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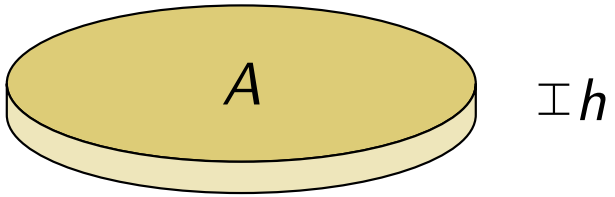

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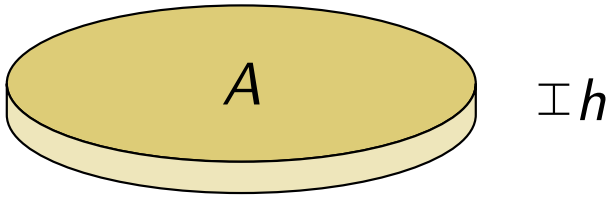

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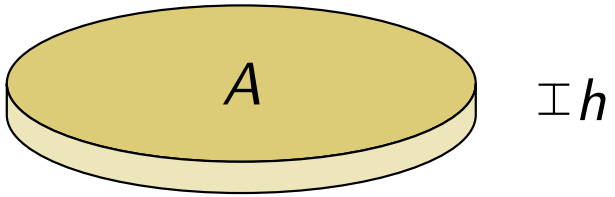

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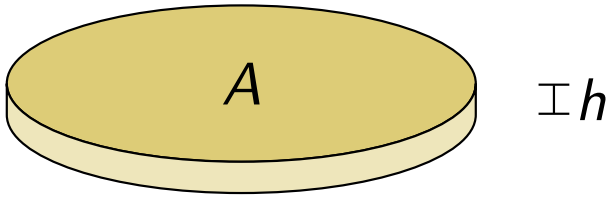

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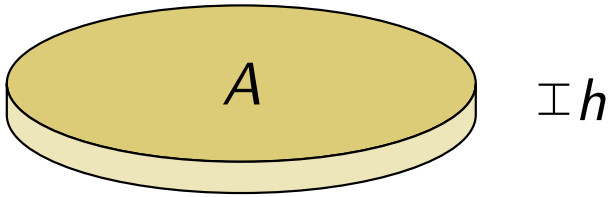

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Implement:

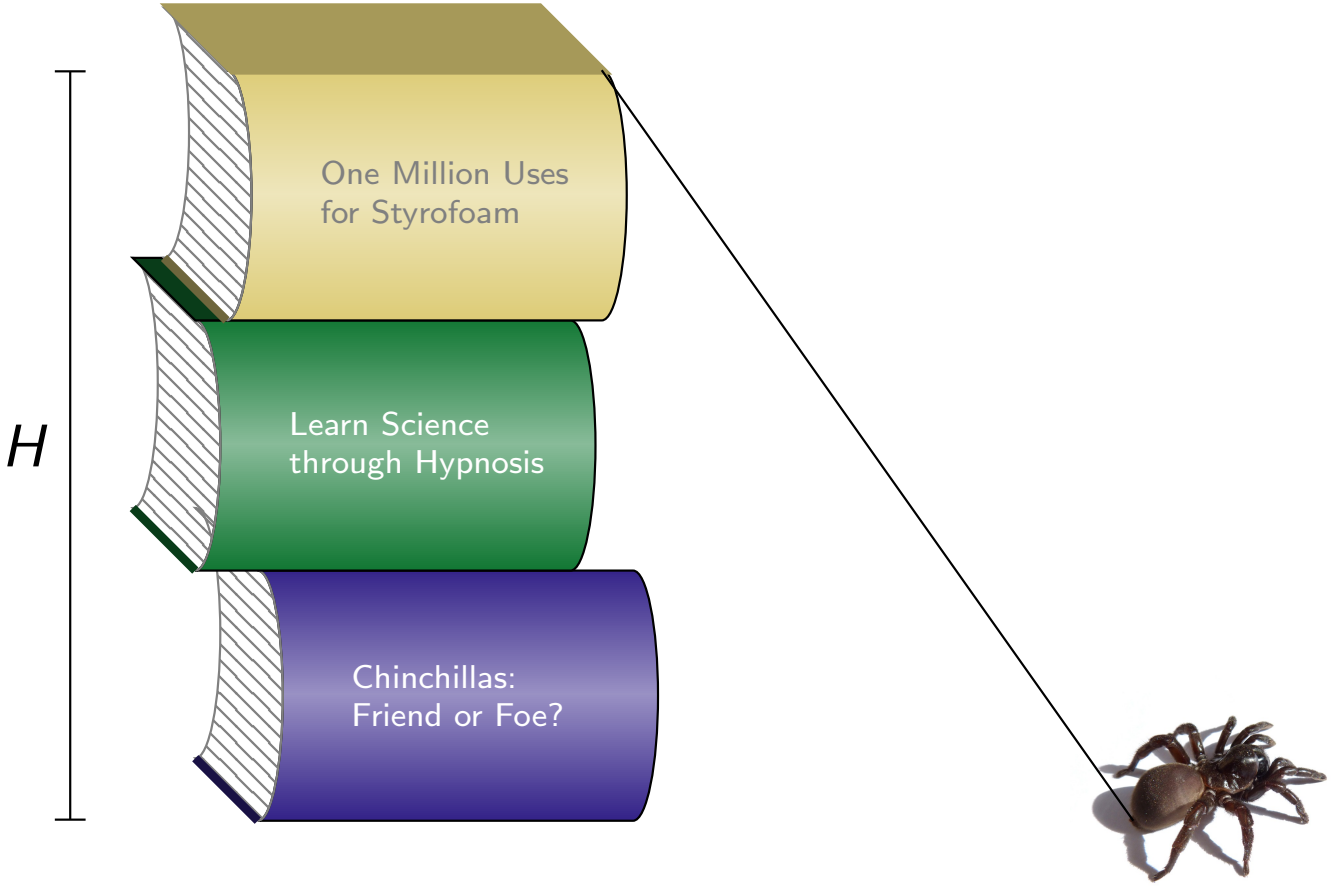
$$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{1000}{0.5\pi}} = \sqrt{\frac{2000}{\pi}}$$

So,

$$\frac{dA}{dt} = 4\pi r = 4\pi\sqrt{\frac{2000}{\pi}} = 80\sqrt{5\pi}$$

Stretching Silk

A spider attaches one end of its silk to a stack of neglected books at a height of H centimetres off a desk, and starts to walk along the desk with a constant rate of k centimetres per second. How fast is the length of silk increasing?



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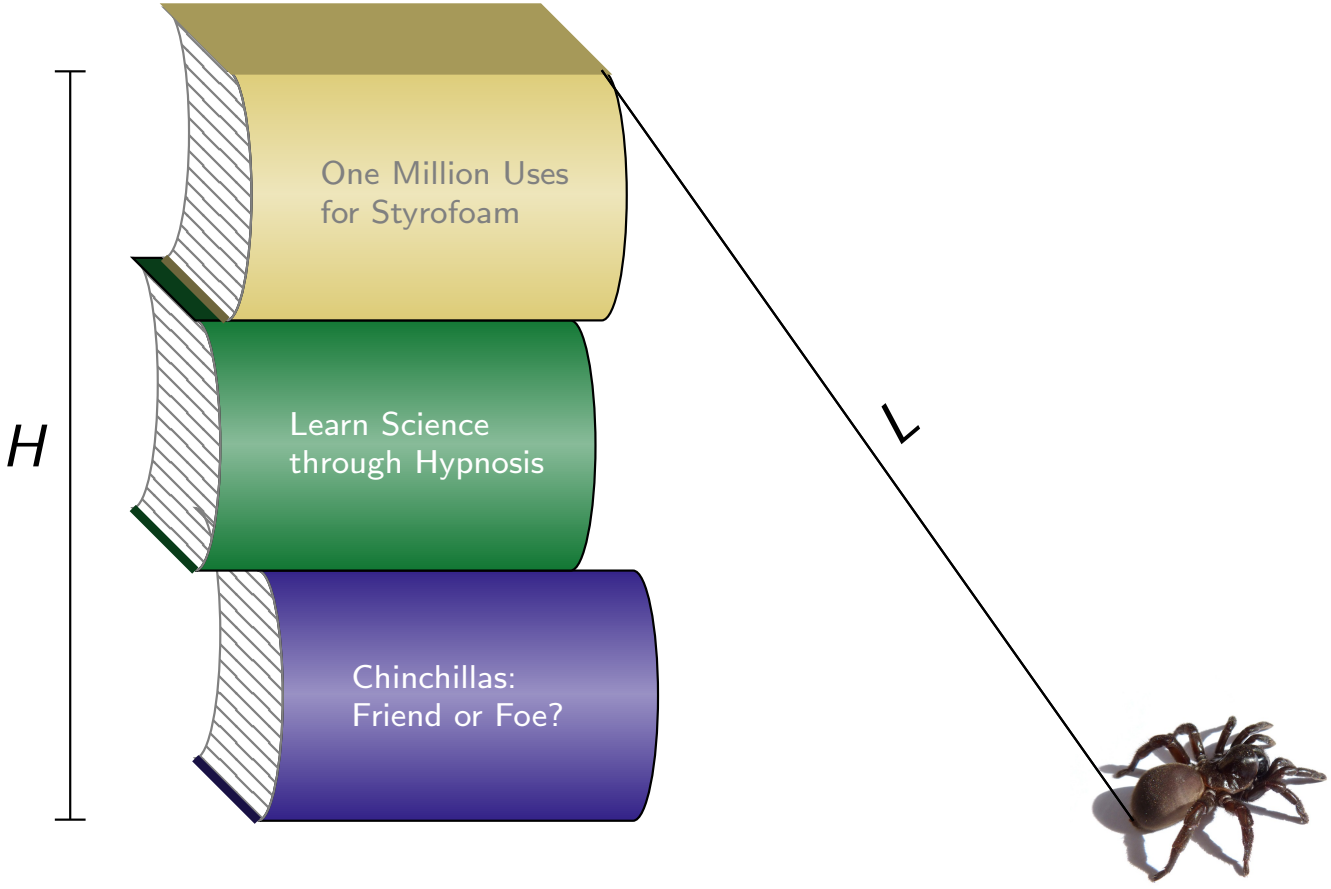

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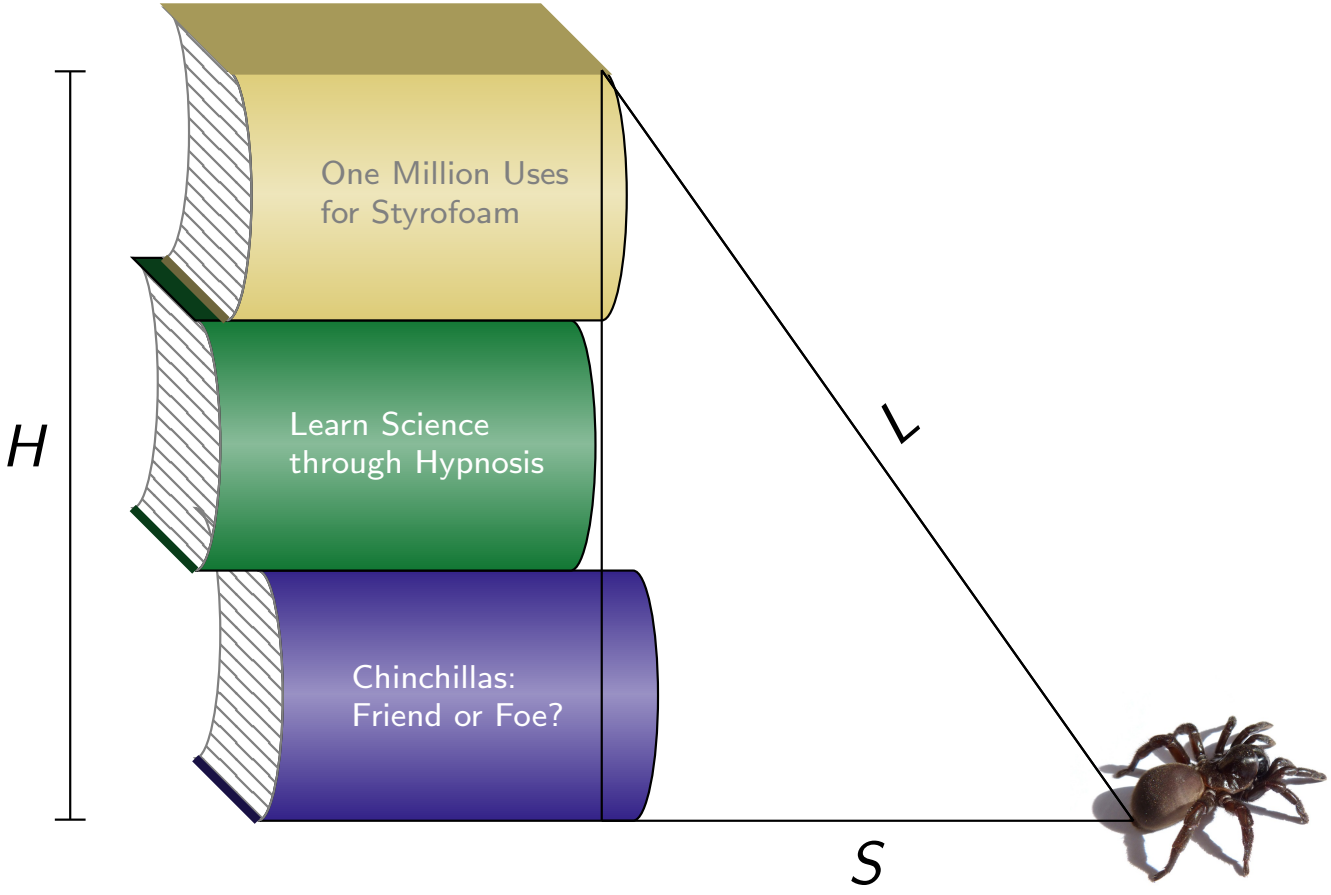

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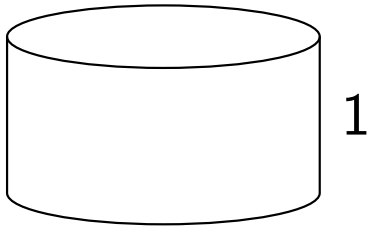
$$S^2 + H^2 = L^2$$
$$\Big(S(t)\Big)^2 + H^2 = \Big(L(t)\Big)^2$$

If the expressions on either side of the equals sign are the same, then they must have the same derivative (with respect to t). We use the chain rule.

$$2\Big(S(t)\Big)\frac{dS}{dt} + 0 = 2\Big(L(t)\Big)\frac{dL}{dt}$$
$$2\Big(S(t)\Big)(k) + 0 = 2\Big(\sqrt{S^2 + H^2}\Big)\frac{dL}{dt}$$
$$2\Big(kt + S_0\Big)(k) + 0 = 2\Big(\sqrt{(kt + S_0)^2 + H^2}\Big)\frac{dL}{dt}$$
$$\frac{dL}{dt} = \frac{k^2 t + kS_0}{\sqrt{(kt + S_0)^2 + H^2}}$$

Creeping Cylinder

A right circular cylinder has a fixed height of 1 metre, and a changing radius r. At time t, its volume is given by $V = \sqrt{t}$.



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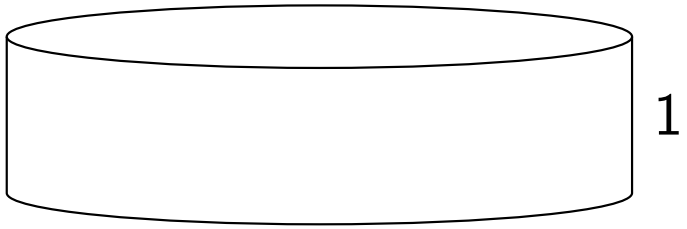

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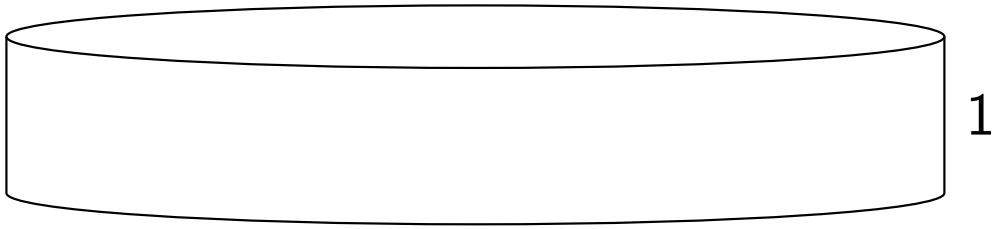

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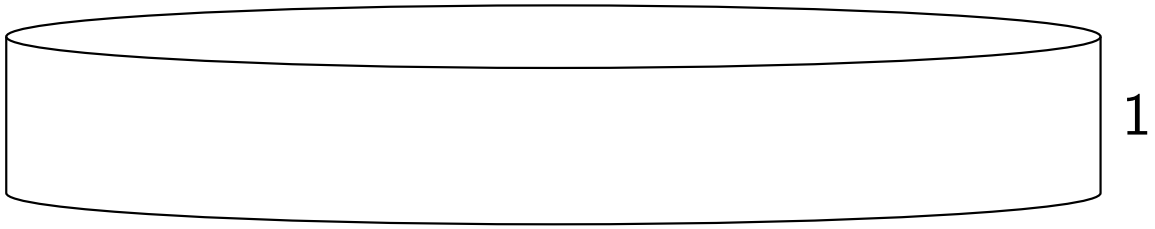

How fast is the surface area of the cylinder increasing when $r = 2$?

Recall a right circular cylinder has volume $V = \pi r^2 h$
and surface area $S = 2\pi r^2 + 2\pi rh$.

Creeping Cylinder

(1) $$\sqrt{t} = V = \pi r^2$$ recall $h = 1$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi r \\ &= 2V + 2\pi r \\ &= 2\sqrt{t} + 2\pi r \end{aligned}$$

(2) $$\frac{dS}{dt} = \frac{1}{\sqrt{t}} + 2\pi \frac{dr}{dt}$$ differentiate

$$\frac{1}{2\sqrt{t}} = \frac{dV}{dt} = 2\pi r \frac{dr}{dt}$$ use (1) to find $\frac{dr}{dt}$

(3) $$\frac{1}{4\pi r\sqrt{t}} = \frac{dr}{dt}$$

$$\frac{dS}{dt} = \frac{1}{\sqrt{t}} + 2\pi \left(\frac{1}{4\pi r\sqrt{t}}\right)$$ plug (3) into (2)

$$= \frac{1}{\sqrt{t}} + \frac{1}{2r\sqrt{t}}$$

$$\frac{dS}{dt} = \frac{1}{\sqrt{t}} + \frac{1}{4\sqrt{t}} = \frac{5}{4\sqrt{t}}$$ if $r = 2$

Creeping Cylinder

So, we need to know what $\sqrt{t}$ is when $r = 2$. Since $\sqrt{t} = V = \pi r^2$, that tells us $\sqrt{t} = 4\pi$. So:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{5}{4\sqrt{t}} = \frac{5}{16\pi}$$

Creeping Cylinder- Solution Method 2

We want to know $\frac{dS}{dt}$. Using the chain rule:

$$\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}$$

We can find $\frac{dS}{dr}$ pretty easily from the equation given for S. (Remember $h = 1$.)

$$S = 2\pi r^2 + 2\pi r$$

$$\frac{dS}{dr} = 4\pi r + 2\pi$$

$$\text{So,} \qquad \frac{dS}{dt} = \underbrace{(4\pi r + 2\pi)}_{\frac{dS}{dr}} \cdot \frac{dr}{dt}$$

Now, we should find $\frac{dr}{dt}$. We know how to relate r to V, and V to t.

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$$\sqrt{t} = V = \pi r^2$$

Differentiating both sides with respect to t:

$$\frac{1}{2\sqrt{t}} = 2\pi r \frac{dr}{dt}$$

When $r = 2$,

$$\frac{1}{2\sqrt{t}} = 4\pi \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{1}{8\pi\sqrt{t}}$$

If $r = 2$, then $V = \pi r^2 = 4\pi$, so $\sqrt{t} = 4\pi$

$$\frac{dr}{dt} = \frac{1}{32\pi^2}$$

Now, can figure out $\frac{dS}{dt}$

$$\begin{aligned}\frac{dS}{dt} &= (4\pi r + 2\pi)\frac{dr}{dt} \\ &= (4\pi \cdot 2 + 2\pi) \cdot \frac{1}{32\pi^2} \\ &= \frac{5}{16\pi}\end{aligned}$$

Computation Practice

Suppose A and B satisfy the relationship

$$A^3 + B^3 + 3A^2B = 15$$

and $\frac{dA}{dt} = 3$. What is $\frac{dB}{dt}$ when $A = 1$ and $B = 2$?
What is $\frac{dA}{dB}$ at the same point?

$$A^3 + B^3 + 3A^2B = 15$$
$$3A^2\frac{dA}{dt} + 3B^2\frac{dB}{dt} + 3A^2\frac{dB}{dt} + 6A\frac{dA}{dt}B = 0$$
$$3(1)^2(3) + 3(2)^2\frac{dB}{dt} + 3(1)^2\frac{dB}{dt} + 6(1)(3)(2) = 0$$
$$\frac{dB}{dt}(12+3) + 45 = 0$$
$$\frac{dB}{dt} = -3$$

For the second part, we use the chain rule.

$$\frac{dA}{dB}\frac{dB}{dt} = \frac{dA}{dt}$$
$$\frac{dA}{dB}(3) = -3$$
$$\frac{dA}{dB} = -1$$

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

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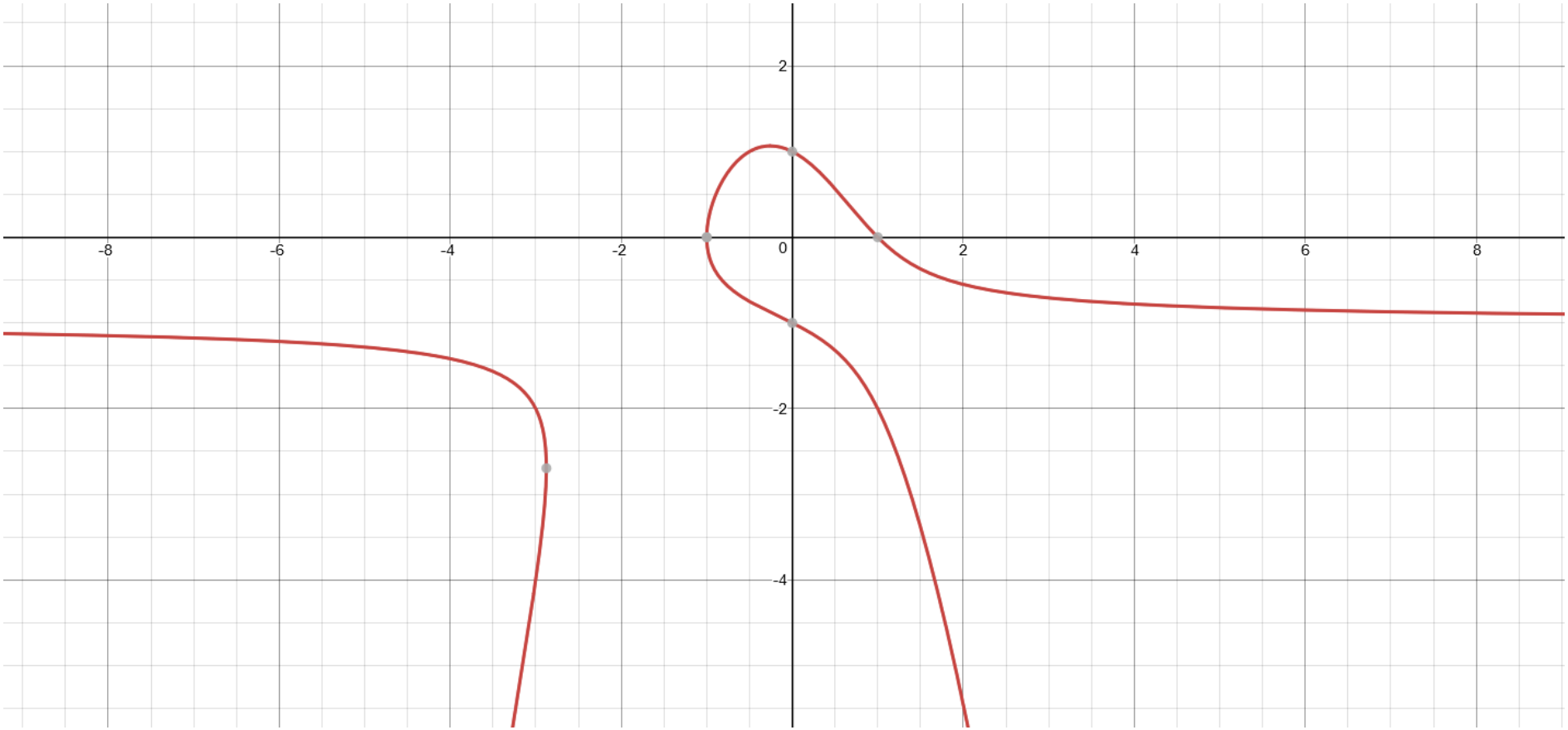

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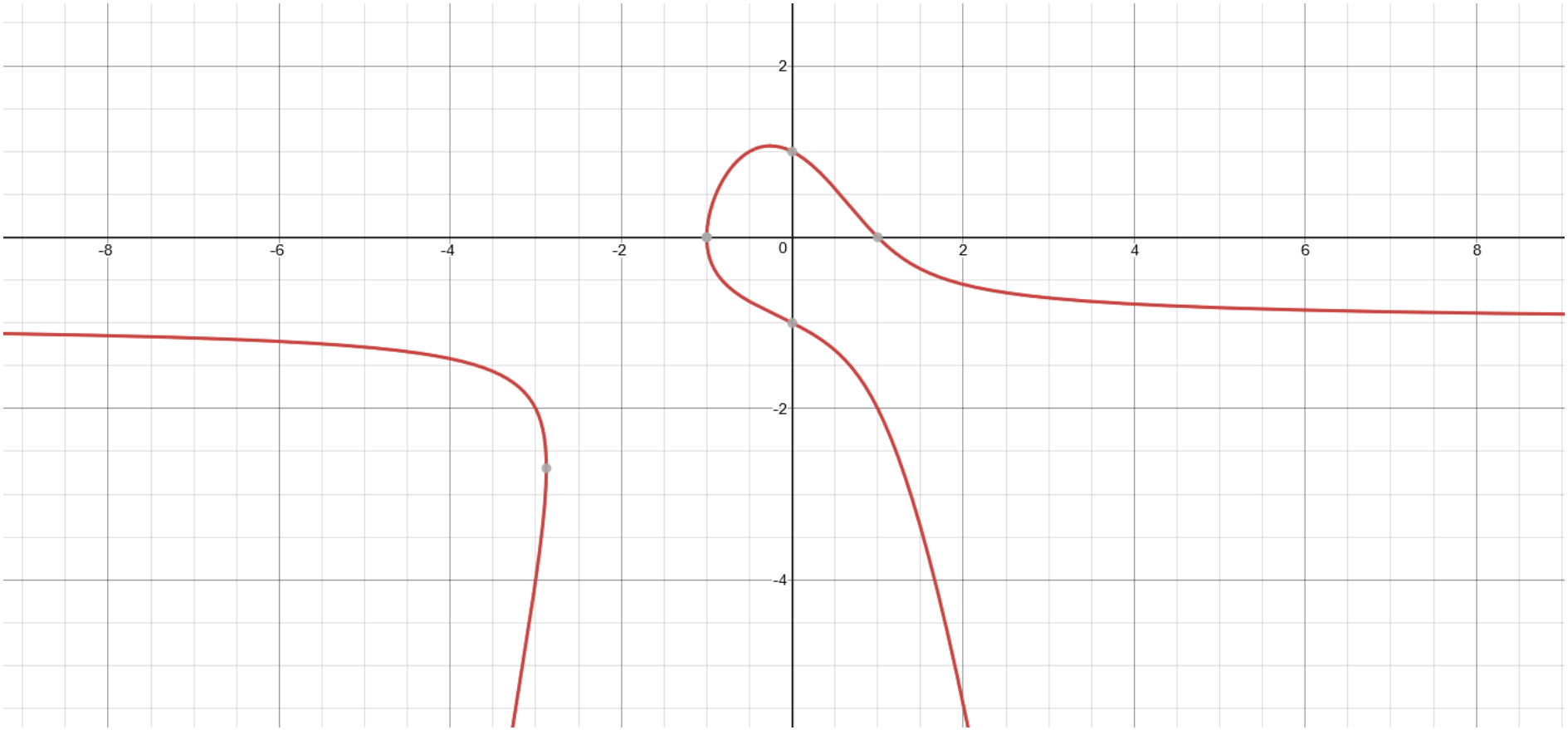

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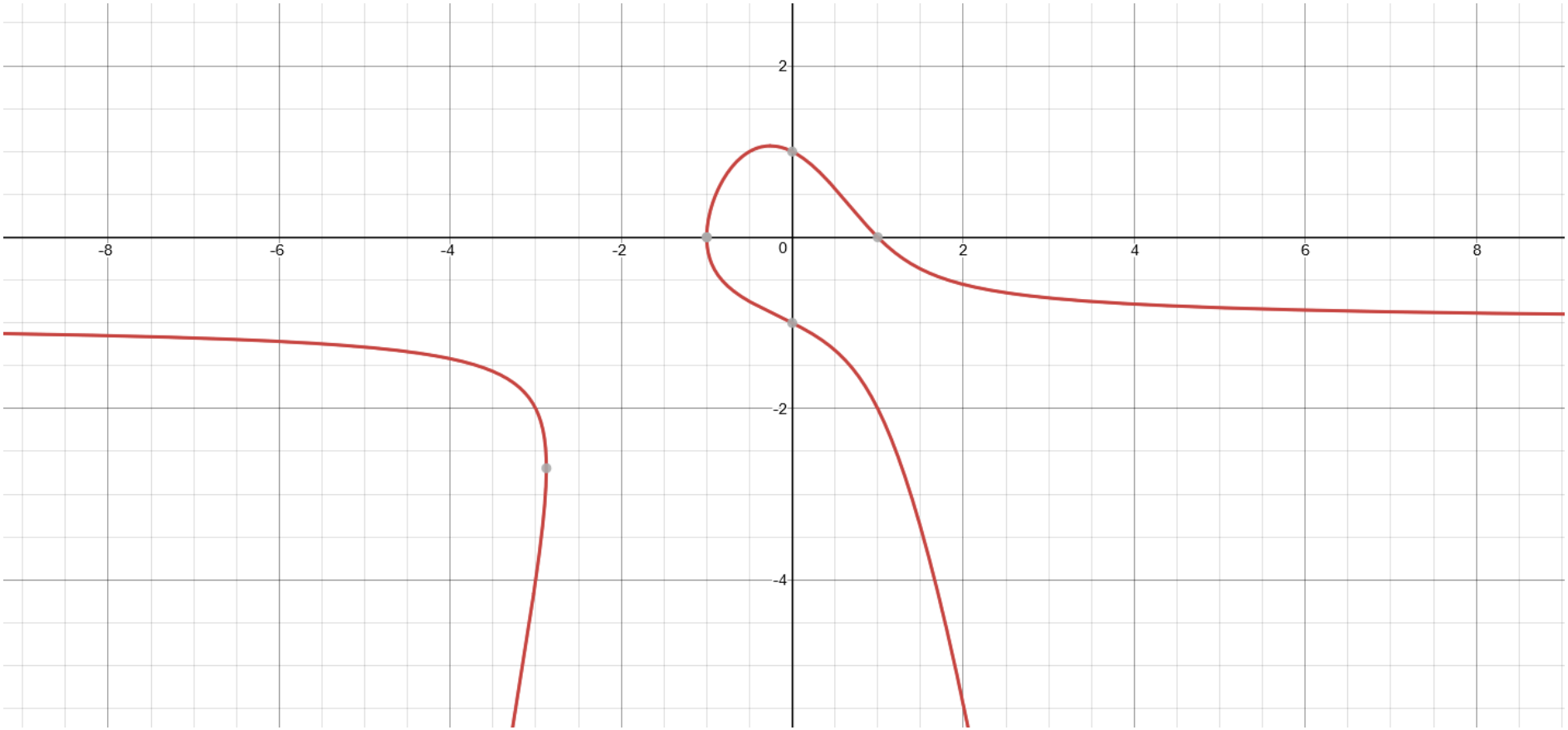

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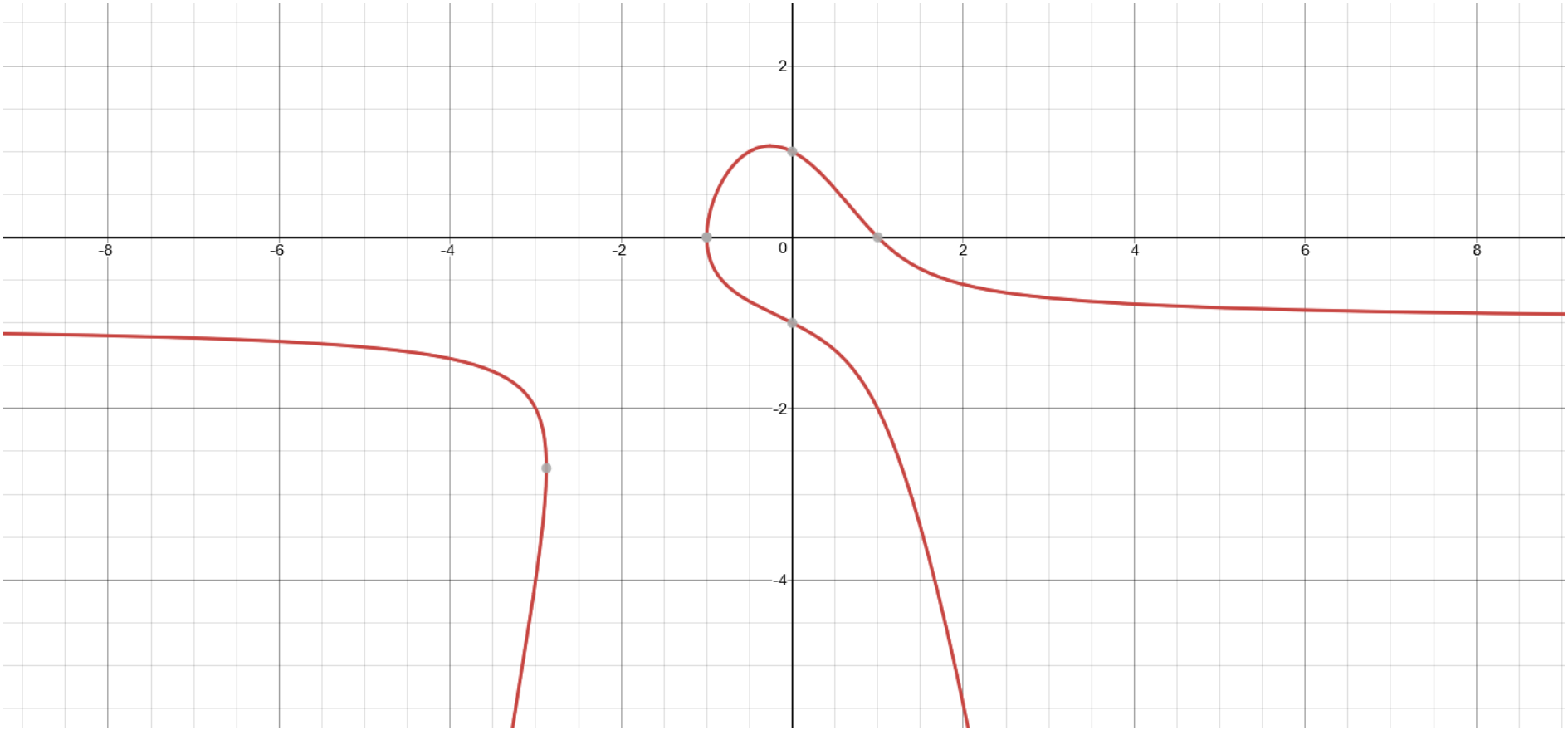

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Consider y as a function of x.

- $\frac{d}{dx}[y] =$

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$$1 = y^2 + x^2 + xy + x^2y$$

Differentiate both sides with respect to x.

$$0 = 2y\frac{dy}{dx} + 2x + \left(x\frac{dy}{dx} + y(1)\right) + \left(x^2\frac{dy}{dx} + 2xy\right)$$

$$0 = \frac{dy}{dx}\left(2y + x + x^2\right) + (2x + y + 2xy)$$

$$-(2x + y + 2xy) = \frac{dy}{dx}\left(2y + x + x^2\right)$$

$$-\frac{2x + y + 2xy}{2y + x + x^2} = \frac{dy}{dx}$$

Implicitly Defined Functions

$$y^2 + x^2 + xy + x^2y = 1$$

$$\frac{dy}{dx} = -\frac{2x + y + 2xy}{2y + x + x^2}$$

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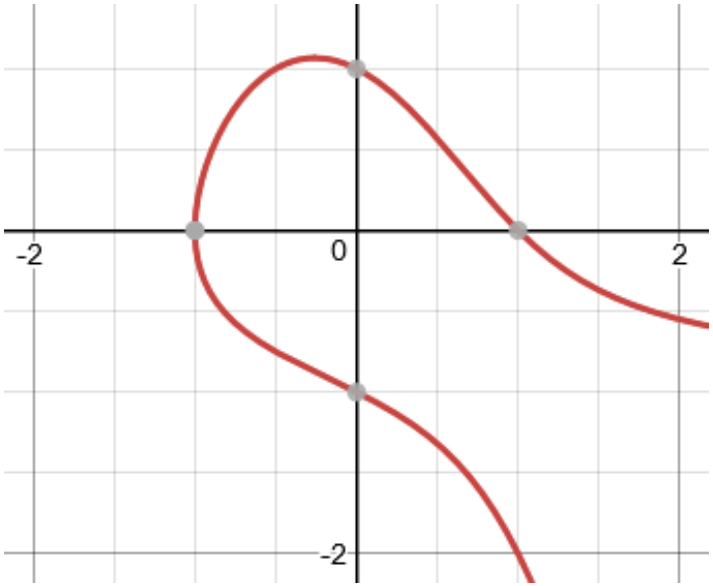
Necessarily, $\frac{dy}{dx}$ depends on **both** y and x. Why?



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$$\left.\frac{dy}{dx}\right|_{(1,0)} = -\frac{2(1) + 0 + 2(1)(0)}{2(0) + 1 + 1} = -\frac{2}{2} = -1$$

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = -\frac{2(1) - 2 + 2(1)(-2)}{2(-2) + 1 + 1} = -2$$

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Points with the same x-value may have different slopes. We need both the x-value and the y-value to figure out which point we're talking about.

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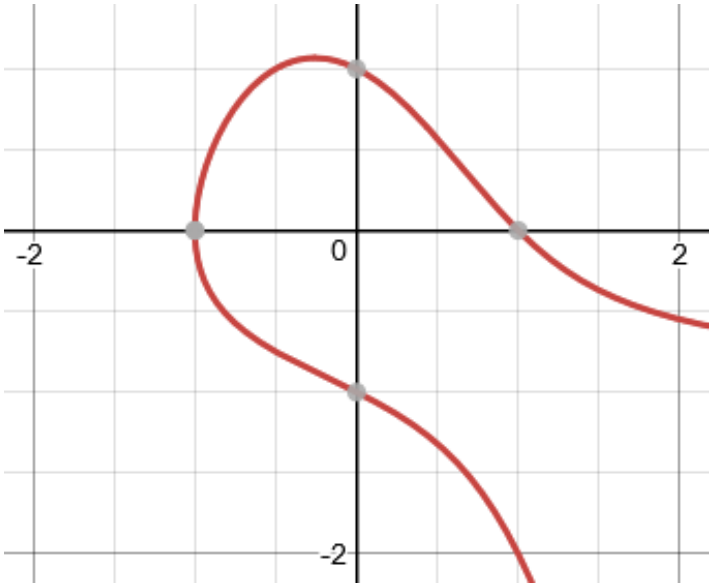

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More Practice

Example 1:

a) Suppose $x^4y + y^4x = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.

b) Suppose $\dfrac{3y^2 + y + y^3}{x^2 + 1} = x$. Find $\frac{dy}{dx}$ when $x = 0$, and the equations of the associated tangent line(s).

(a)

$$
\begin{aligned}
x^4y + y^4x &= 2 \\
4x^3y + x^4\frac{dy}{dx} + y^4 + 4y^3\frac{dy}{dx}x &= 0 \\
4 + \frac{dy}{dx} + 1 + 4\frac{dy}{dx} &= 0 \\
5\frac{dy}{dx} &= -5 \\
\frac{dy}{dx} &= -1
\end{aligned}
$$

(b)

$$\frac{3y^2+y+y^3}{x^2+1}=x$$

$$\frac{(x^2+1)\left(6y\frac{dy}{dx}+\frac{dy}{dx}+3y^2\frac{dy}{dx}\right)-(3y^2+y+y^3)(2x)}{(x^2+1)^2}=1$$

$$\left(6y\frac{dy}{dx}+\frac{dy}{dx}+3y^2\frac{dy}{dx}\right)=1$$

We need to know y.

$$3y^2+y+y^3=0$$

$$y(y^2+3y+1)=0$$

$$y=0, y=\frac{-3\pm\sqrt{9-4}}{2}=\frac{-3\pm\sqrt{5}}{2}$$

At the point $(0, 0)$:

$$\left(6y\frac{dy}{dx} + \frac{dy}{dx} + 3y^2\frac{dy}{dx}\right) = 1$$
$$\frac{dy}{dx} = 1$$

So, the equation of the tangent line is $y = x$.

At the point $\left(0, \frac{-3-\sqrt{5}}{2}\right)$:

$$\left(6y\frac{dy}{dx} + \frac{dy}{dx} + 3y^2\frac{dy}{dx} + 1\right) = 1$$

$$\frac{dy}{dx}\left(6\left(\frac{-3-\sqrt{5}}{2}\right) + 3\left(\frac{-3-\sqrt{5}}{2}\right)^2 + 1\right) = 1$$

$$\frac{dy}{dx}\left(-9 - 3\sqrt{5} + \frac{3}{4}\left(14 + 6\sqrt{5}\right) + 1\right) = 1$$

$$\frac{dy}{dx}\left(1.5\sqrt{5} + 2.5\right) = 1$$

$$\frac{dy}{dx} = \frac{2}{3(2+\sqrt{5})}$$

So, the equation of the tangent line is

$$y + \frac{3+\sqrt{5}}{2} = \frac{2x}{3\sqrt{5}+5}$$

At the point $\left(0, \frac{-3+\sqrt{5}}{2}\right)$:

$$\left(6y\frac{dy}{dx} + \frac{dy}{dx} + 3y^2\frac{dy}{dx} + 1\right) = 1$$

$$\frac{dy}{dx}\left(6\left(\frac{-3+\sqrt{5}}{2}\right) + 3\left(\frac{-3+\sqrt{5}}{2}\right)^2 + 1\right) = 1$$

$$\frac{dy}{dx}\left(-9 + 3\sqrt{5} + \frac{3}{4}\left(14 - 6\sqrt{5}\right) + 1\right) = 1$$

$$\frac{dy}{dx}\left(-1.5\sqrt{5} + 2.5\right) = 1$$

$$\frac{dy}{dx} = \frac{2}{5 - 3\sqrt{5}}$$

So, the equation of the tangent line is

$$y - \frac{-3+\sqrt{5}}{2} = \frac{2x}{5 - 3\sqrt{5}}$$

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We found $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ using Pascal's Triangle.

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Implicit:

$$\begin{aligned} y &= \sqrt{x} \\ y^2 &= x \\ 2y\frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{2y} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{1/2-1} \end{aligned}$$

Differentiate $y = x^{m/n}$, where m and n are whole numbers.

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$$
\begin{aligned}
y^n &= x^m \\
ny^{n-1}\frac{dy}{dx} &= mx^{m-1}\frac{dy}{dx} \\
\frac{dy}{dx} &= \frac{mx^{m-1}}{ny^{n-1}} \\
&= \frac{mx^{m-1}}{n\left(x^{m/n}\right)^{n-1}} \\
&= \frac{m}{n}\left(\frac{x^{m-1}}{x^{m-m/n}}\right) \\
&= \frac{m}{n}\left(x^{m-1-m+m/n}\right) \\
&= \frac{m}{n}\left(x^{m/n-1}\right)
\end{aligned}
$$