talking about.
Points with the same $x$-value may have different slopes. We need both the $x$-value and the $y$-value to figure out which point we’re talking about.
Example 1:

(a) Suppose \( x^4 y + y^4 x = 2 \). Find \( \frac{dy}{dx} \) at the point \((1, 1)\).

(b) Suppose \( \frac{3y^2 + y + y^3}{x^2 + 1} = x \). Find \( \frac{dy}{dx} \) when \( x = 0 \), and the equations of the associated tangent line(s).
(a)

\[ x^4 y + y^4 x = 2 \]

\[ 4x^3 y + x^4 \frac{dy}{dx} + y^4 + 4y^3 \frac{dy}{dx} x = 0 \]

\[ 4 + \frac{dy}{dx} + 1 + 4 \frac{dy}{dx} = 0 \]

\[ 5 \frac{dy}{dx} = -5 \]

\[ \frac{dy}{dx} = -1 \]
(b) \[
\frac{3y^2 + y + y^3}{x^2 + 1} = x
\]
\[
(x^2 + 1) \left( 6y \frac{dy}{dx} + \frac{dy}{dx} + 3y^2 \frac{dy}{dx} \right) - (3y^2 + y + y^3)(2x)
\]
\[
= 1
\]
\[
= 1
\]
\[
\left( 6y \frac{dy}{dx} + \frac{dy}{dx} + 3y^2 \frac{dy}{dx} \right) = 1
\]

We need to know \( y \).

\[
3y^2 + y + y^3 = 0
\]
\[
y(y^2 + 3y + 1) = 0
\]
\[
y = 0, y = \frac{-3 \pm \sqrt{9 - 4}}{2} = \frac{-3 \pm \sqrt{5}}{2}
\]
At the point \((0, 0)\):

\[
\left(6y \frac{dy}{dx} + \frac{dy}{dx} + 3y^2 \frac{dy}{dx}\right) = 1
\]

\[
\frac{dy}{dx} = 1
\]

So, the equation of the tangent line is \(y = x\).
At the point \( (0, \frac{-3 - \sqrt{5}}{2}) \):

\[
6y \frac{dy}{dx} + \frac{dy}{dx} + 3y^2 \frac{dy}{dx} + 1 = 1
\]

\[
\frac{dy}{dx} \left( 6 \left( \frac{-3 - \sqrt{5}}{2} \right) + 3 \left( \frac{-3 - \sqrt{5}}{2} \right)^2 + 1 \right) = 1
\]

\[
\frac{dy}{dx} \left( -9 - 3\sqrt{5} + \frac{3}{4} \left( 14 + 6\sqrt{5} \right) + 1 \right) = 1
\]

\[
\frac{dy}{dx} \left( 1.5\sqrt{5} + 2.5 \right) = 1
\]

\[
\frac{dy}{dx} = \frac{2}{3(2 + \sqrt{5})}
\]

So, the equation of the tangent line is

\[
y + \frac{3 + \sqrt{5}}{2} = \frac{2x}{3\sqrt{5} + 5}
\]
At the point \( (0, \frac{-3 + \sqrt{5}}{2}) \):

\[
\left( 6y \frac{dy}{dx} + \frac{dy}{dx} + 3y^2 \frac{dy}{dx} + 1 \right) = 1
\]

\[
\frac{dy}{dx} \left( 6 \left( \frac{-3 + \sqrt{5}}{2} \right) + 3 \left( \frac{-3 + \sqrt{5}}{2} \right)^2 + 1 \right) = 1
\]

\[
\frac{dy}{dx} \left( -9 + 3\sqrt{5} + \frac{3}{4} \left( 14 - 6\sqrt{5} \right) + 1 \right) = 1
\]

\[
\frac{dy}{dx} \left( -1.5\sqrt{5} + 2.5 \right) = 1
\]

\[
\frac{dy}{dx} = \frac{2}{5 - 3\sqrt{5}}
\]

So, the equation of the tangent line is

\[
y - \frac{-3 + \sqrt{5}}{2} = \frac{2x}{5 - 3\sqrt{5}}
\]
Earlier:
We found \( \frac{d}{dx}(x^n) = nx^{n-1} \) using Pascal’s Triangle.
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That worked for **whole-number powers**.
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We found \( \frac{d}{dx}(x^n) = nx^{n-1} \) using Pascal’s Triangle.
That worked for \textbf{whole-number powers}.

Suppose we want to find \( \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) \).
\textbf{Why} does it work the same way?
Earlier:
We found $\frac{d}{dx}(x^n) = nx^{n-1}$ using Pascal’s Triangle.

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Suppose we want to find $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2})$.
**Why** does it work the same way?

Implicit:

\[
y = \sqrt{x}
\]
\[
y^2 = x
\]
Earlier:
We found \( \frac{d}{dx}(x^n) = nx^{n-1} \) using Pascal’s Triangle.

That worked for **whole-number powers**.

Suppose we want to find \( \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) \).

**Why** does it work the same way?

Implicit:

\[
\begin{align*}
y &= \sqrt{x} \\
y^2 &= x \\
2y \frac{dy}{dx} &= 1
\end{align*}
\]
Earlier:
We found $\frac{d}{dx}(x^n) = nx^{n-1}$ using Pascal’s Triangle.

That worked for **whole-number powers**.

Suppose we want to find $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2})$.
**Why** does it work the same way?

Implicit:

\[
\begin{align*}
y &= \sqrt{x} \\
y^2 &= x \\
2y \frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{1}{2y} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{1/2-1}
\end{align*}
\]
Differentiate \( y = x^{m/n} \), where \( m \) and \( n \) are whole numbers.
Differentiate $y = x^{m/n}$, where $m$ and $n$ are whole numbers.

\[
\begin{align*}
    y^n &= x^m \\
    ny^{n-1} \frac{dy}{dx} &= mx^{m-1} \frac{dy}{dx} \\
    \frac{dy}{dx} &= \frac{mx^{m-1}}{ny^{n-1}} \\
    &= \frac{mx^{m-1}}{n \left( x^{m/n} \right)^{n-1}} \\
    &= \frac{m}{n} \left( \frac{x^{m-1}}{x^{m-m/n}} \right) \\
    &= \frac{m}{n} \left( x^{m-1-m+m/n} \right) \\
    &= \frac{m}{n} \left( x^{m/n-1} \right)
\end{align*}
\]