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- Product and quotient rules
- Chain rule

Power Rule

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Tangent Line

$$\frac{\mathrm{d}}{\mathrm{d}x}\{x^n\} = nx^{n-1}$$

Example 1: Write an equation of the tangent line described.

- Tangent line to the function $y = x^2$ at the point $(-5, 25)$.

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The line with slope m, passing through the point (x_1, y_1), has equation

$$(y - y_1) = m(x - x_1)$$

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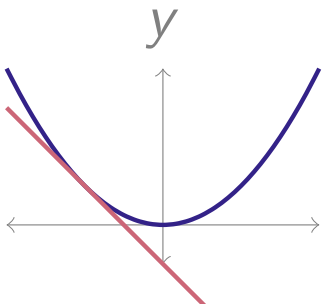

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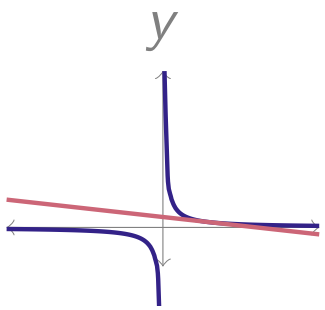

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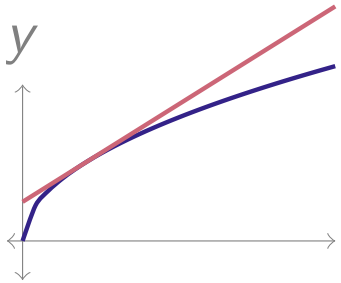

Solution 1: Write an equation of the tangent line described.

- Tangent line to the function $y = x^2$ at the point $(-5, 25)$.
  Using the power rule, we find the derivative of this function is $2x$, so when $x = -5$, its derivative is $2(-5) = -10$. Now we can describe the tangent line in point-slope form with $m = -10$, $x_1 = -5$, and $y_1 = 25$:
  $$y - 25 = -10(x + 5)$$
- Tangent line to the function $y = \frac{1}{x}$ at the point $(3, \frac{1}{3})$.
  Note $y = x^{-1}$. Using the power rule, we find the derivative of this function is $-x^{-2}$, so when $x = 3$, its derivative is $-(3^{-2}) = -\frac{1}{9}$. Now we can describe the tangent line in point-slope form with $m = -\frac{1}{9}$, $x_1 = 3$, and $y_1 = \frac{1}{3}$:
  $$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$
- Tangent line to the function $y = \sqrt{x}$ at the point $x = 4$.
  Note $y = x^{1/2}$, and if $x = 4$, then $y = 2$. Using the power rule, we find the derivative of this function is $\frac{1}{2}x^{-1/2}$, so when $x = 4$, its derivative is $\frac{1}{2} \cdot \frac{1}{4^{1/2}} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. Now we can describe the tangent line in point-slope form with $m = \frac{1}{4}$, $x_1 = 4$, and $y_1 = 2$:
  $$y - 2 = \frac{1}{4}(x - 4)$$

The derivative is a linear operator

The derivative satisfies several convenient properties, among them:

1. a constant multiplying a function can be brought outside the derivative; and
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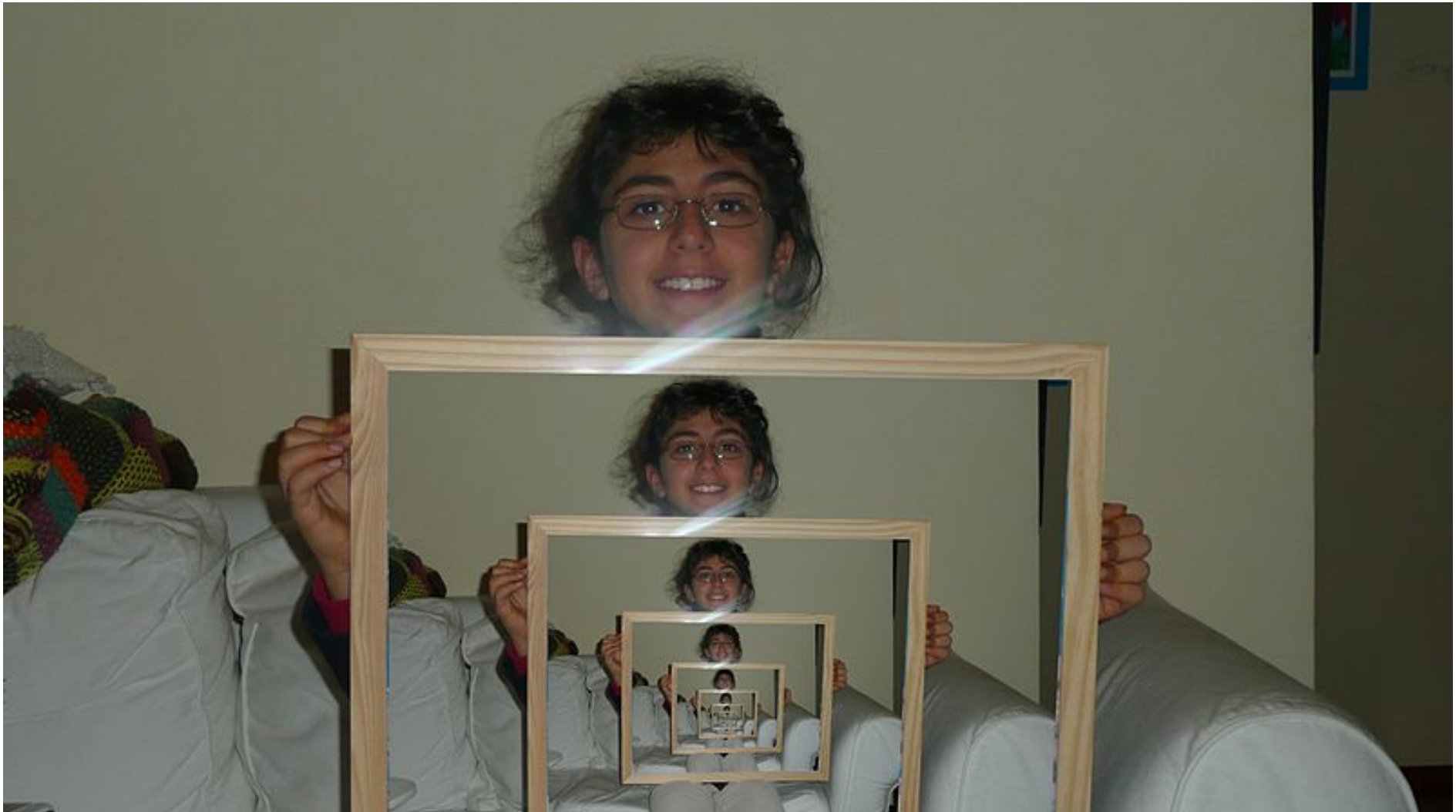

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The derivative of a derivative is called the **second derivative**, written

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Similarly, the derivative of a second derivative is a third derivative, etc.

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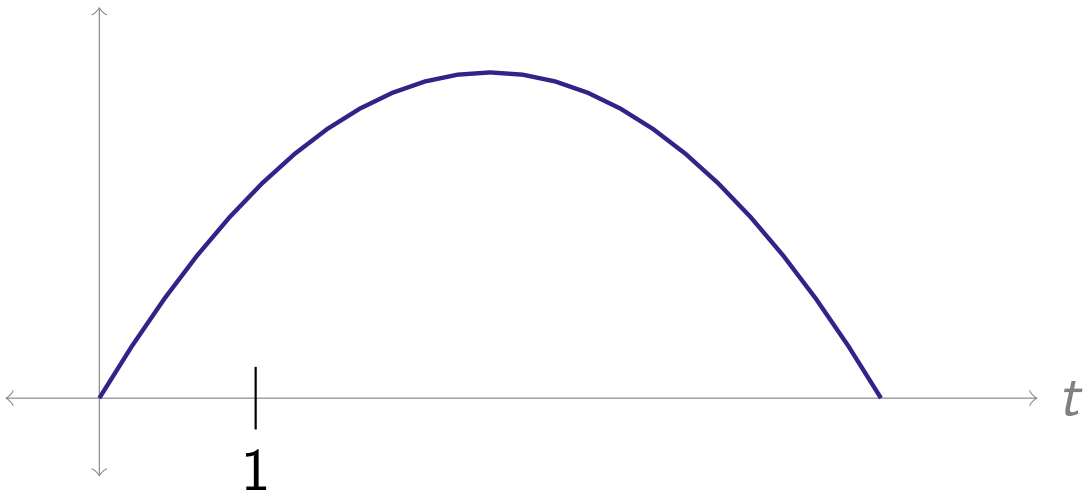

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$$s'(1) = 5 - 2(1) = 3 = \text{vel} \qquad -2 = \text{acc}$$

Concept Check

True or False: If $f'(1) = 18$, then $f''(1) = 0$, since the derivative of the constant 18 is 0.

Which of the following is always true of a QUADRATIC polynomial $f(x)$?

A. $f(0) = 0$
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We can only determine antiderivatives up to some additive constant.

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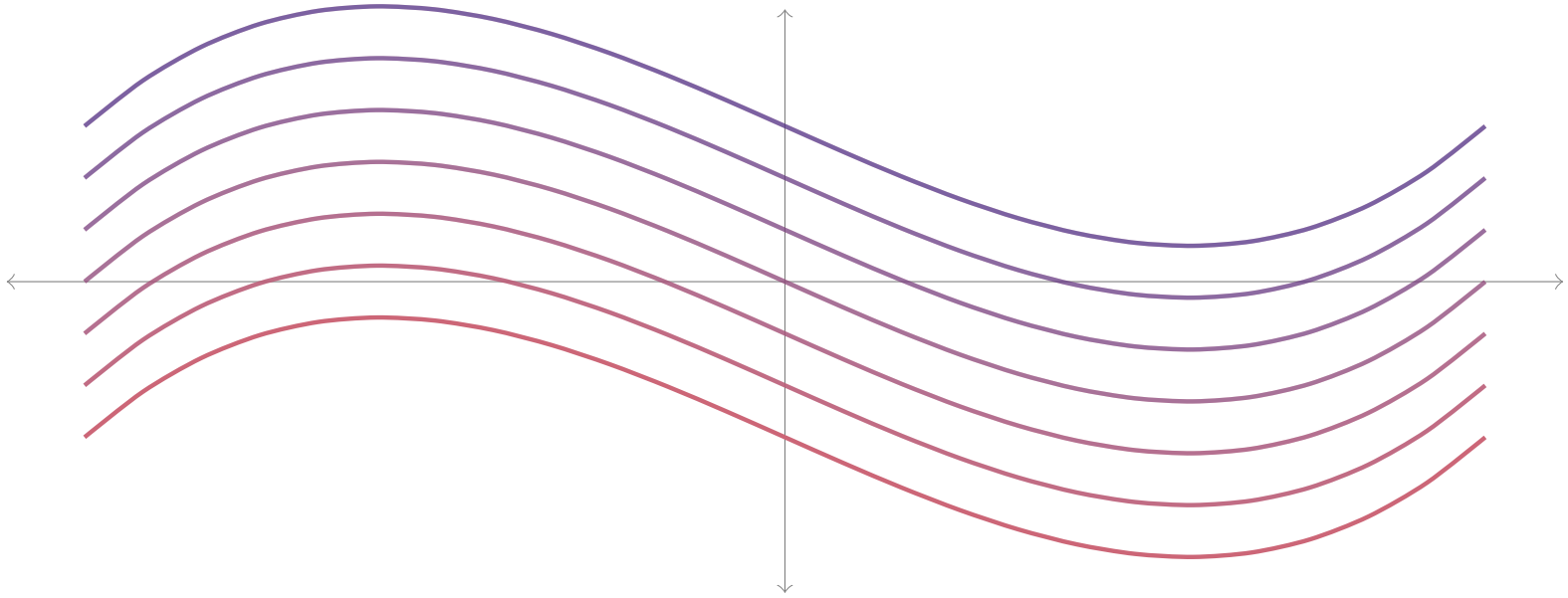

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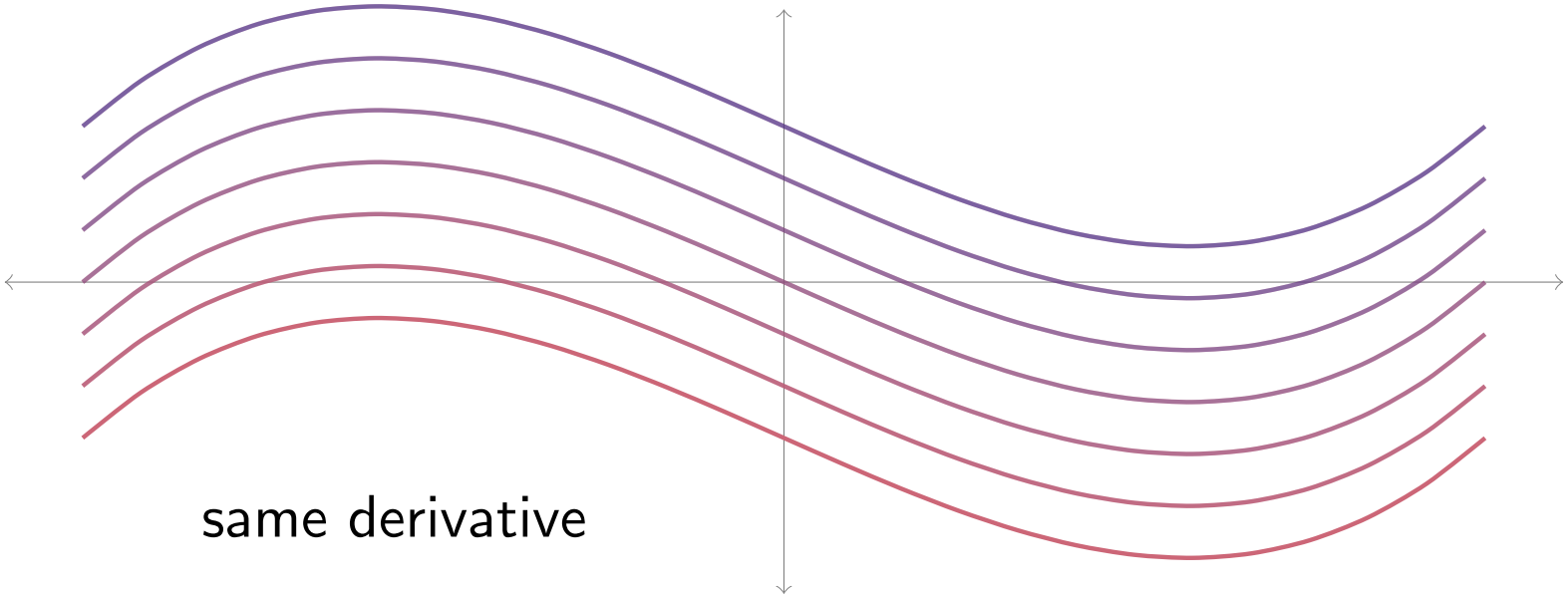

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The antiderivative of x is $\frac{1}{2}x^2 + C$.

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Max Pixel

Antiderivatives of Power Functions and Polynomials

The dorsal fin of a breaching orca accelerates at -10 m/sec^2.

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   $s\left(\frac{C}{10}\right) - s\left(\frac{C}{10} - 1\right) = 5$ m

Facing a Long Problem

Keep in mind:

- **What (exactly) are you doing?** (Can you describe it precisely?)
- **Why are you doing it?** (How does it fit into the solution?)
- **How does it help you?** (What will you do with the outcome when you obtain it?)

Alan H. Schoenfeld, *Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics*, 1992 (link)

Product and Quotient Rules

- True or false:

$$\frac{\mathrm{d}}{\mathrm{d}x}\{x^2 + x^3\} = \frac{\mathrm{d}}{\mathrm{d}x}\{x^2\} + \frac{\mathrm{d}}{\mathrm{d}x}\{x^3\} = 2x + 3x^2$$

- True or false:

$$\frac{\mathrm{d}}{\mathrm{d}x}\{7x^3\} = \frac{\mathrm{d}}{\mathrm{d}x}\{7\} \cdot \frac{\mathrm{d}}{\mathrm{d}x}\{x^3\} = 0 \cdot 3x^2 = 0$$

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Where the Product Rule Comes From

You won't be assessed on this–it's only for background knowledge

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big\{f(x)g(x)\Big\} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

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fancy algebra

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$$= \boxed{f(x)g'(x)+g(x)f'(x)}$$

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$$f'(x) = (x^2 + 5)(6x^5 - 2x + 3) + (x^6 - x^2 + 3x)(2x)$$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{f(x)}{g(x)}\right\} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \quad \text{(def of deriv)}$$

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$$= \boxed{\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}}$$

Product and Quotient Rules

Theorem

If $f(x)$ and $g(x)$ are differentiable, and $g(x) \neq 0$, then:

- $\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)g(x)\} = f(x)g'(x) + g(x)f'(x)$ product rule
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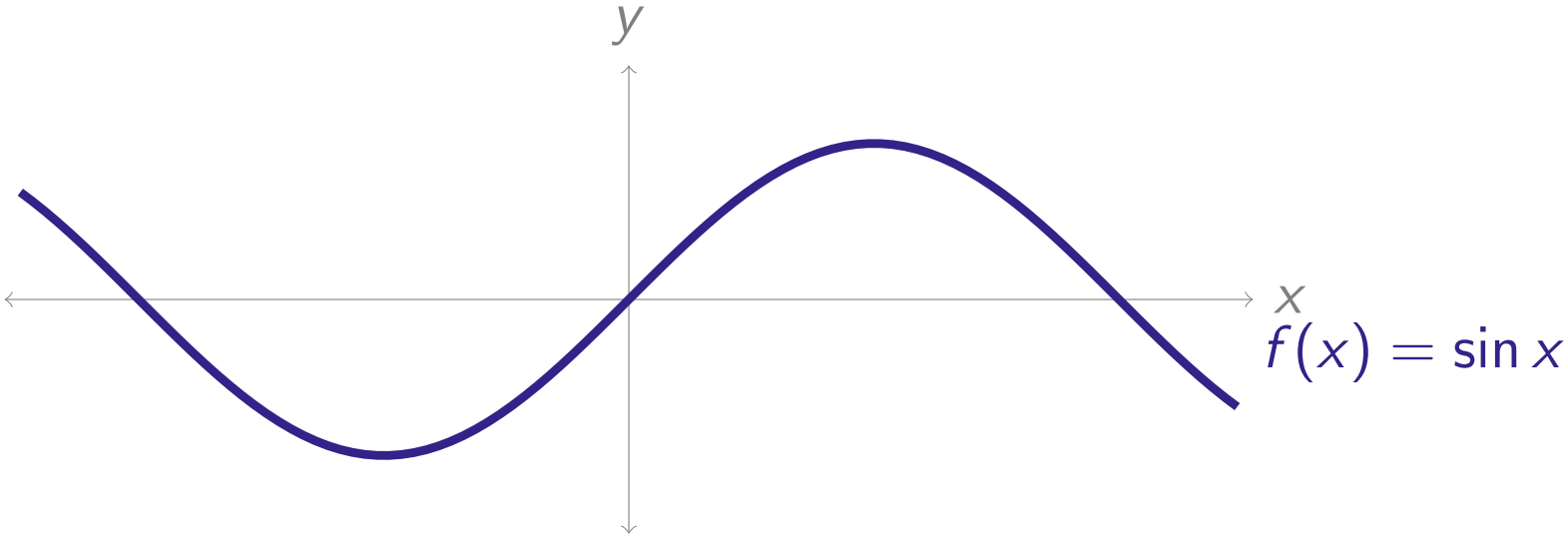
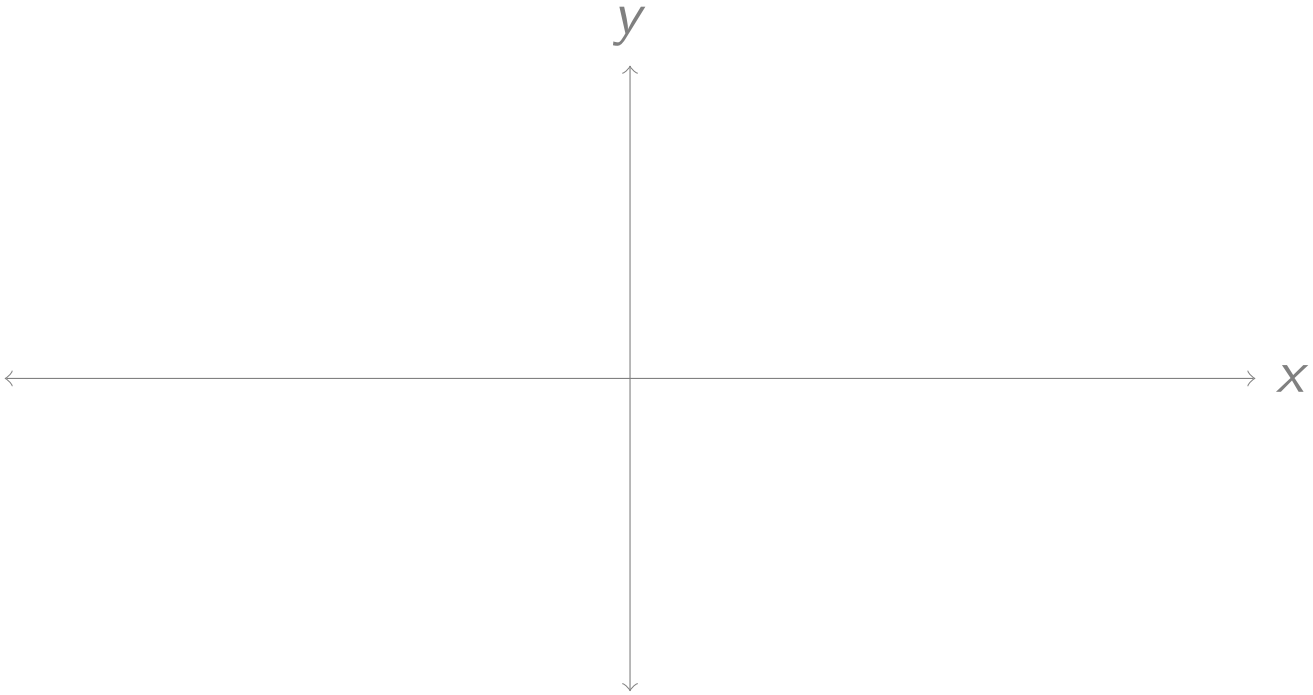
$$g(x) = (x^3+1)(x^2+2) \qquad g'(x) = (x^3+1)(2x) + (x^2+2)(3x^2)$$

$$= 5x^4 + 6x^2 + 2x$$

Intuition: $\sin x$ versus $\sin(2x)$





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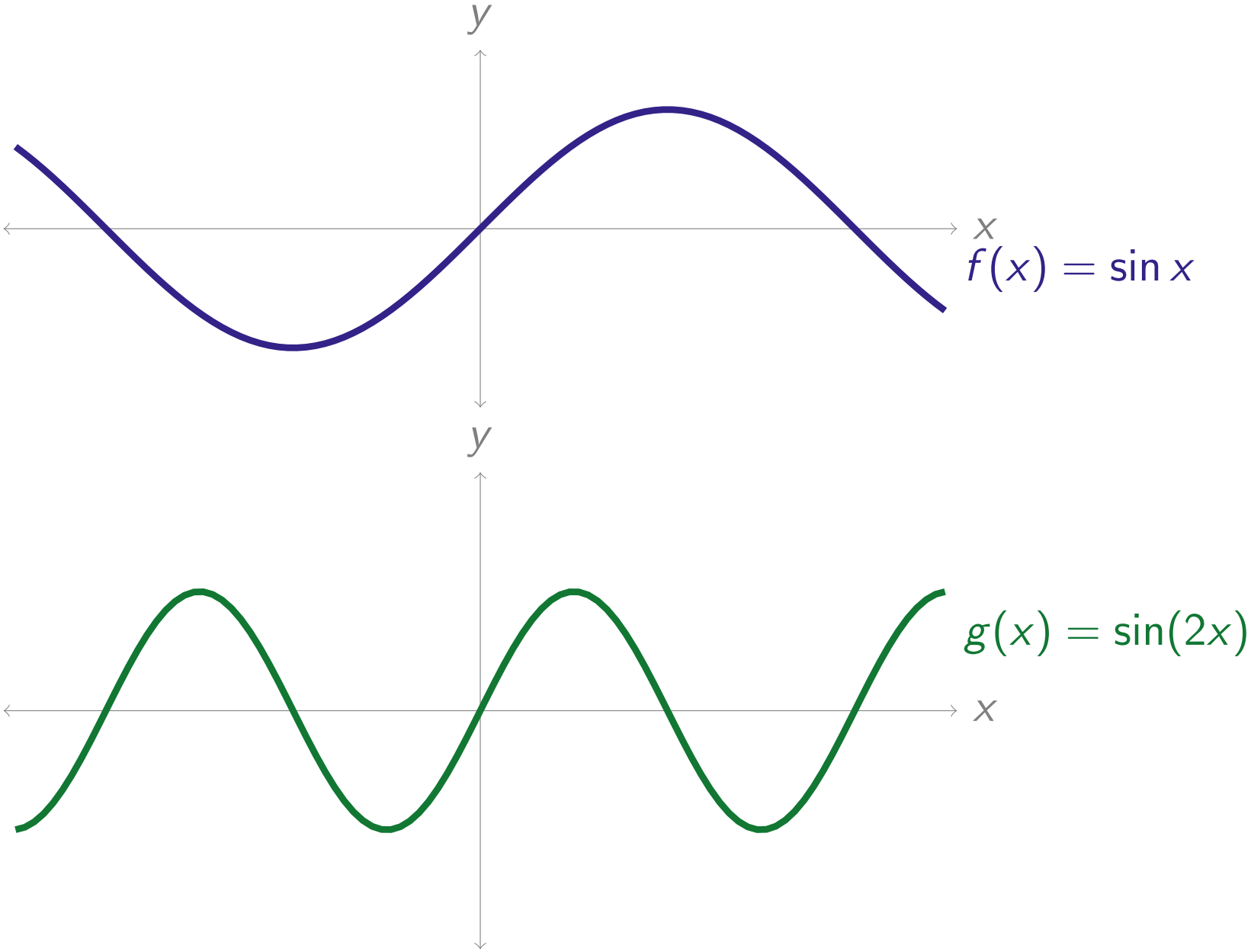

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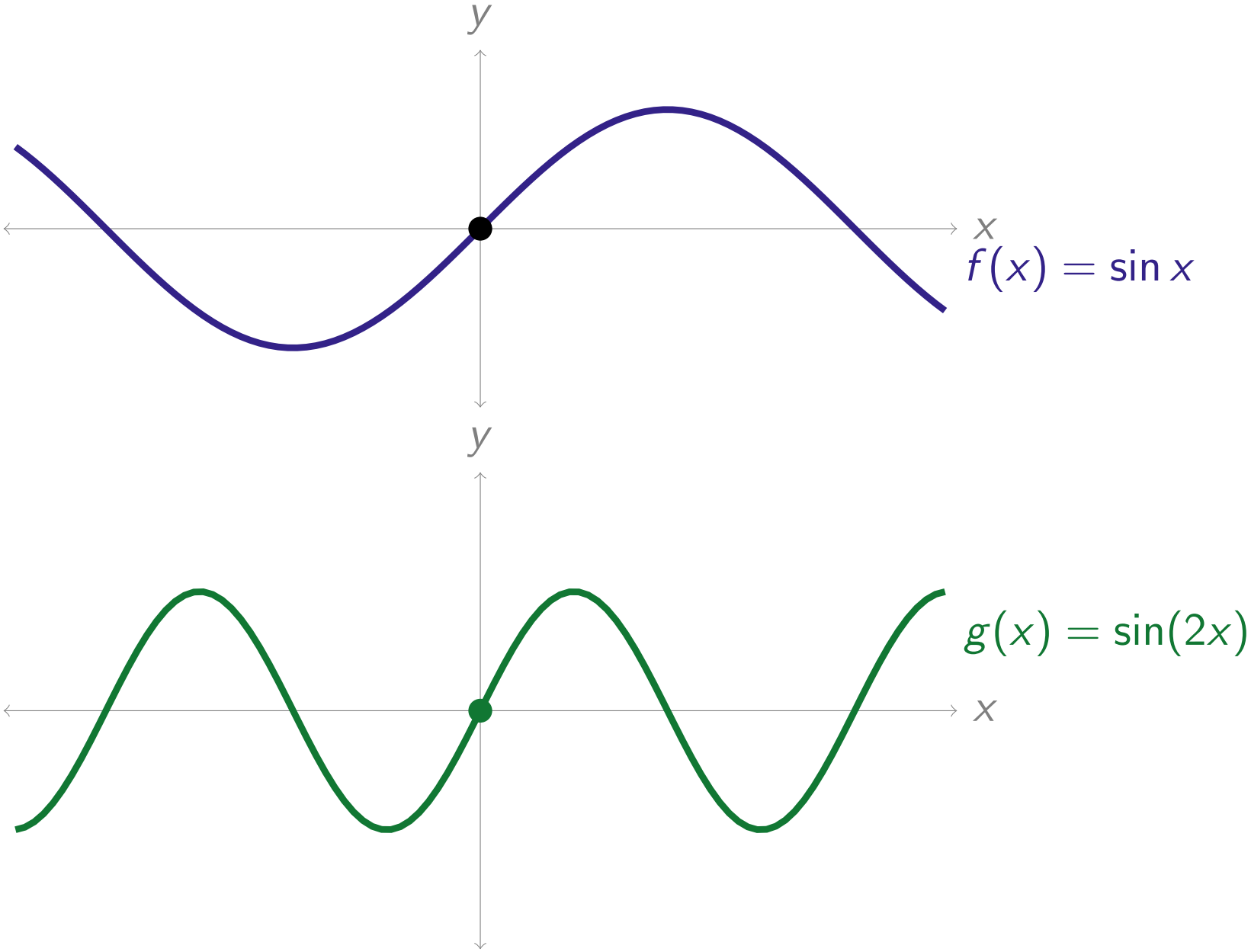

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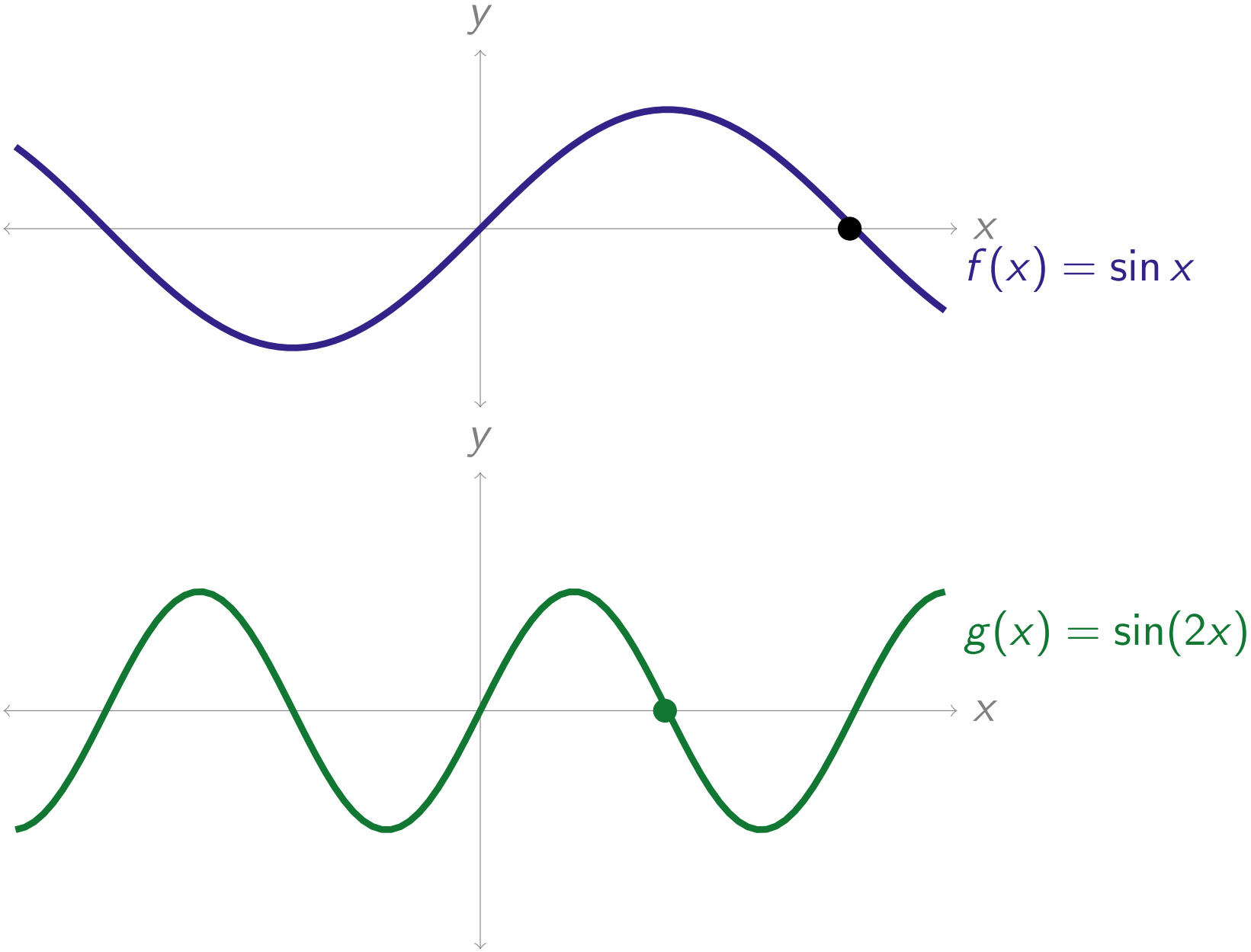

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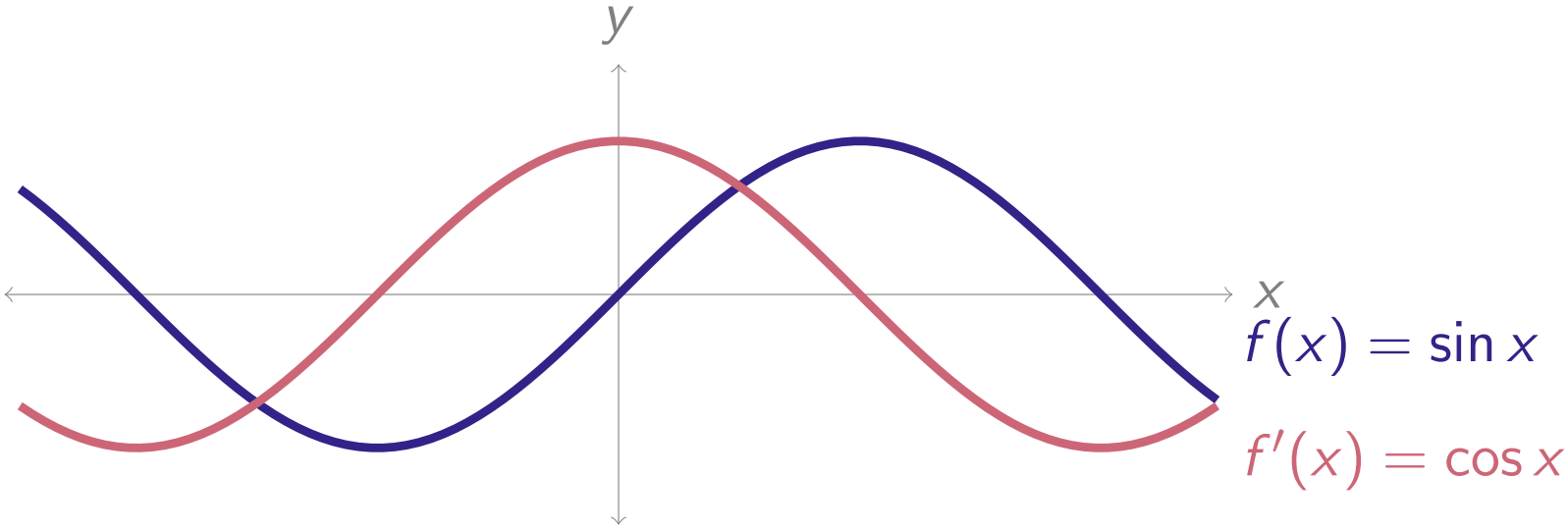

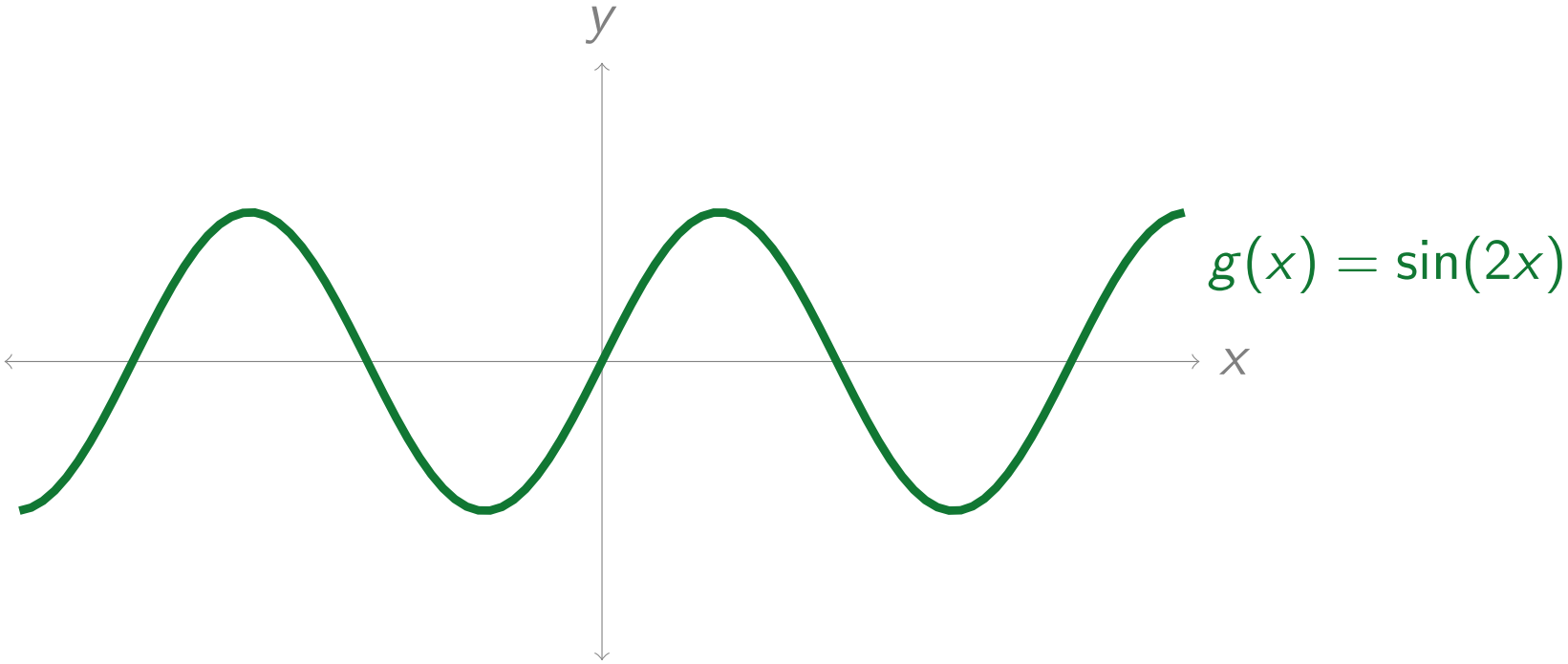

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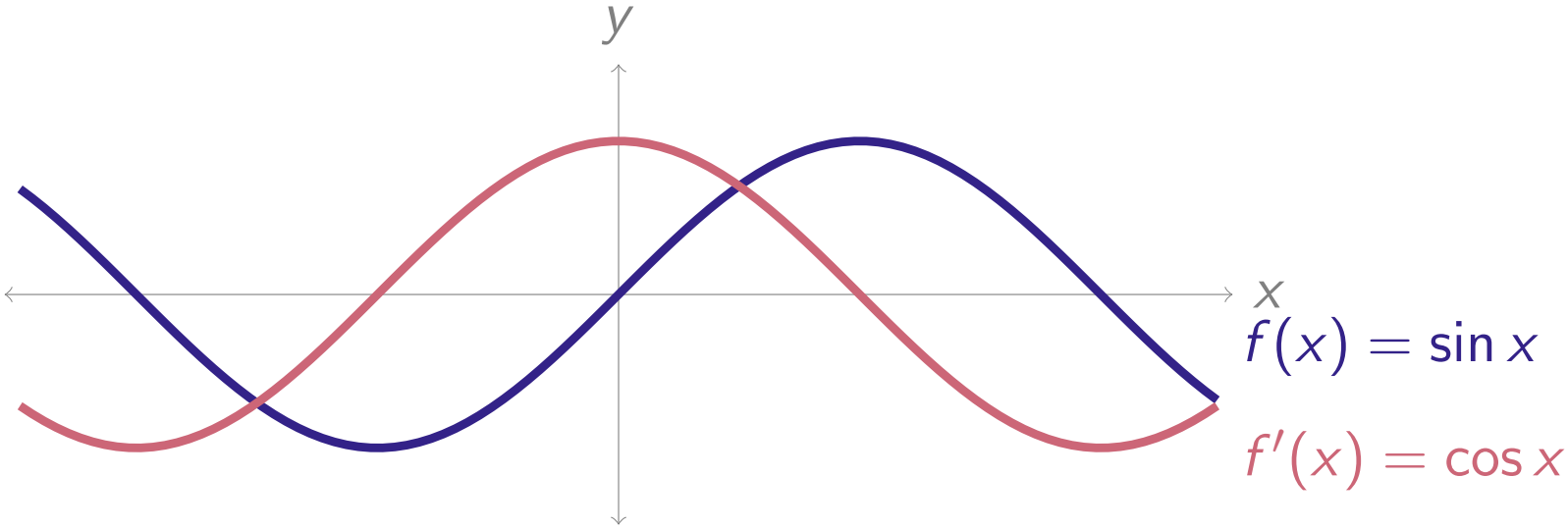

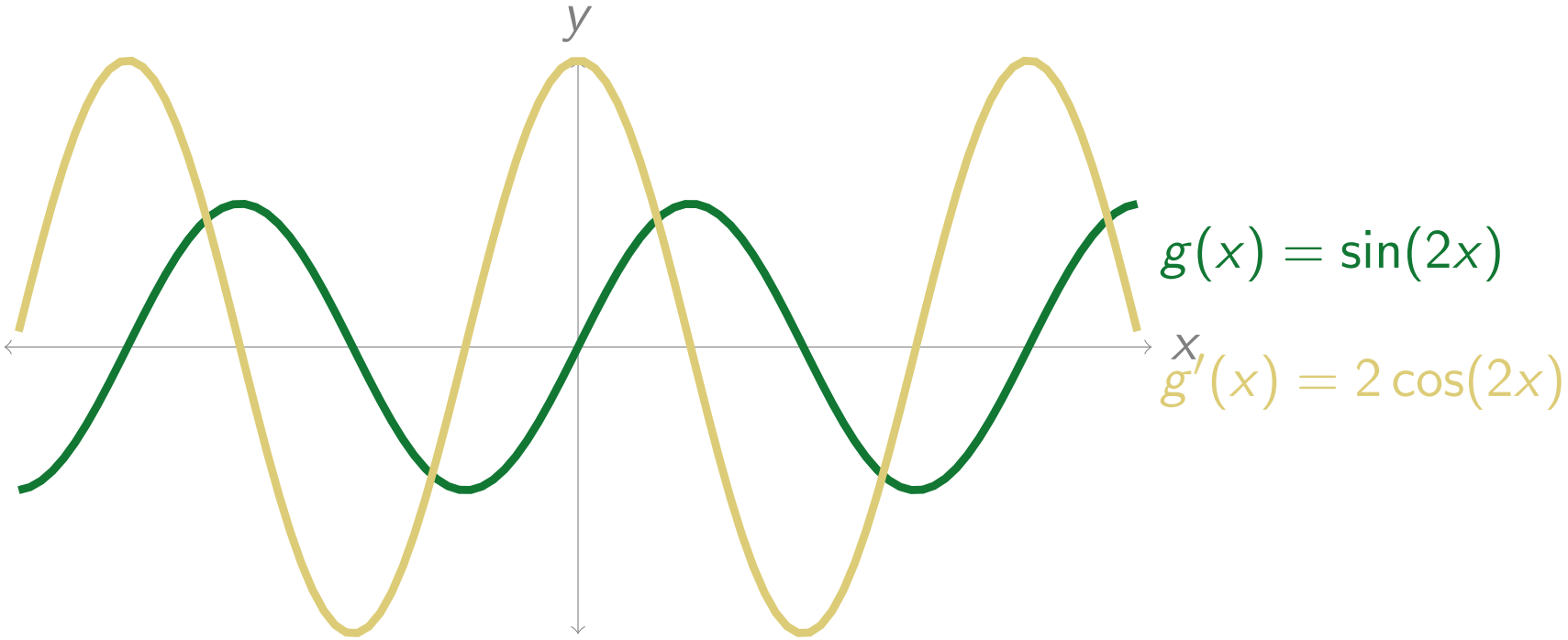

Compound Functions

Kelp Population

k	kelp population
u	urchin population
o	otter population

Kelp Population

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$k = k(u)$

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In the case of kelp, $\dfrac{dk}{do} = \dfrac{dk}{du}\dfrac{du}{do}$

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$$= 6\left(\boxed{\frac{v}{v^3+1}}\right)^5 \cdot \frac{-2v^3+1}{(v^3+1)^2}$$

Chain Rule Practice

1. Differentiate $f(x) = (x^2 + x^3)^4$
2. Differentiate $g(x) = \sqrt{\dfrac{x}{x+1}}$
3. Antidifferentiate $h'(x) = 5(x^3 + x^2)^4(3x^2 + 2x)$

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   By inspection, $h(x) = (x^3 + x^2)^5$

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When $t = 0$, what it the object's position? Its velocity?

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Velocity: B Position: C

Acceleration

Position

If an object accelerates at a constant rate a, then its position is given by

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Example 4:

(a) You toss a ball in the air with an initial upwards velocity of 3 metres per second. Assuming the acceleration due to gravity is a constant $a = -9$ m/sec^2, how long is the ball in the air before it falls back in your hand?
(b) You toss a ball in the air and the acceleration due to gravity is constant. If the ball falls back into your hand two seconds later, at what time did the ball reach its highest point?

Acceleration

Solution 4:

(a) You toss a ball in the air with an initial upwards velocity of 3 metres per second. Assuming the acceleration due to gravity is a constant $a = -9$ m/sec^2, how long is the ball in the air before it falls back in your hand?

If your hand is at height 0, and you throw the ball at time $t = 0$, then $s(t) = -\frac{9}{2}t^2 + 3t = t\left(3 - \frac{9}{2}t\right)$, so the ball is back at your hand when $0 = \left(3 - \frac{9}{2}t\right)$, i.e. at time $t = \frac{2}{3}$. The ball is in the air for two-thirds of a second.

(b) You toss a ball in the air and the acceleration due to gravity is constant. If the ball falls back into your hand two seconds later, at what time did the ball reach its highest point

If a is constant, then the position function is a parabola, and parabolas are symmetric, so it reached its highest point after 1 second.

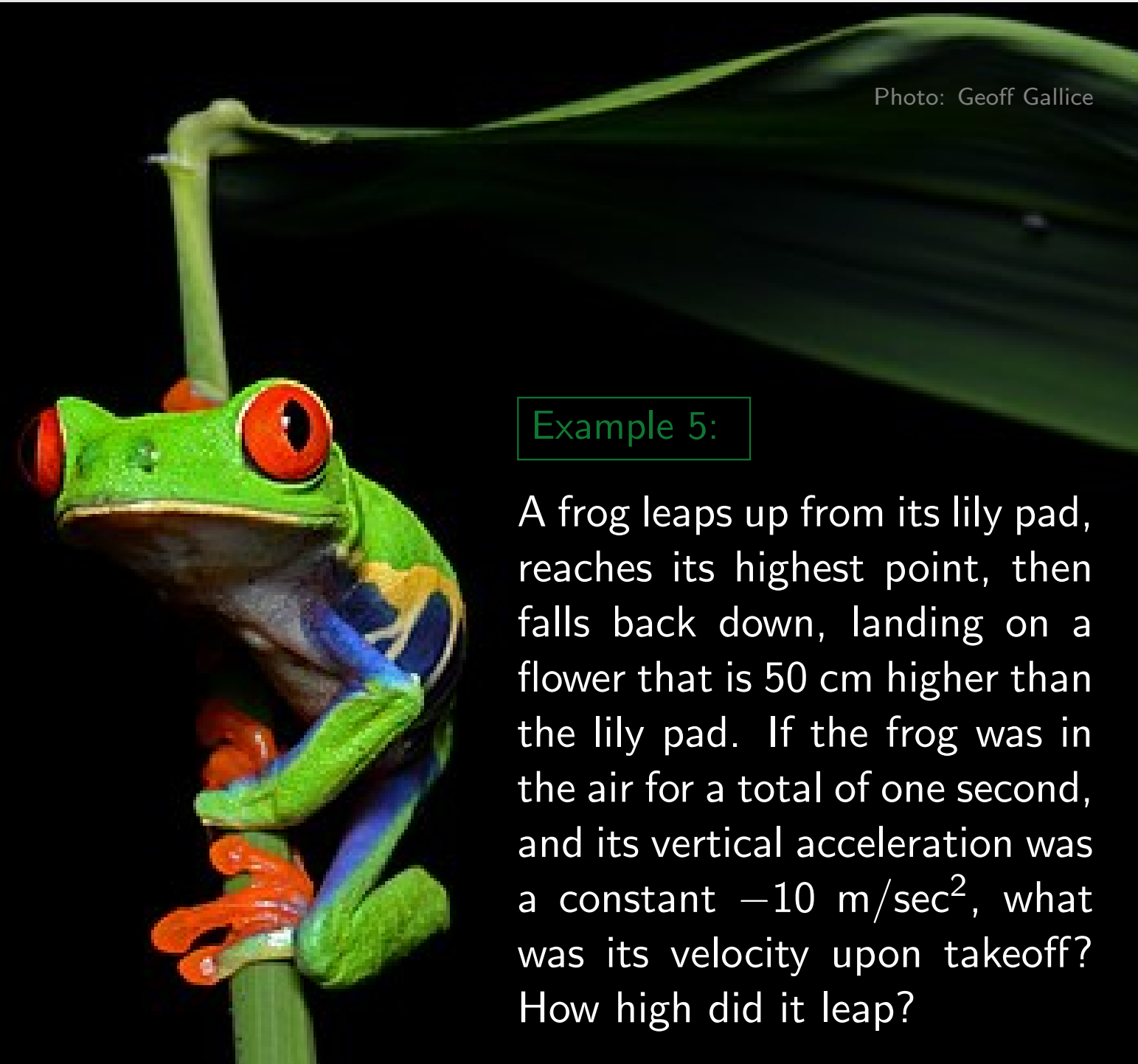


Photo: Geoff Gallice

Example 5:

A frog leaps up from its lily pad, reaches its highest point, then falls back down, landing on a flower that is 50 cm higher than the lily pad. If the frog was in the air for a total of one second, and its vertical acceleration was a constant -10 m/sec^2, what was its velocity upon takeoff? How high did it leap?

The position of the frog is

$$s(t) = \frac{1}{2}(-10)t^2 + v_0 t = -5t^2 + v_0 t,$$

if we take its initial height to be 0.
After one second, its height is

$$s(1) = -5 + v_0 t = \frac{1}{2}$$

Solving, we see its initial velocity was

$$v_0 = 5.5 \text{ m/sec}$$

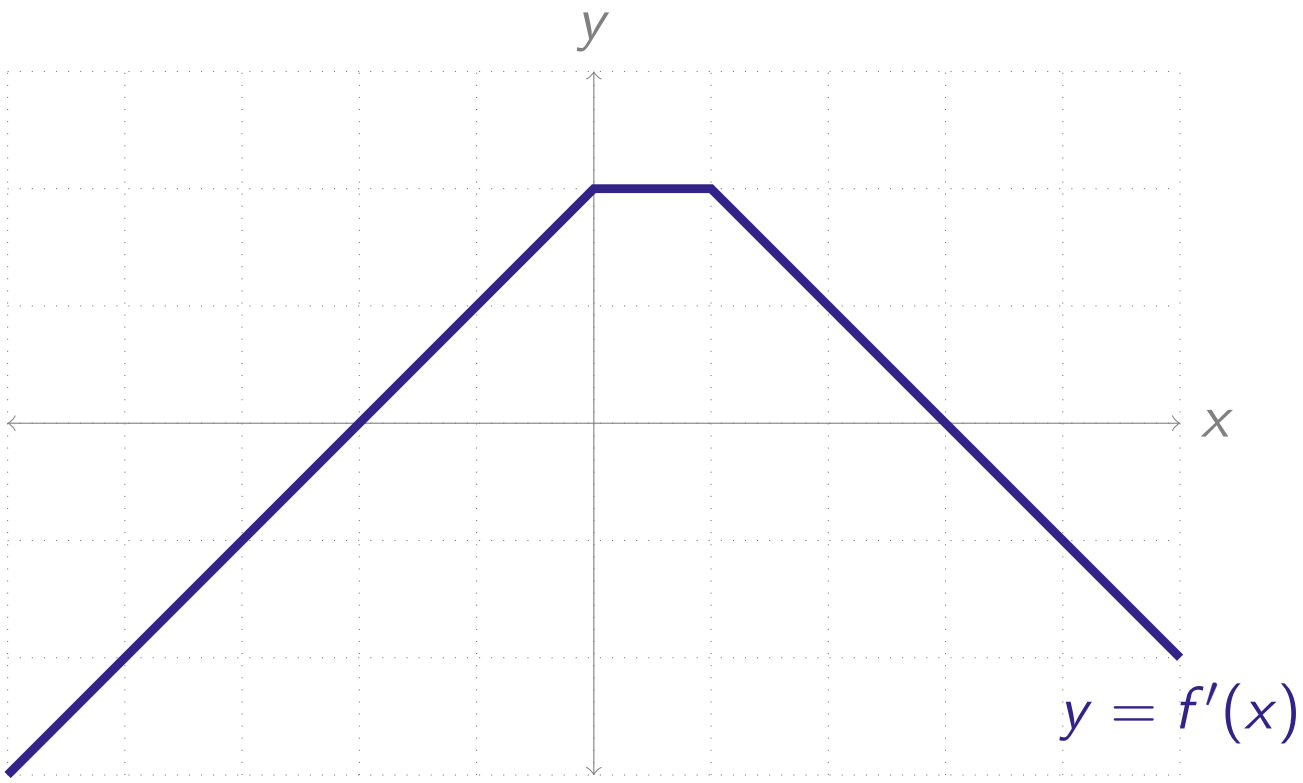
To find its maximum height, we observe that this happens when its velocity is zero. Differentiating,

$$s'(t) = -10t + v_0 = -10t + 5.5$$

So, its velocity is zero when $t = 0.55$. At this point, its height is

$$s(0.55) = -5(0.55)^2 + 5.5(0.55) = 1.5125 \text{ m}$$

Sketching Antiderivatives



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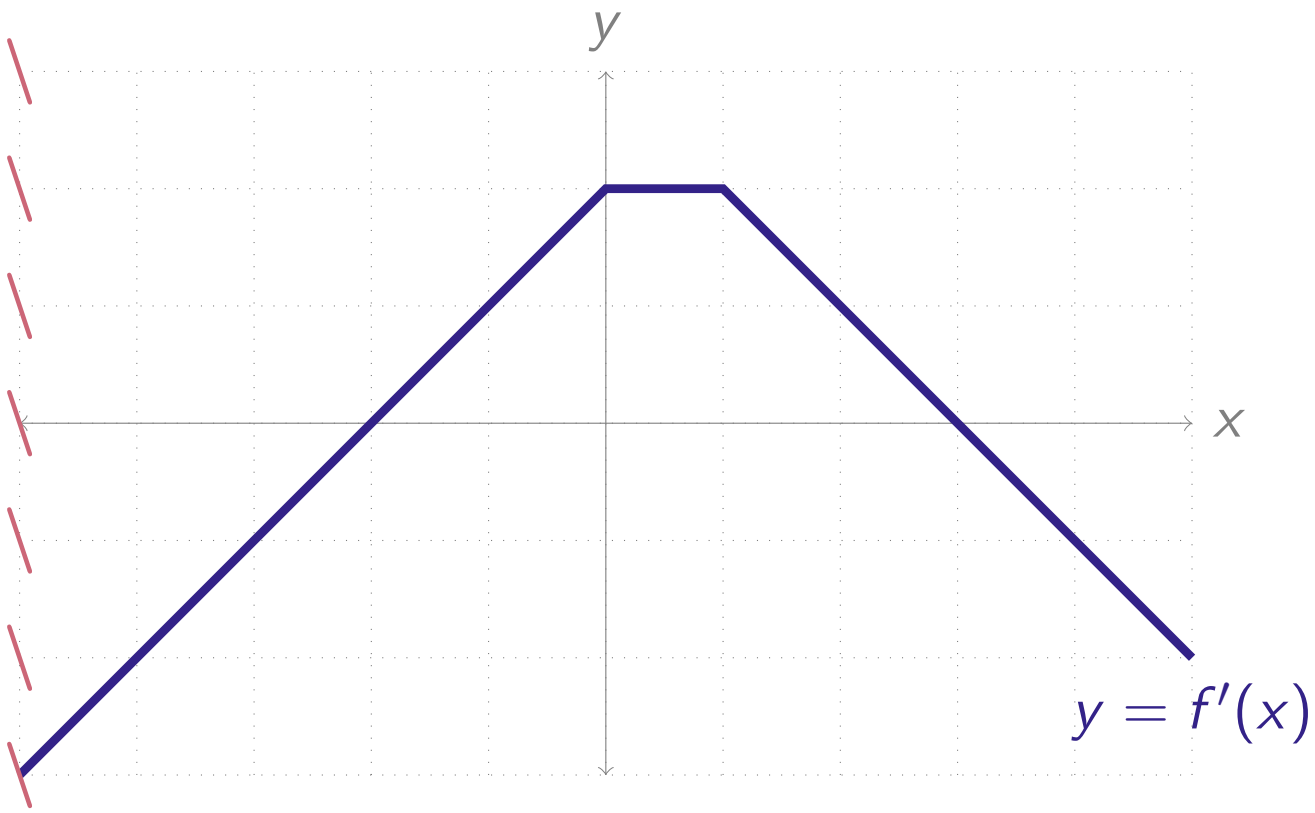

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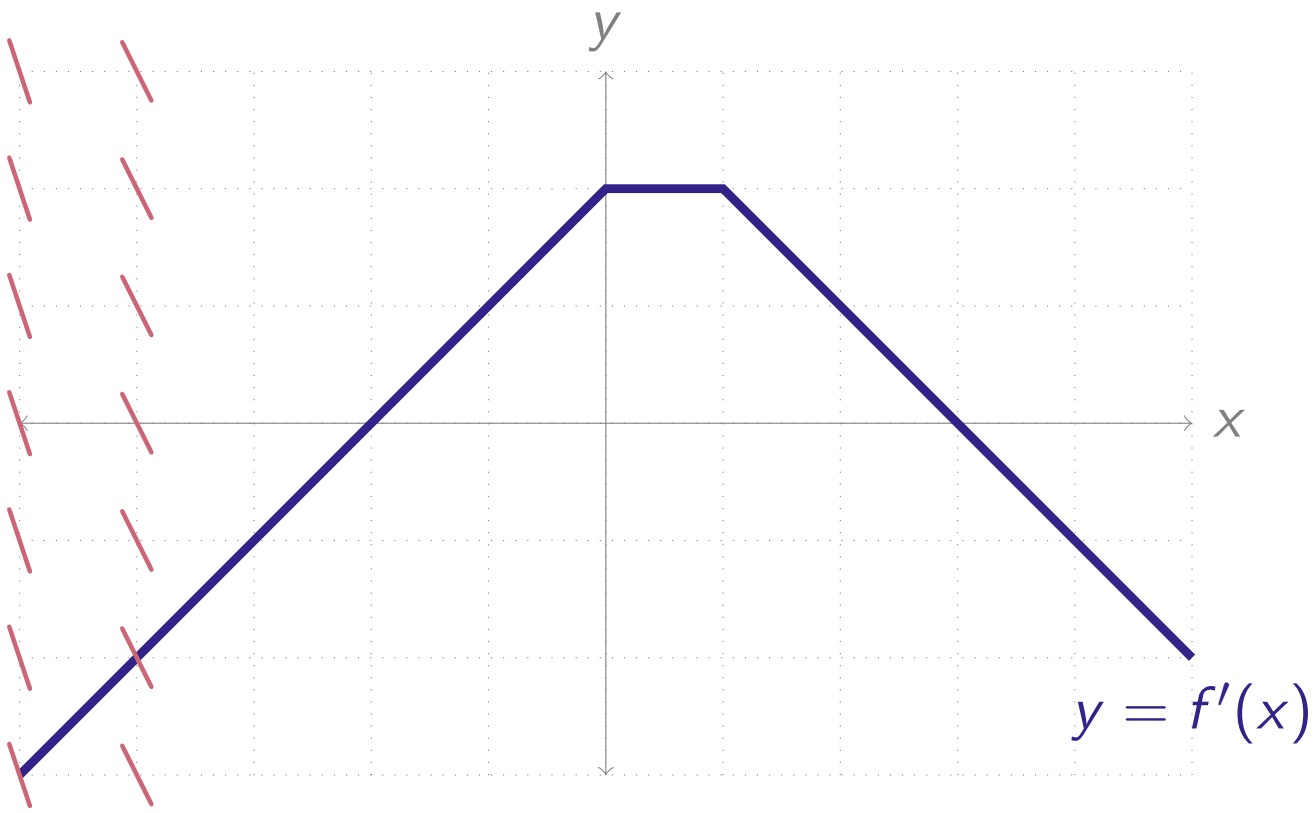

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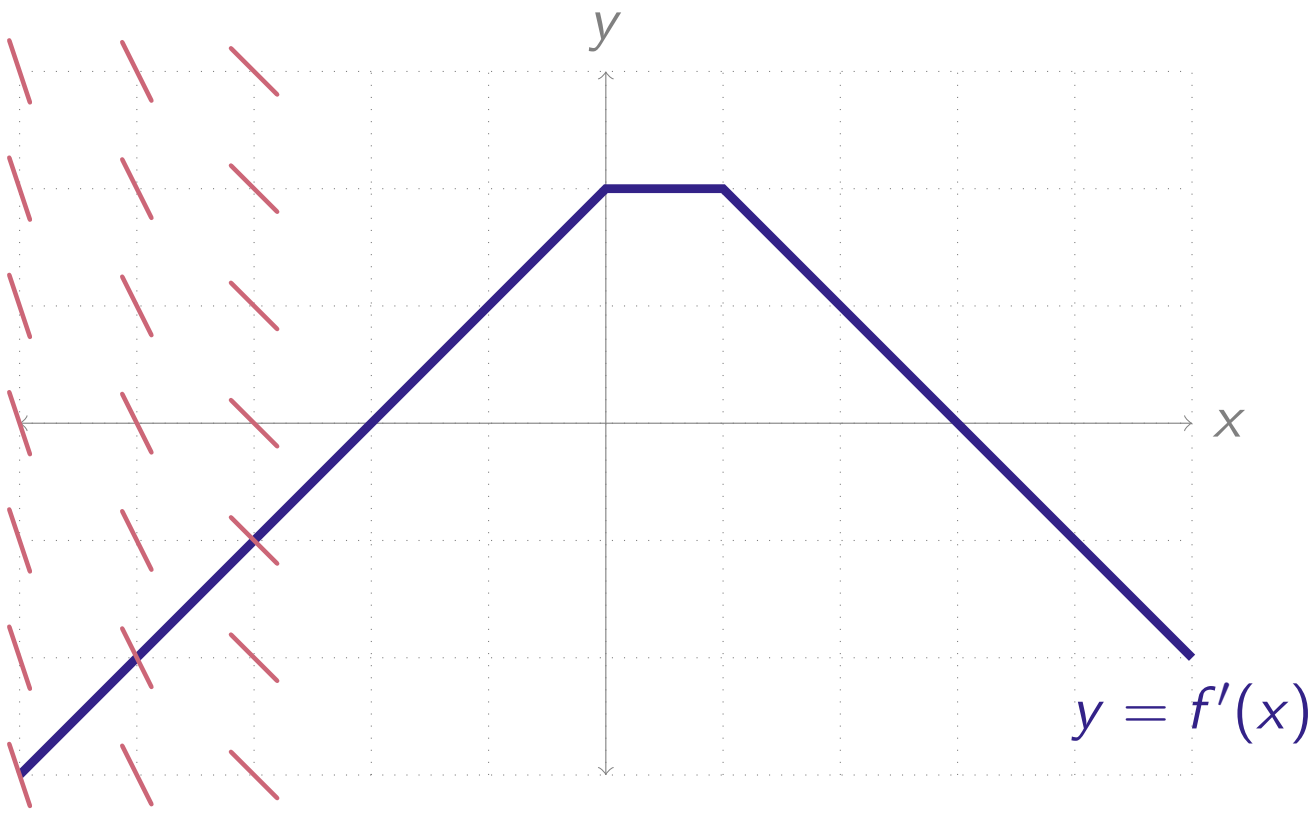

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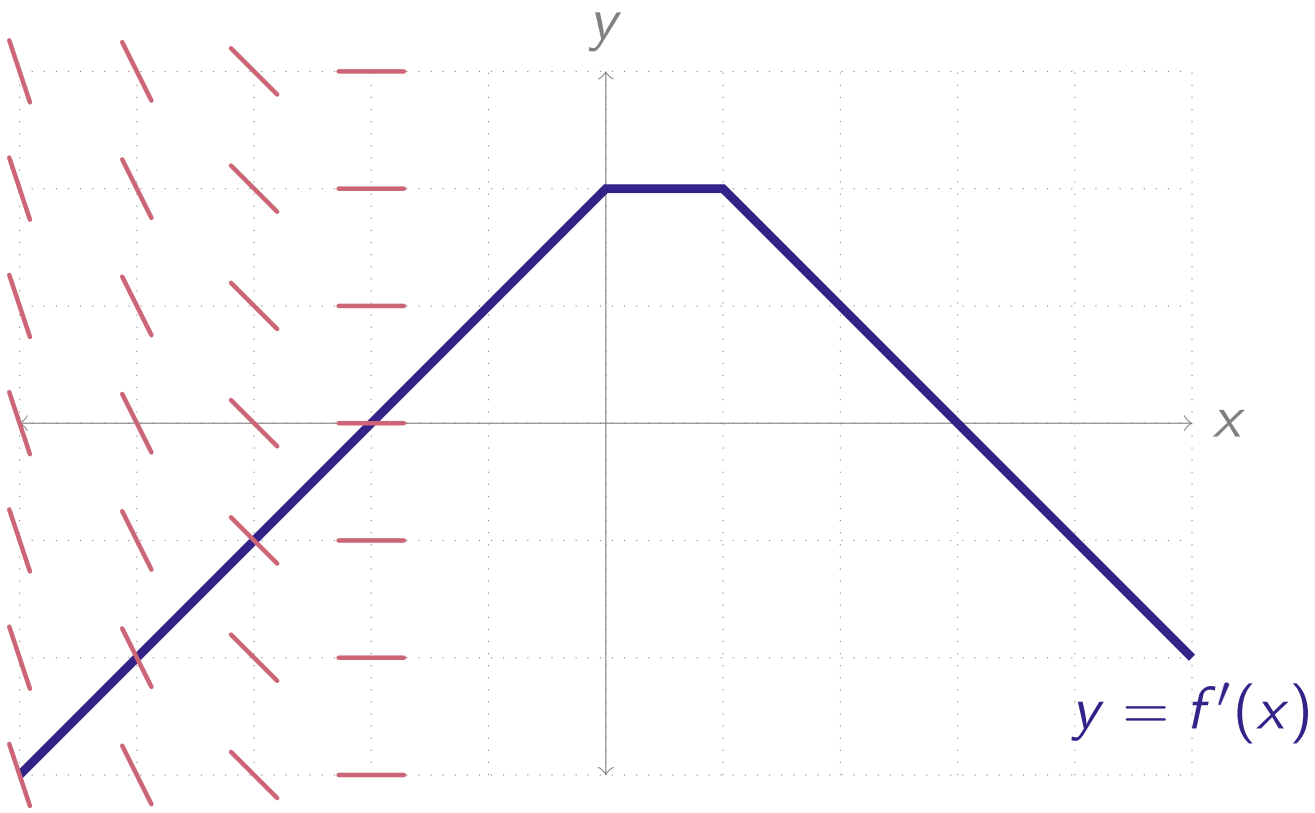

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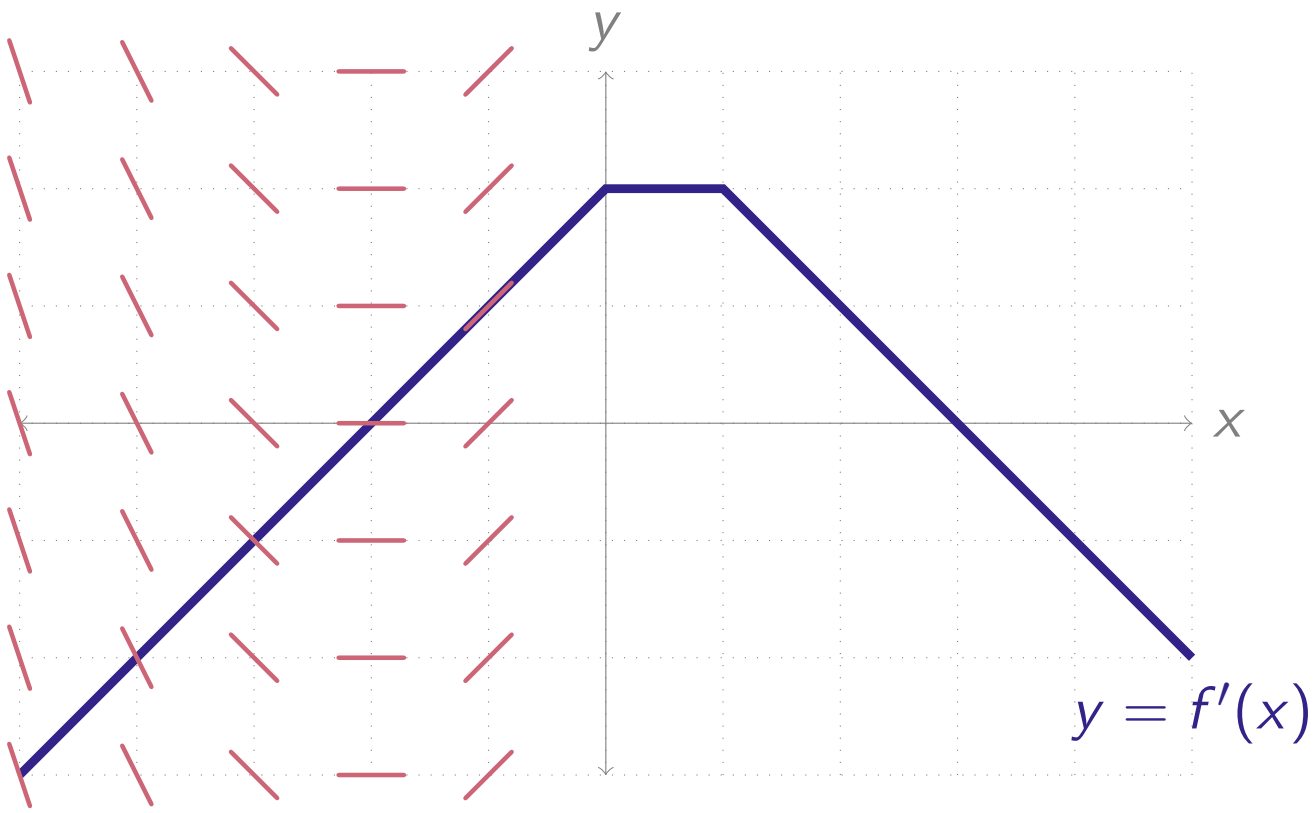

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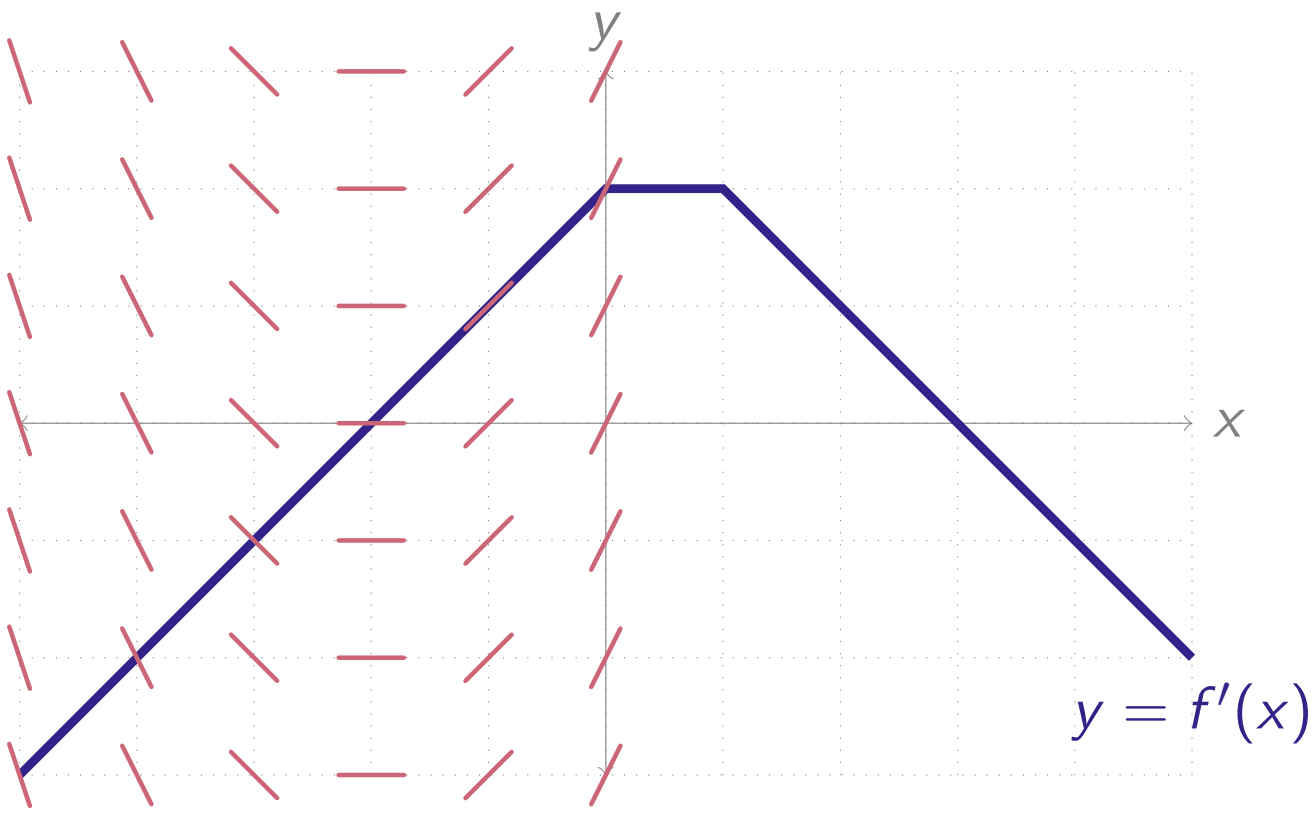

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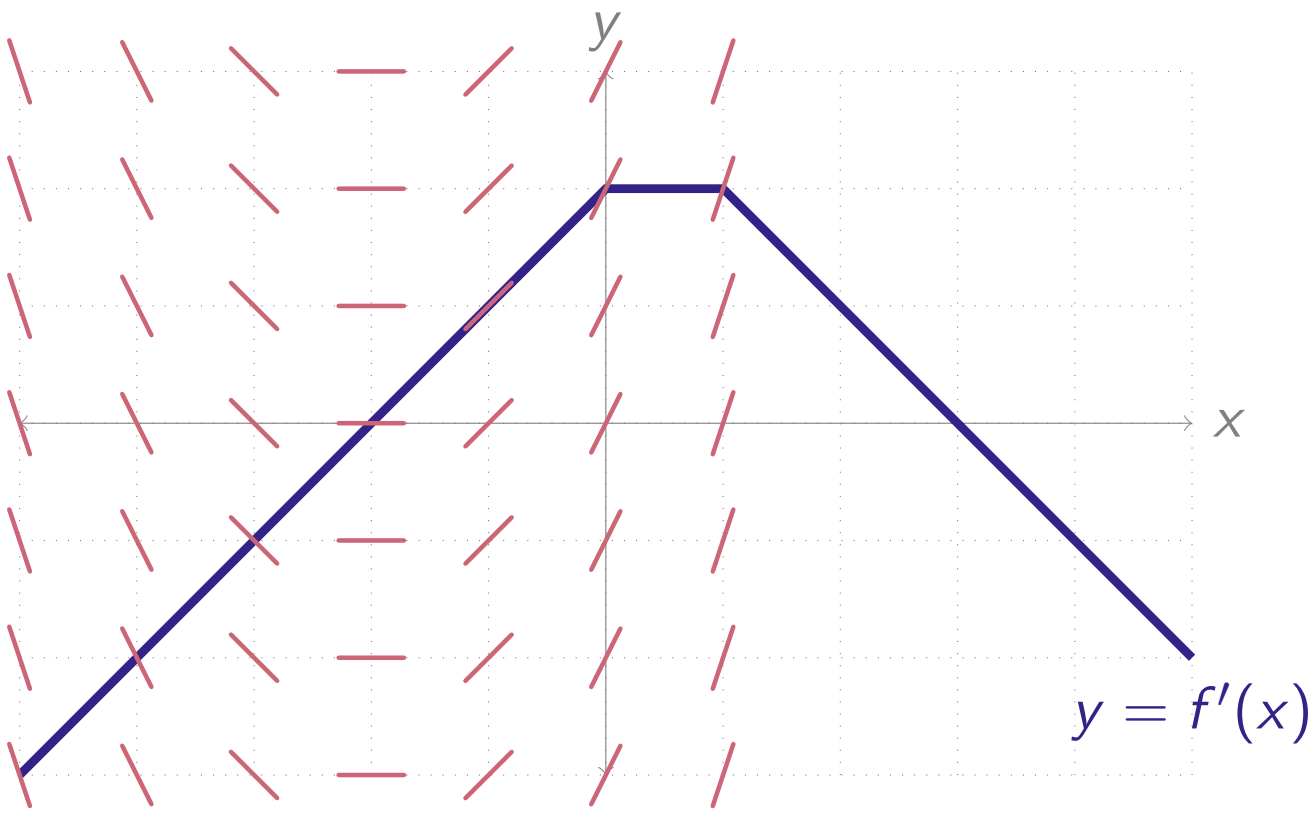

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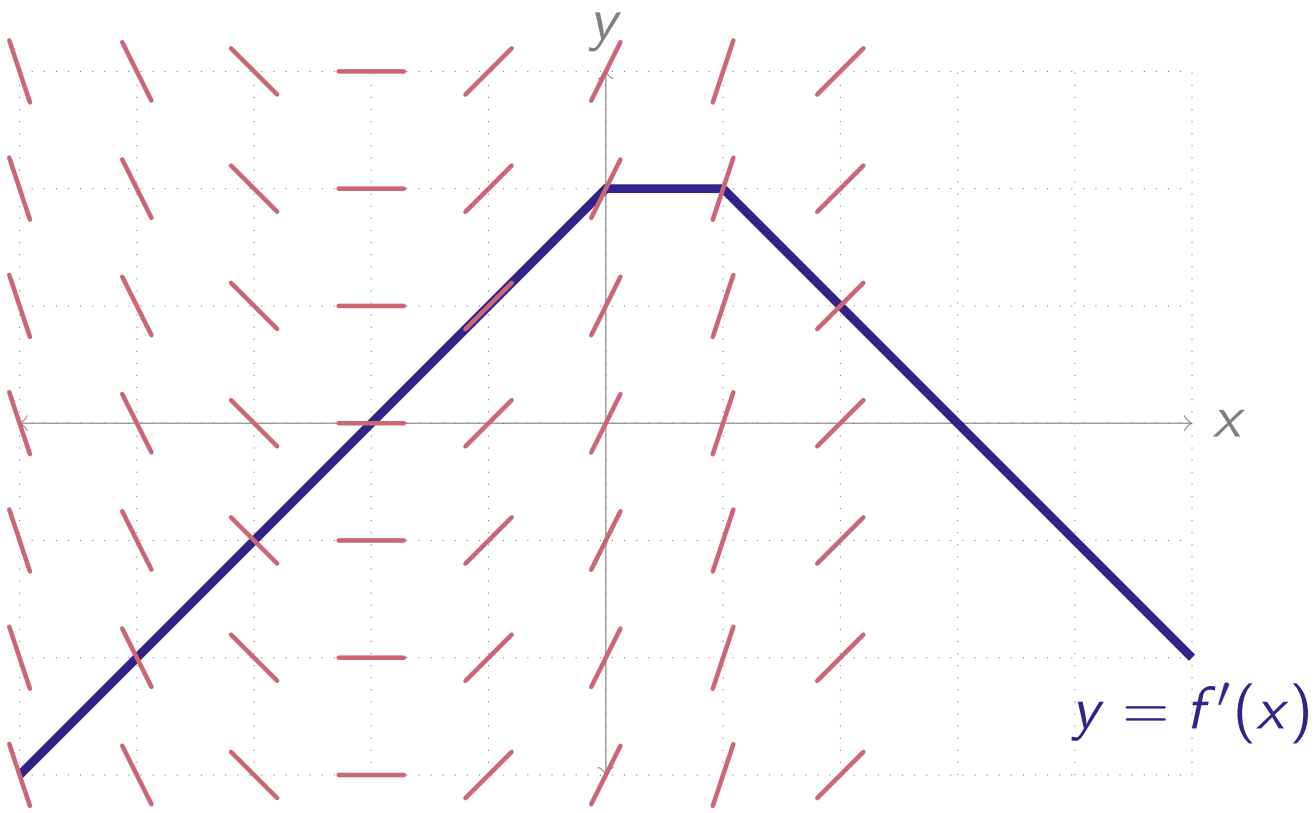

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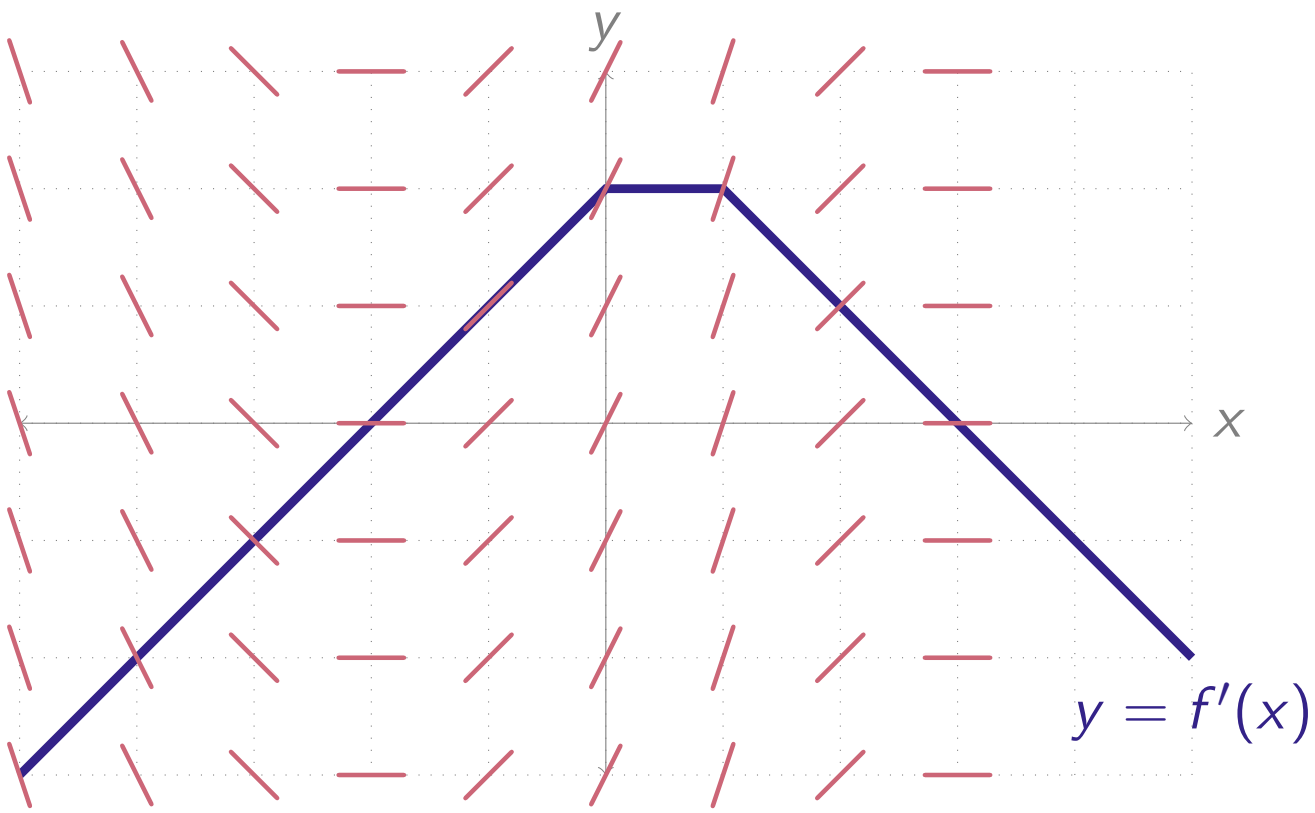

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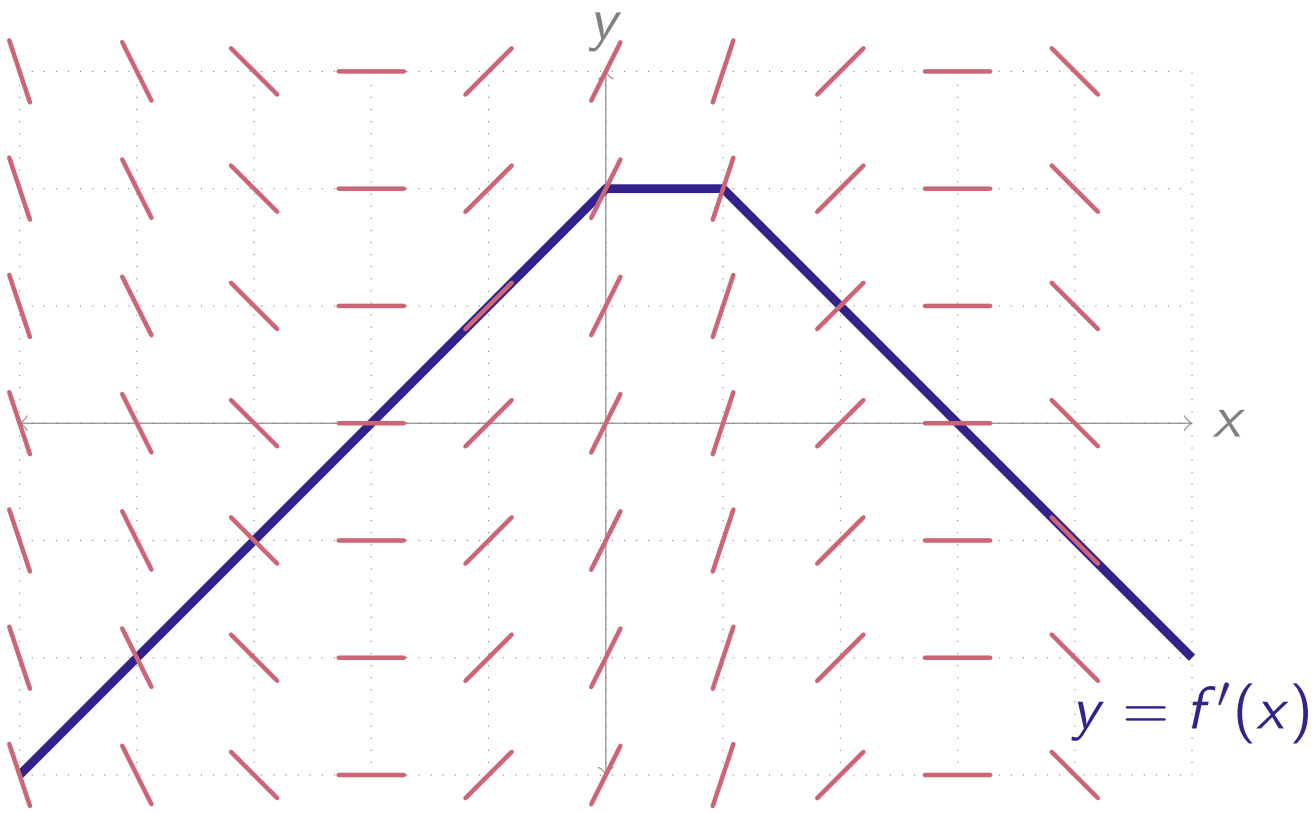

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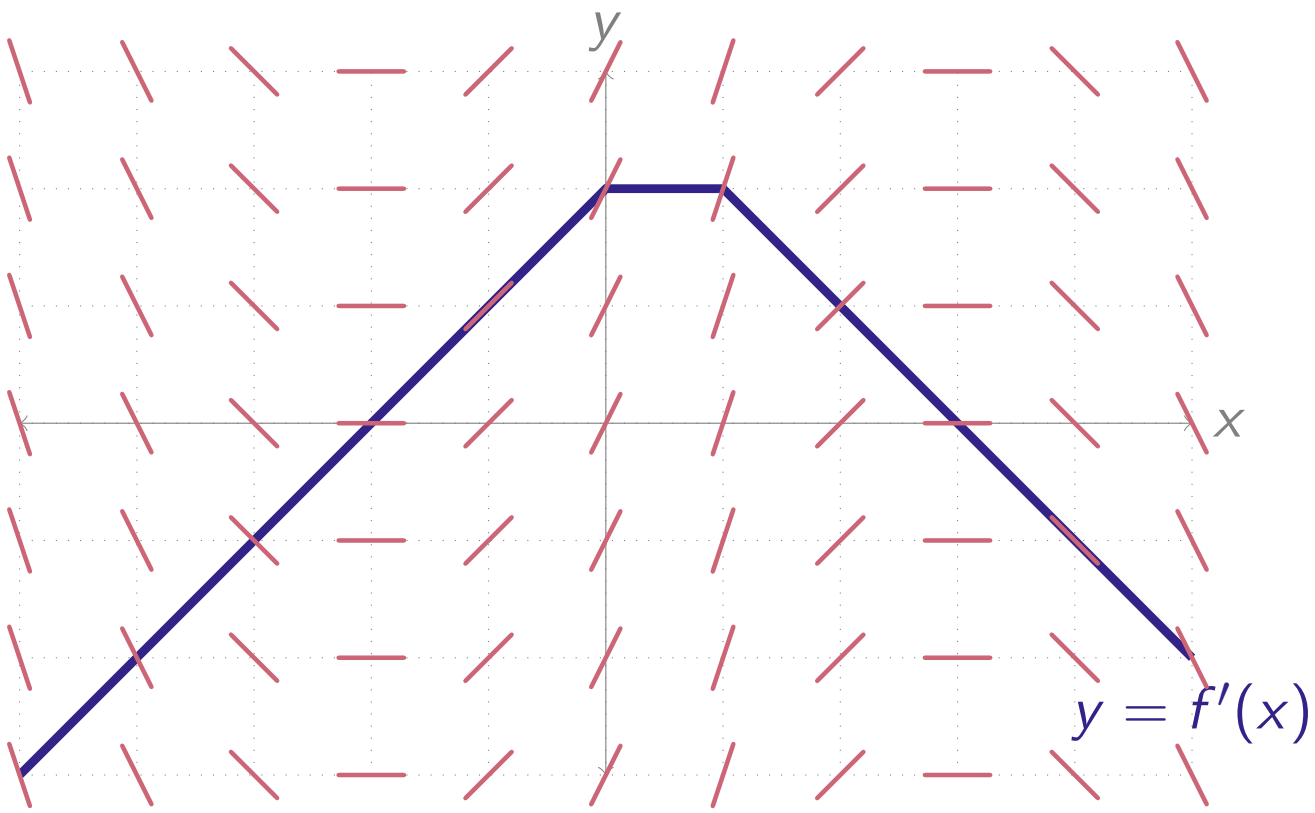

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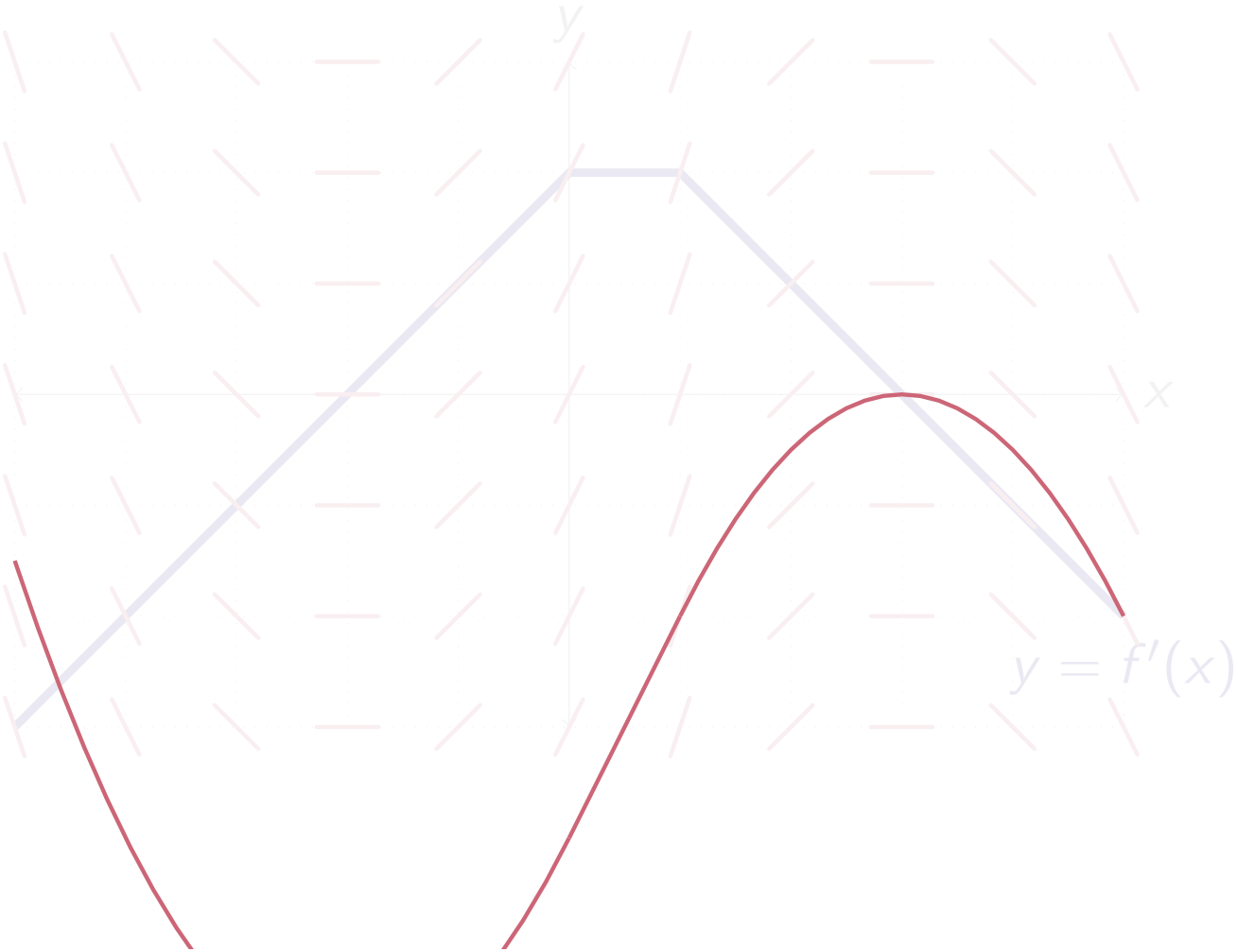

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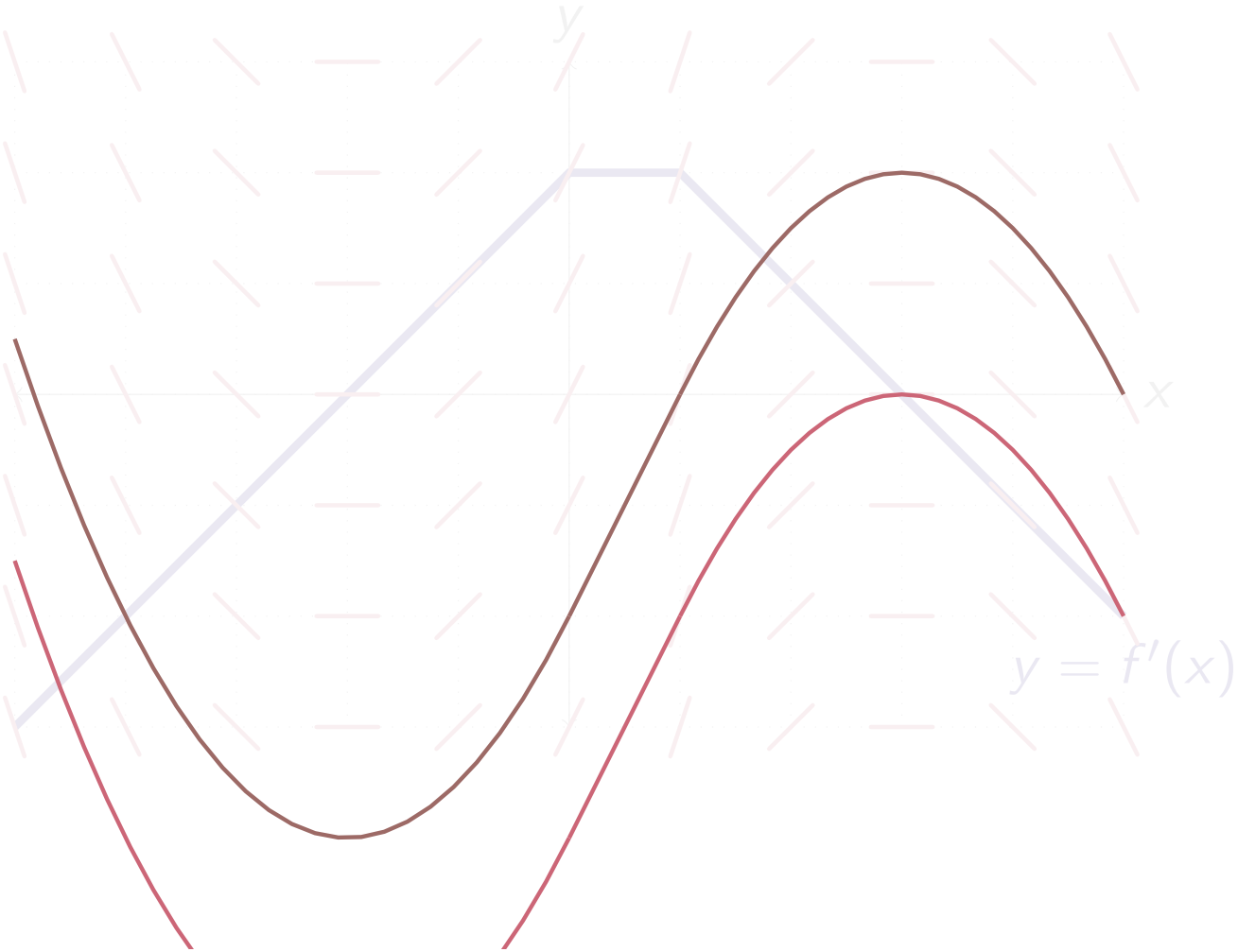

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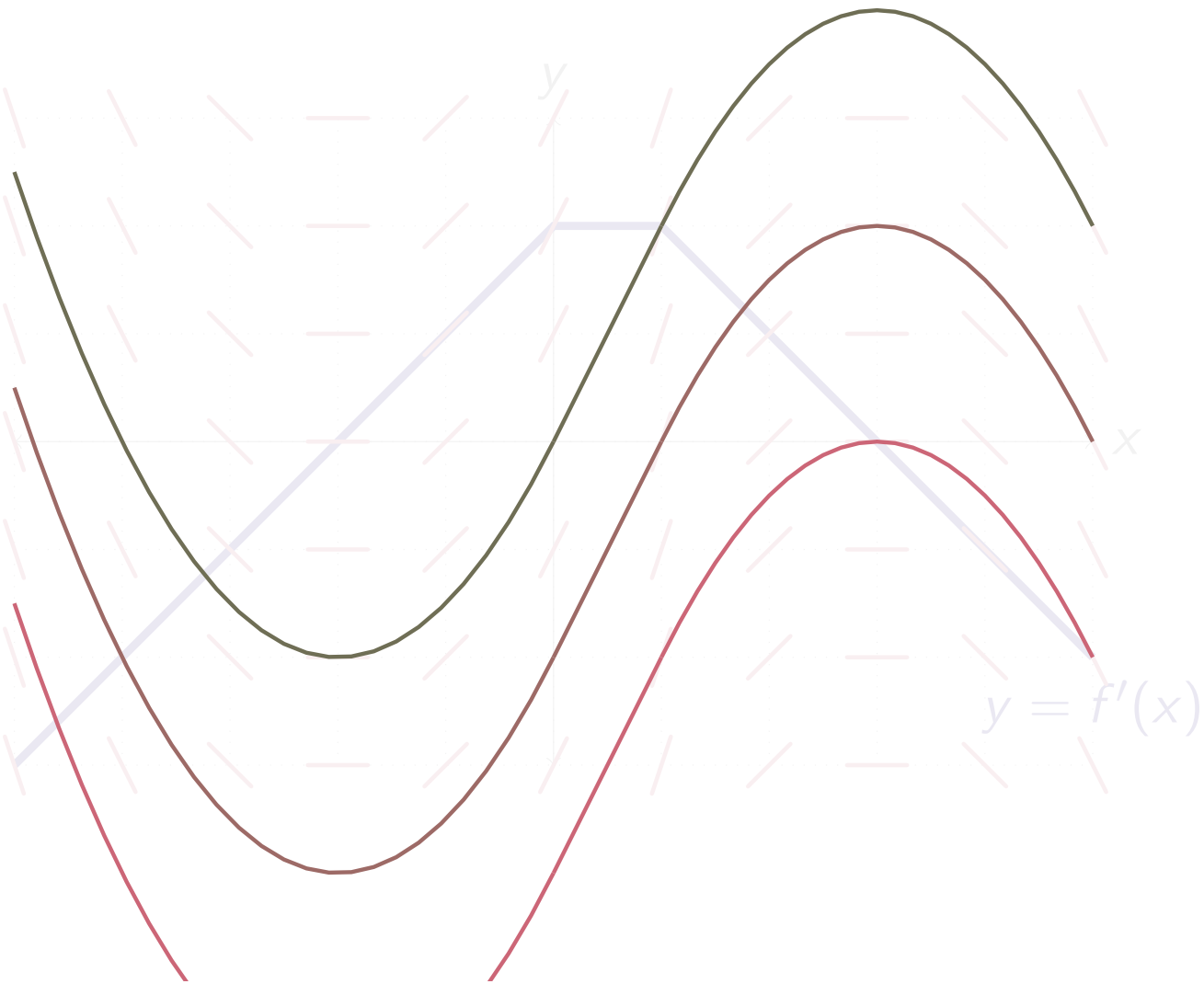

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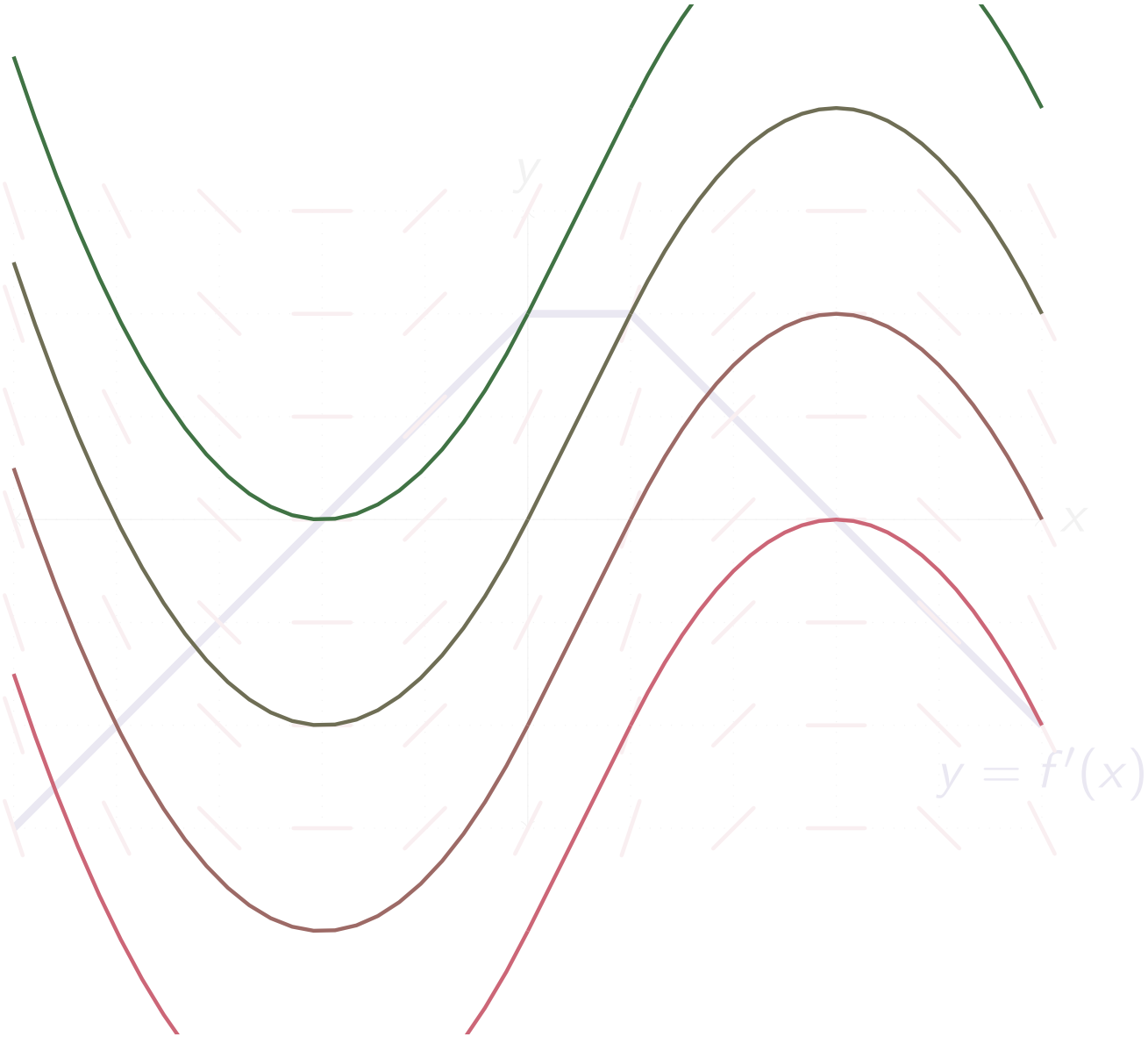

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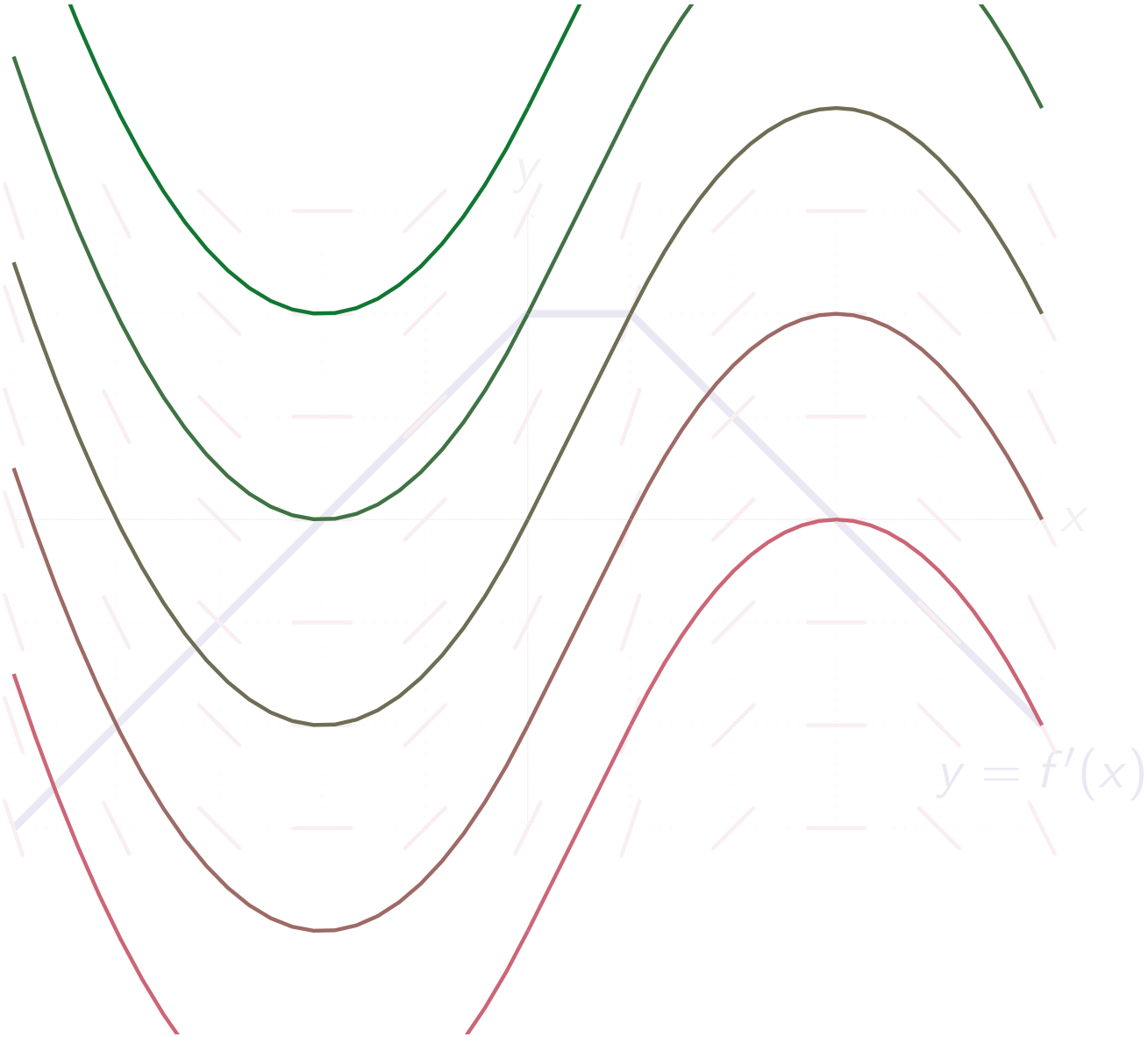

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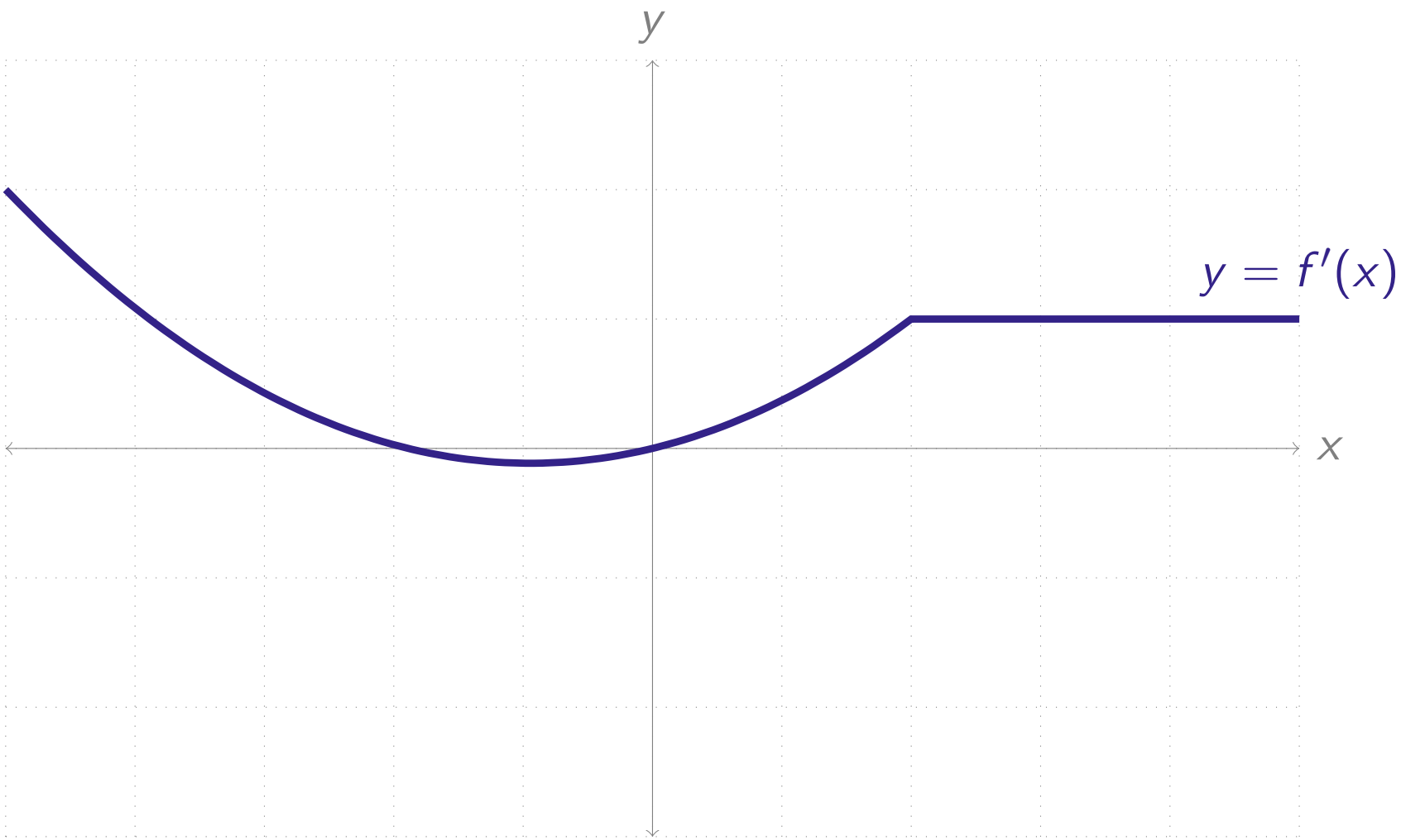

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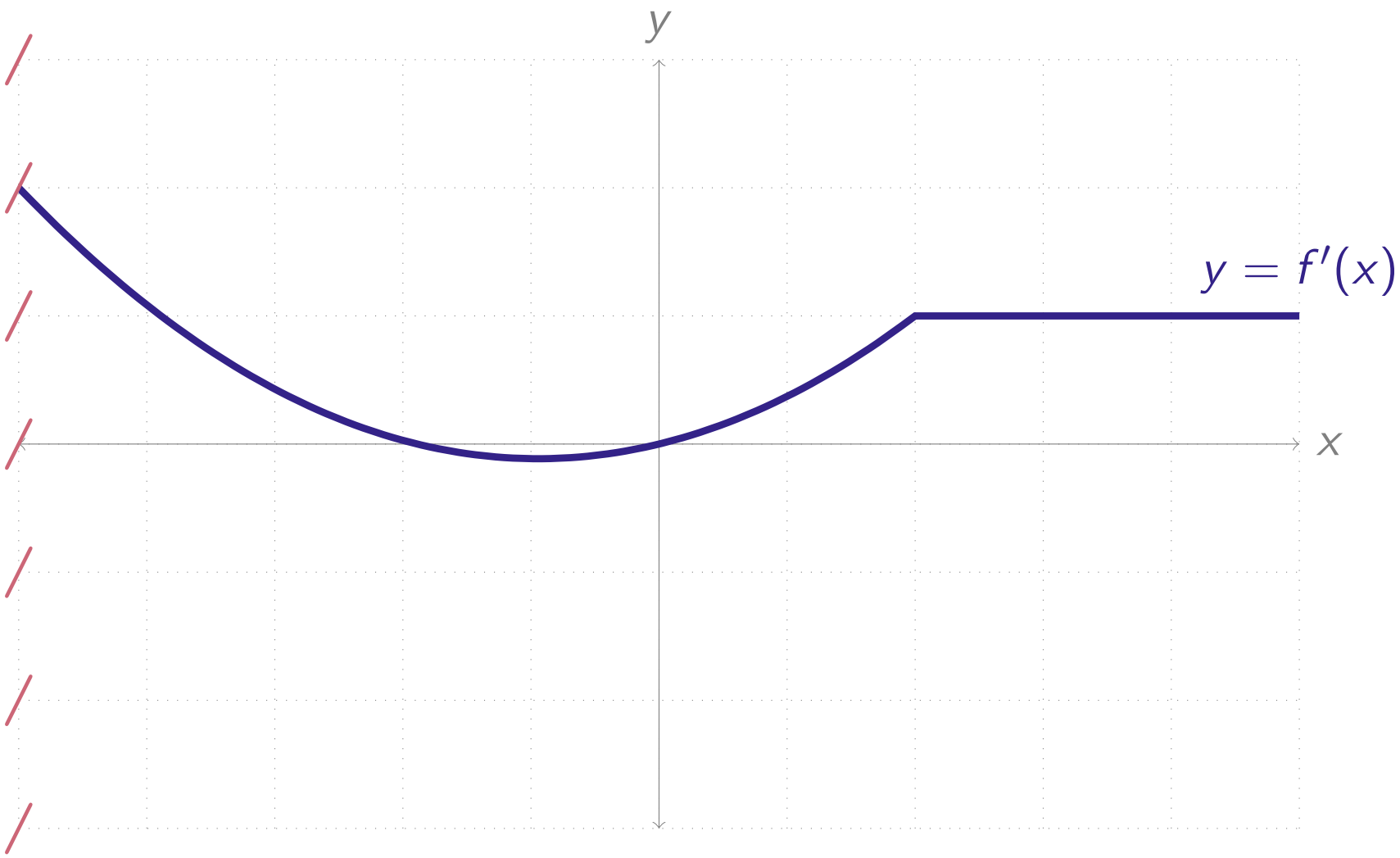

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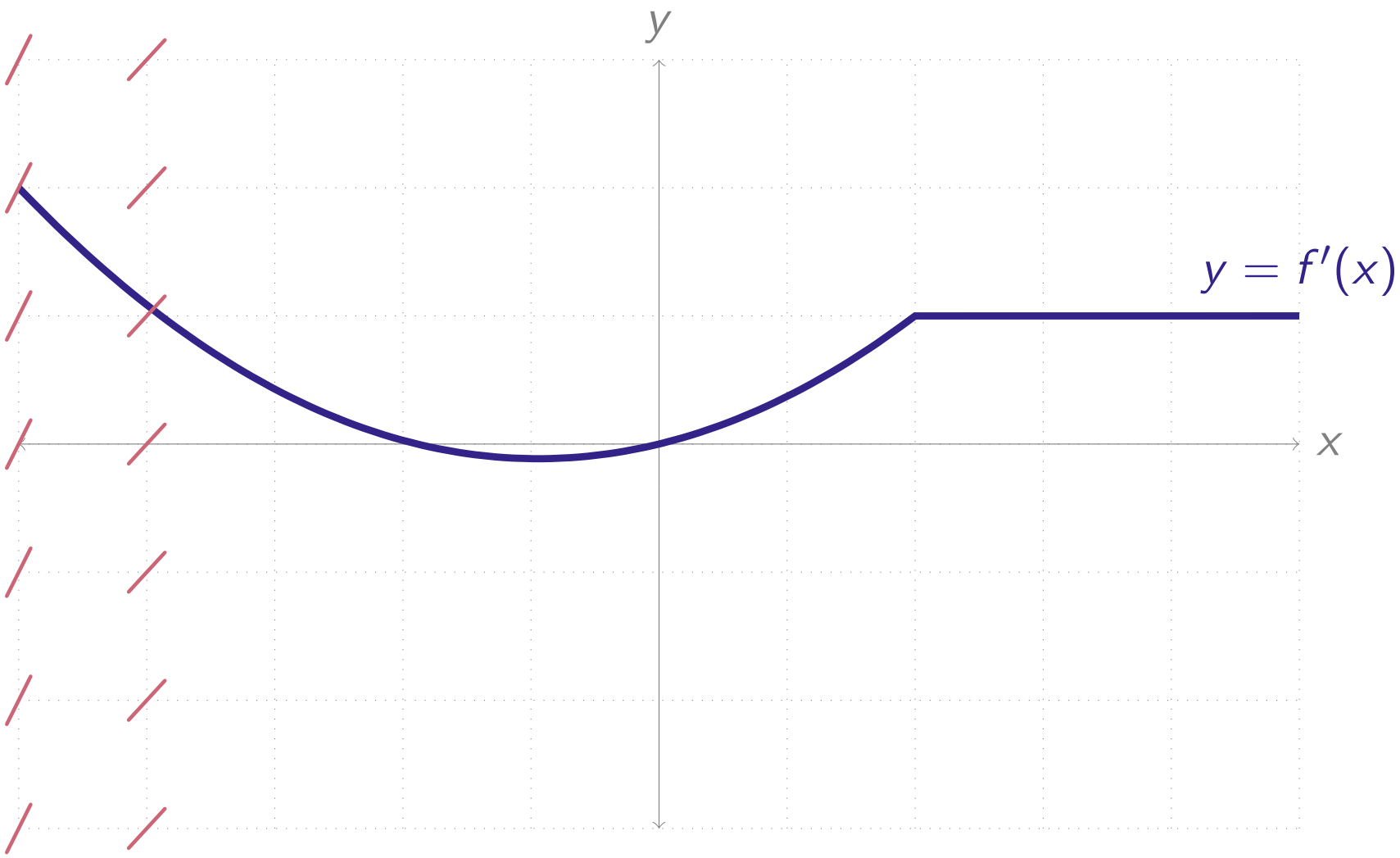

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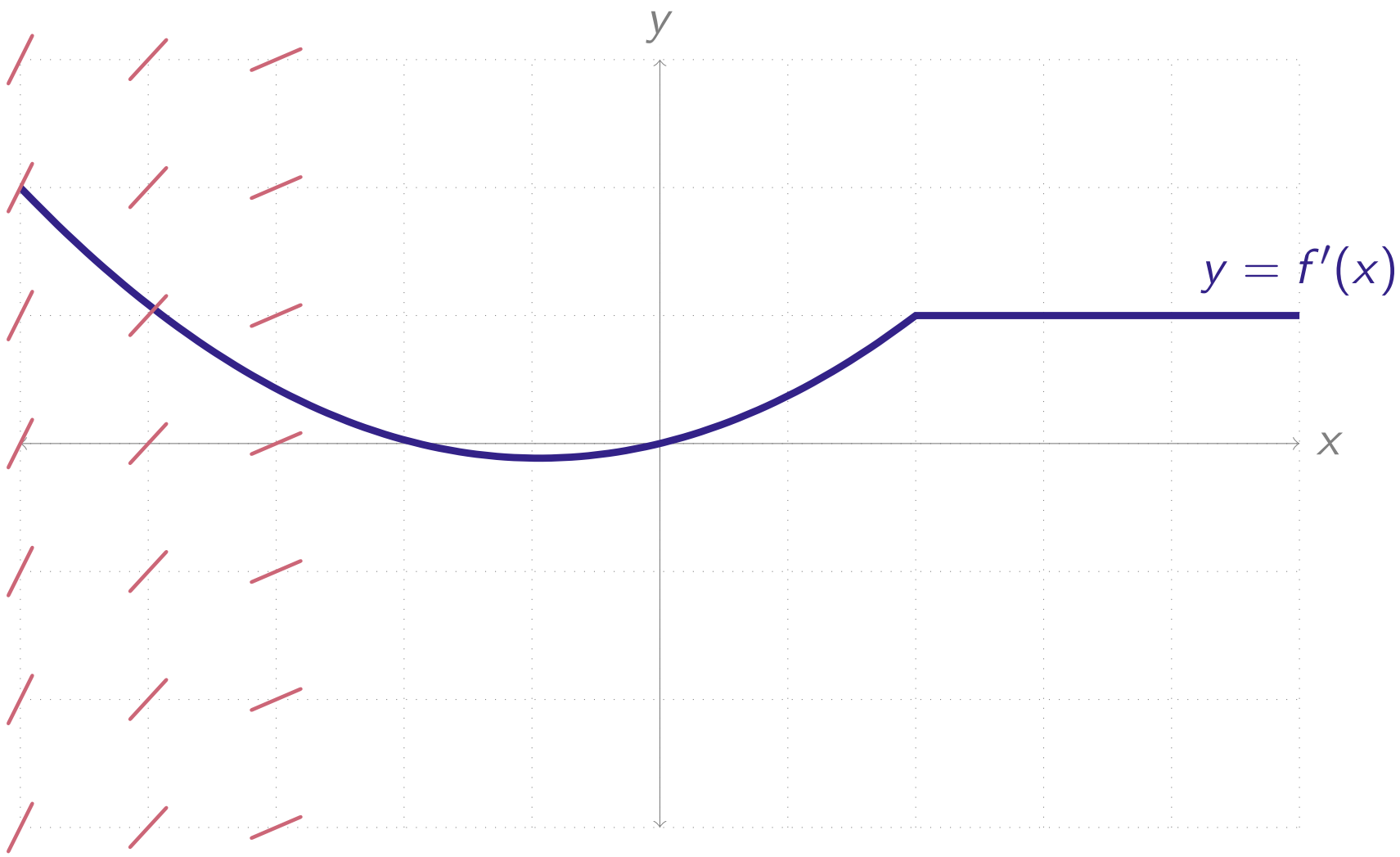

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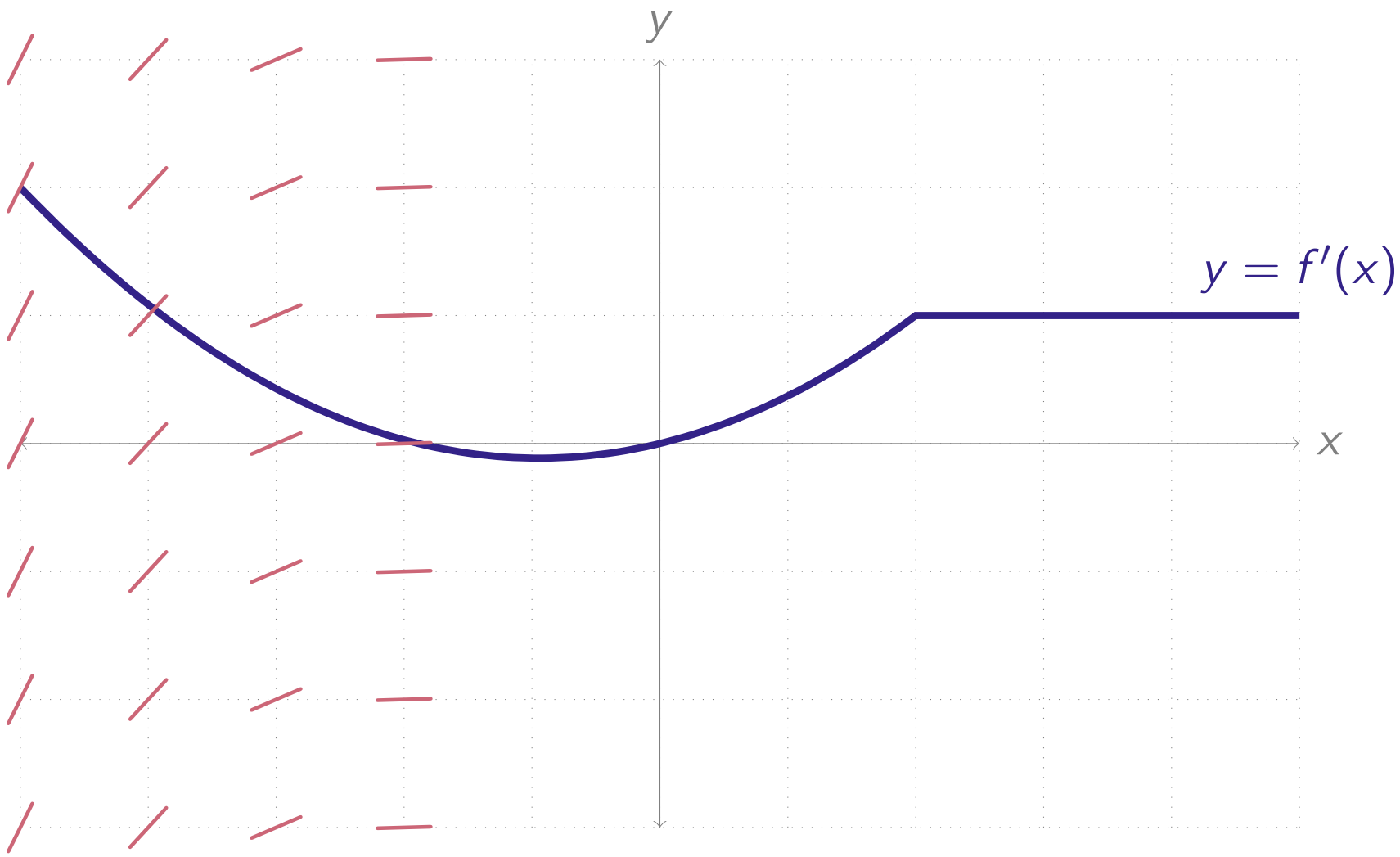

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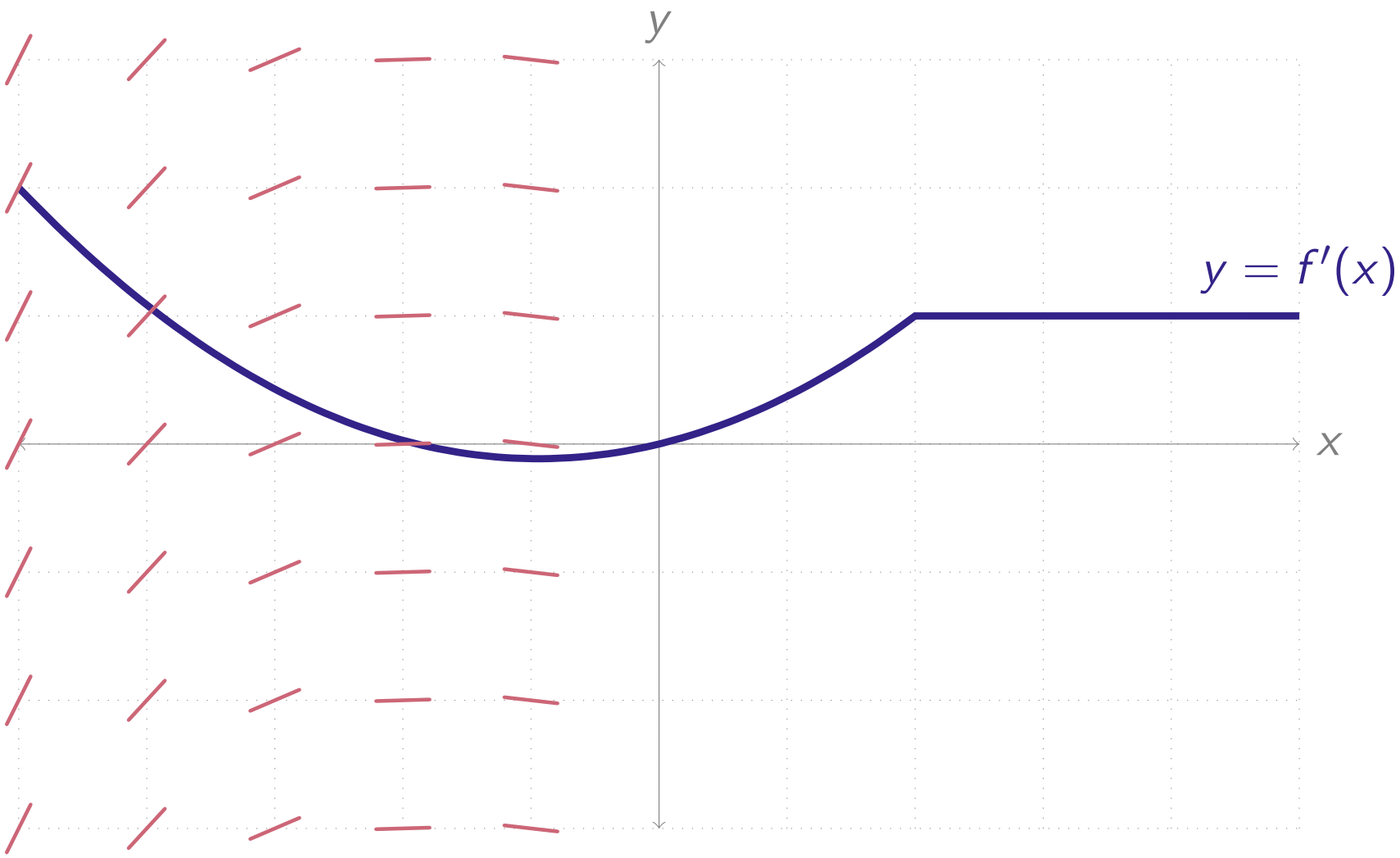

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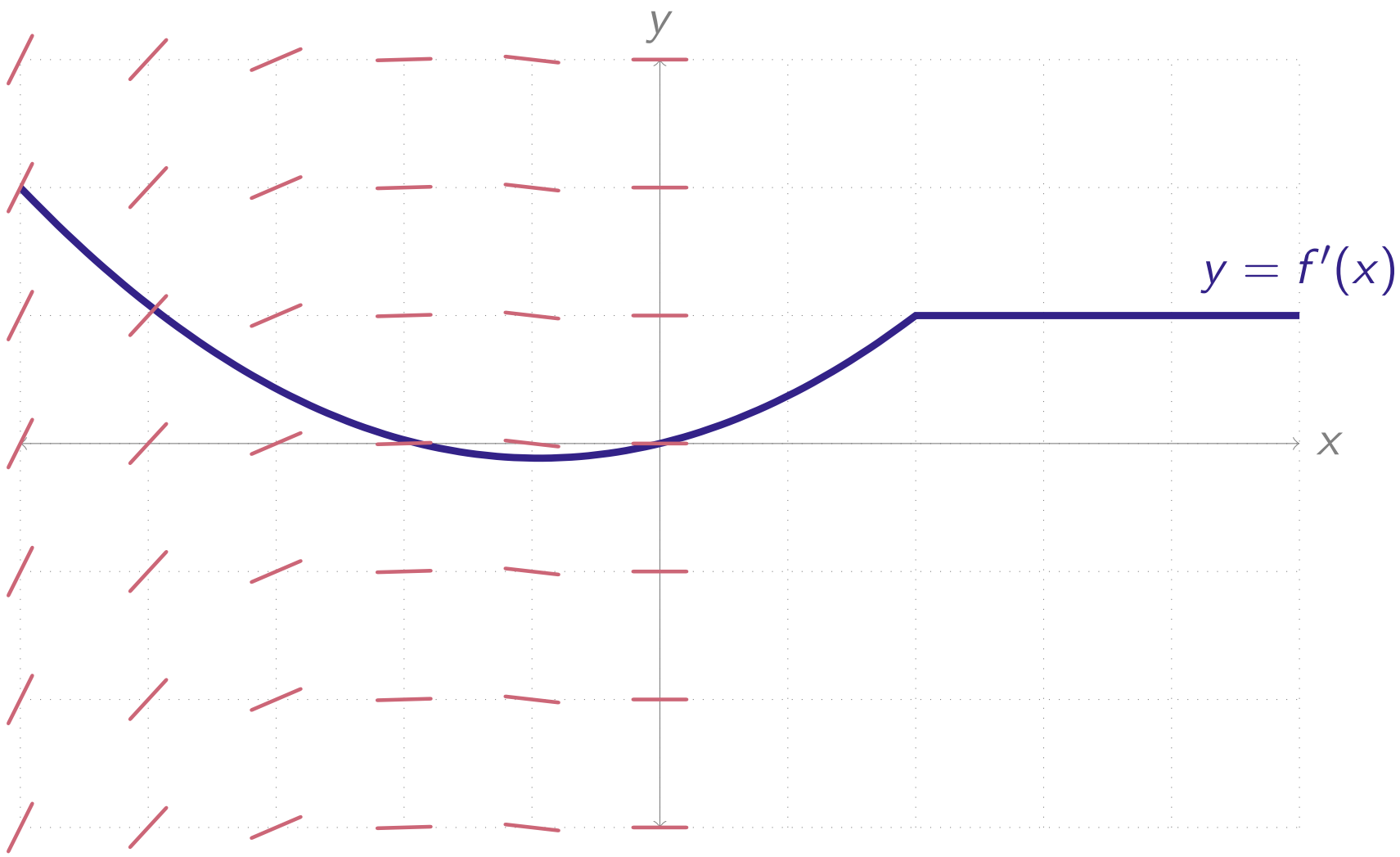

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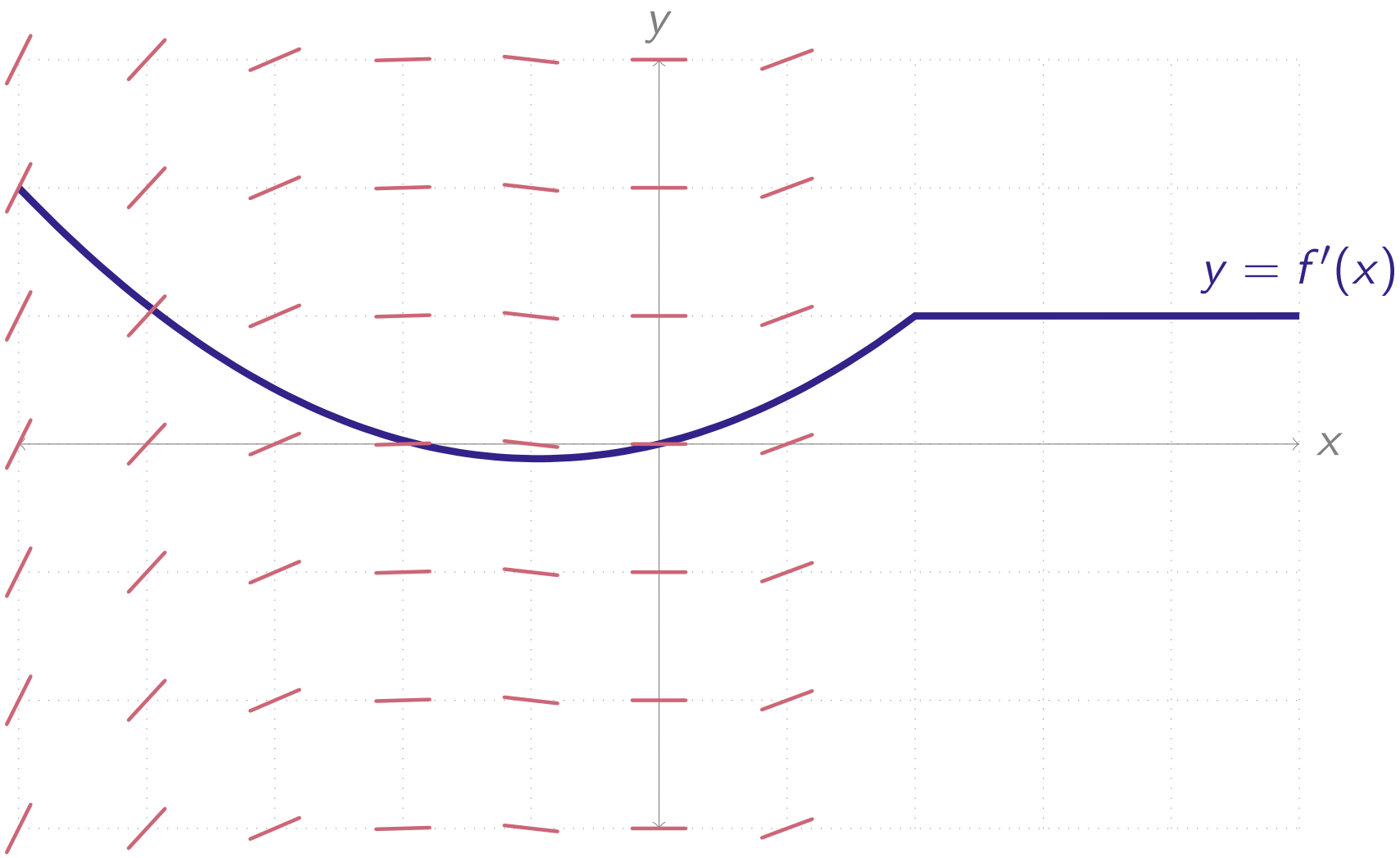

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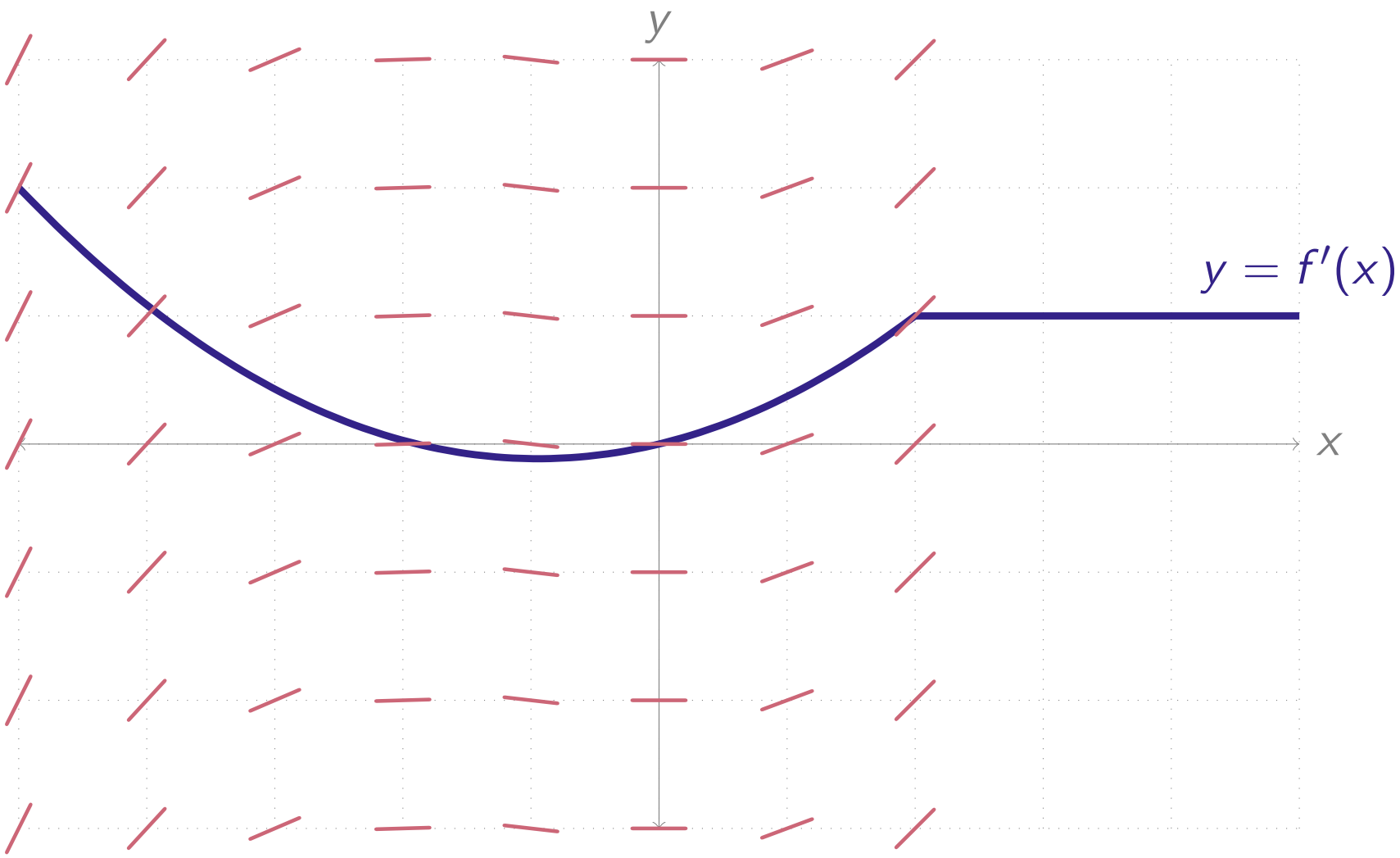

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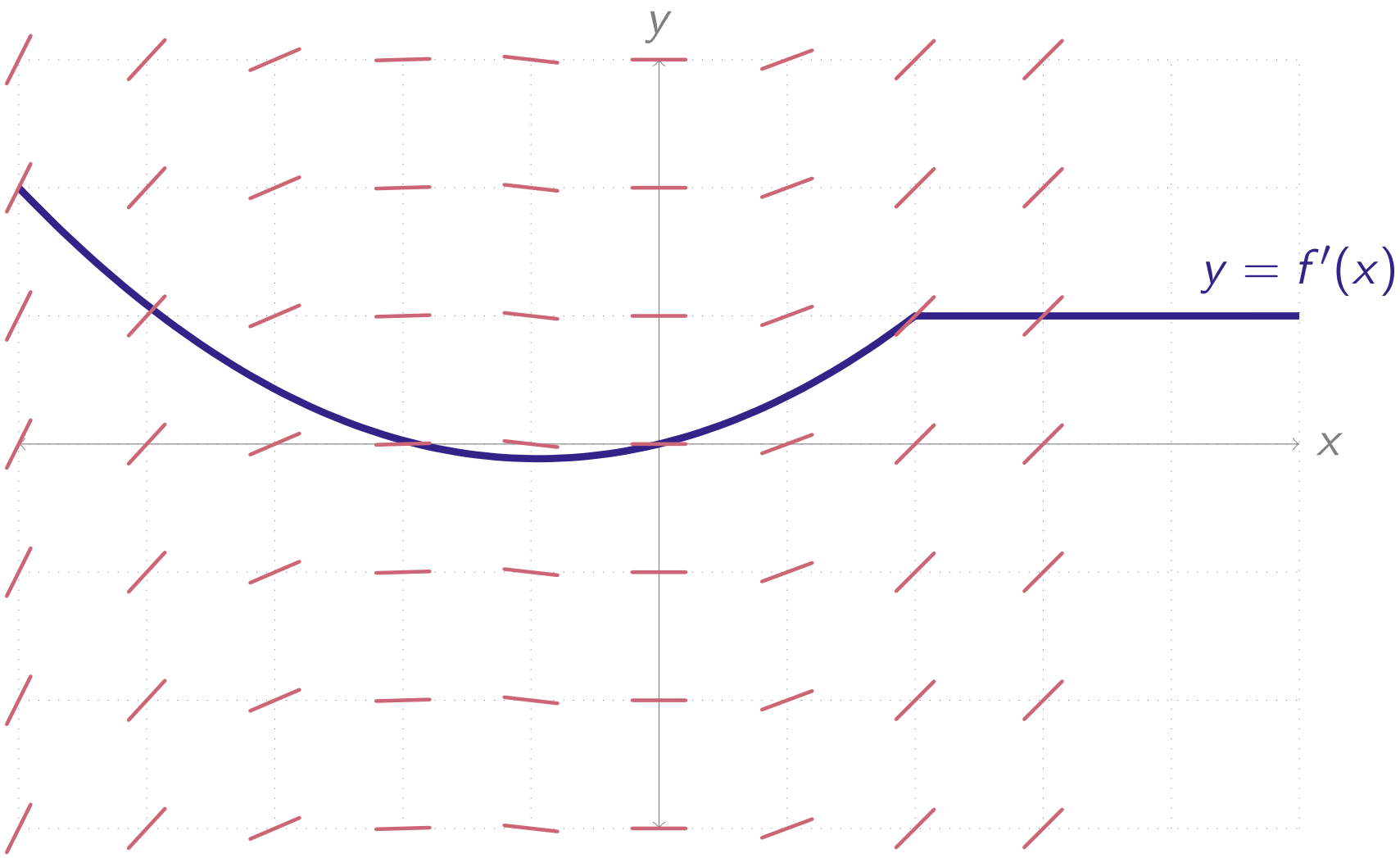

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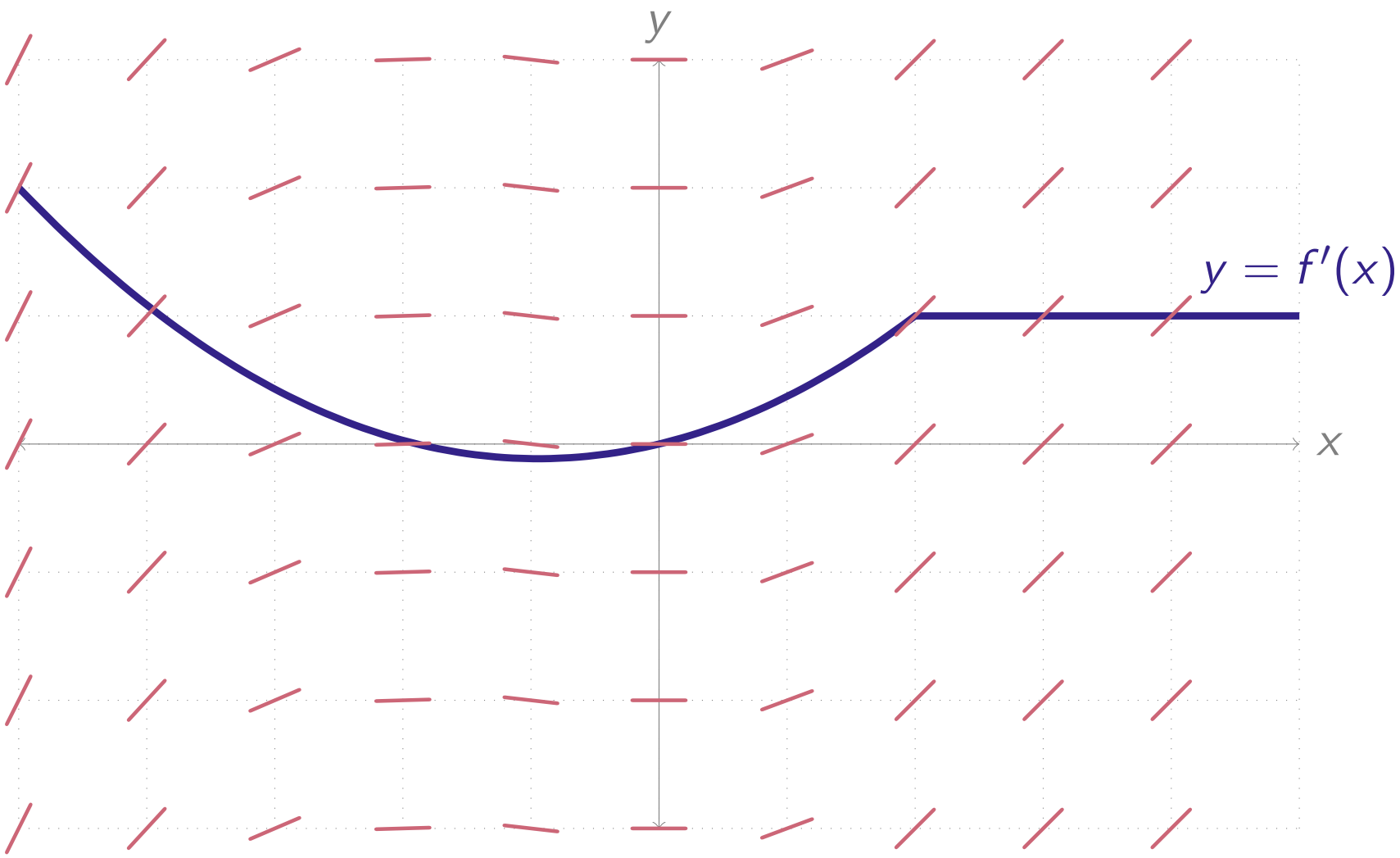

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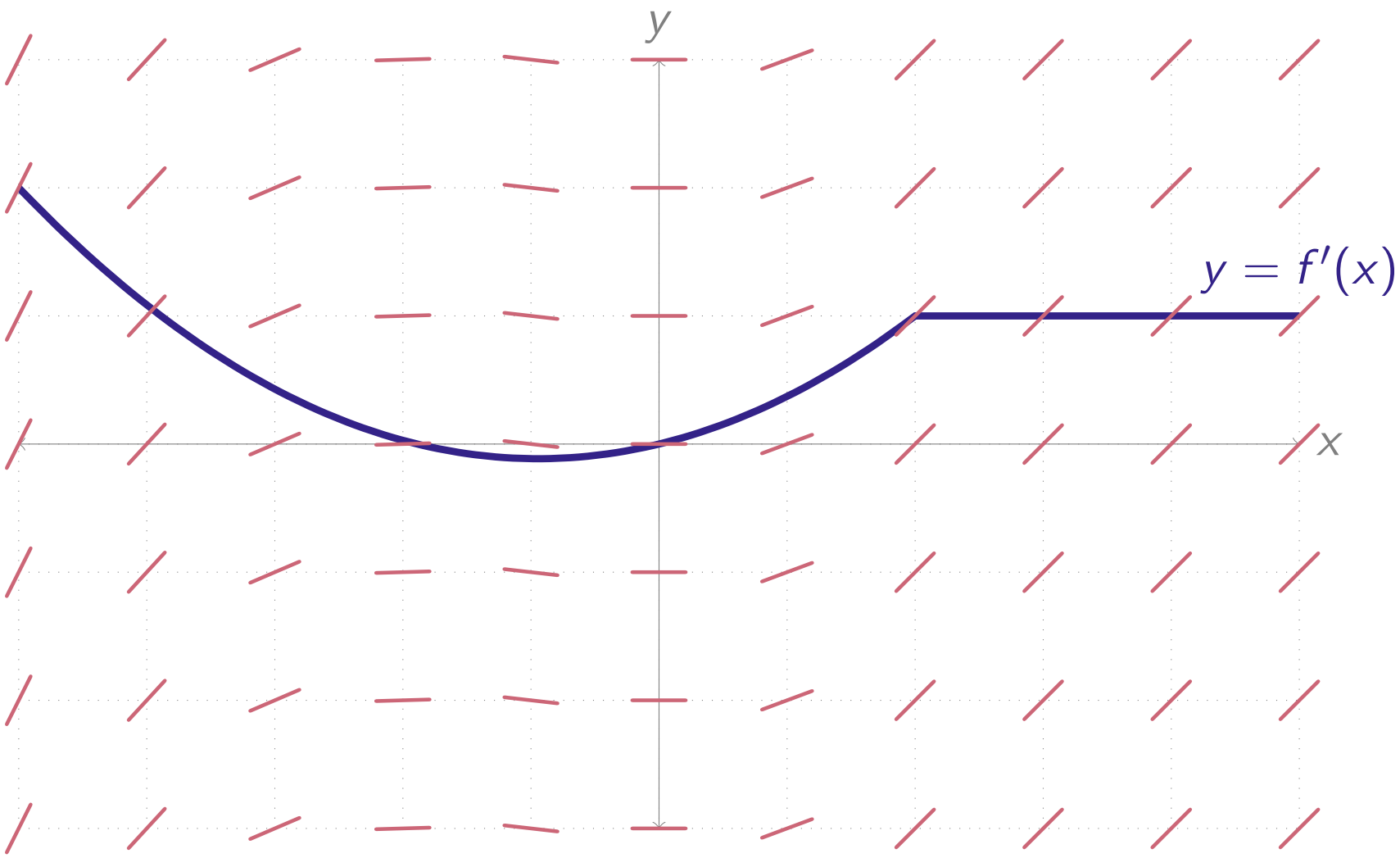

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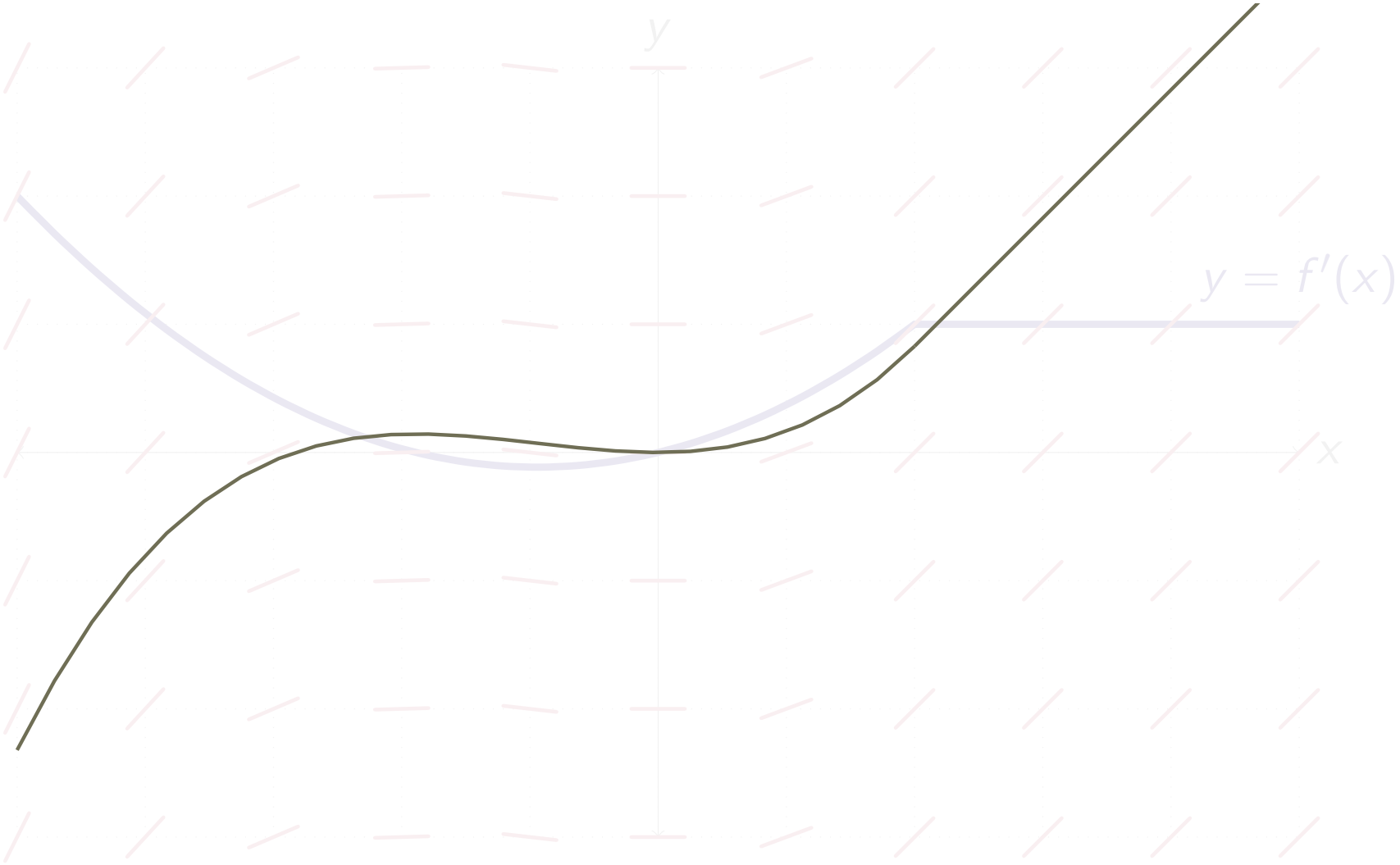

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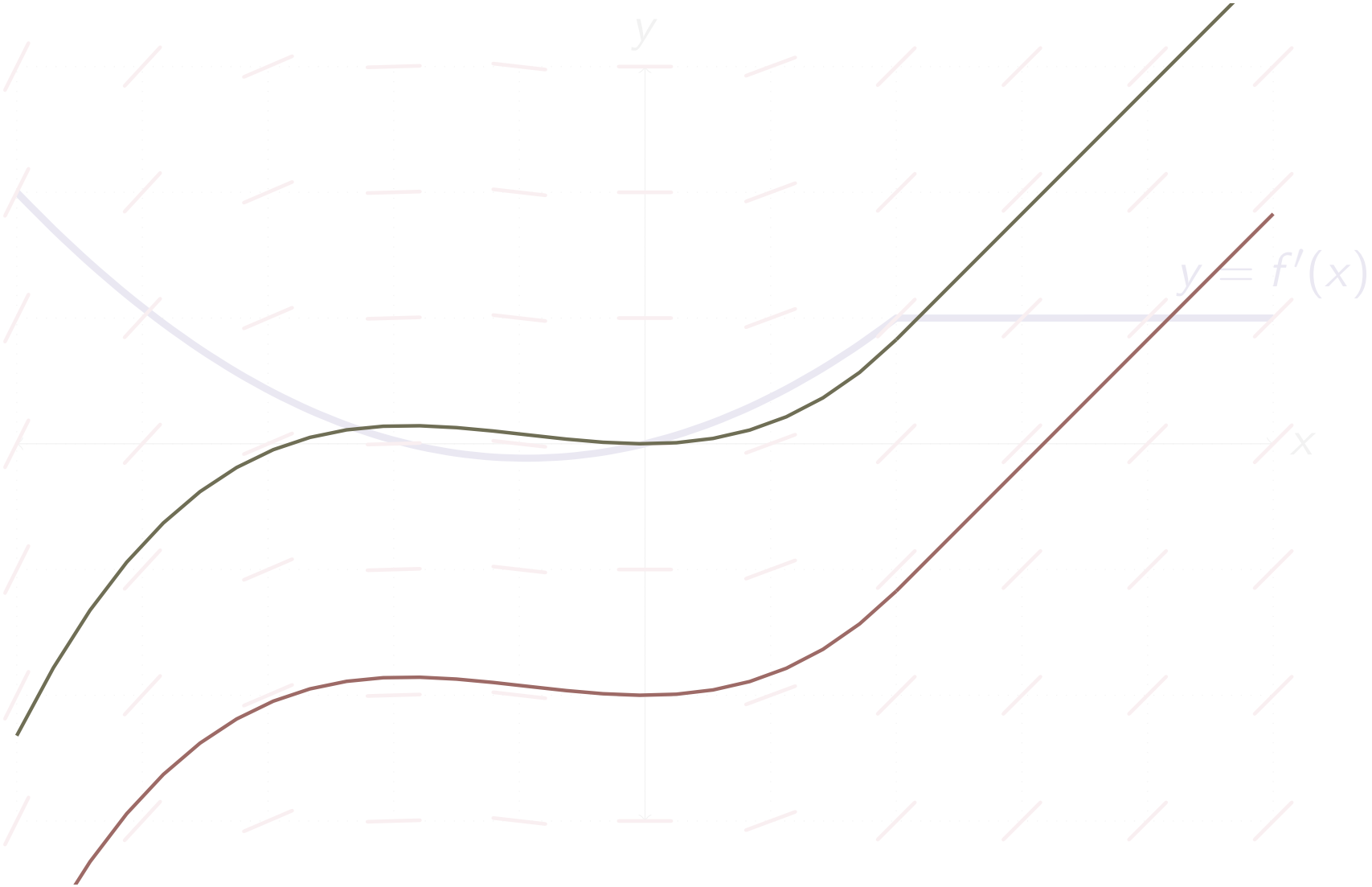

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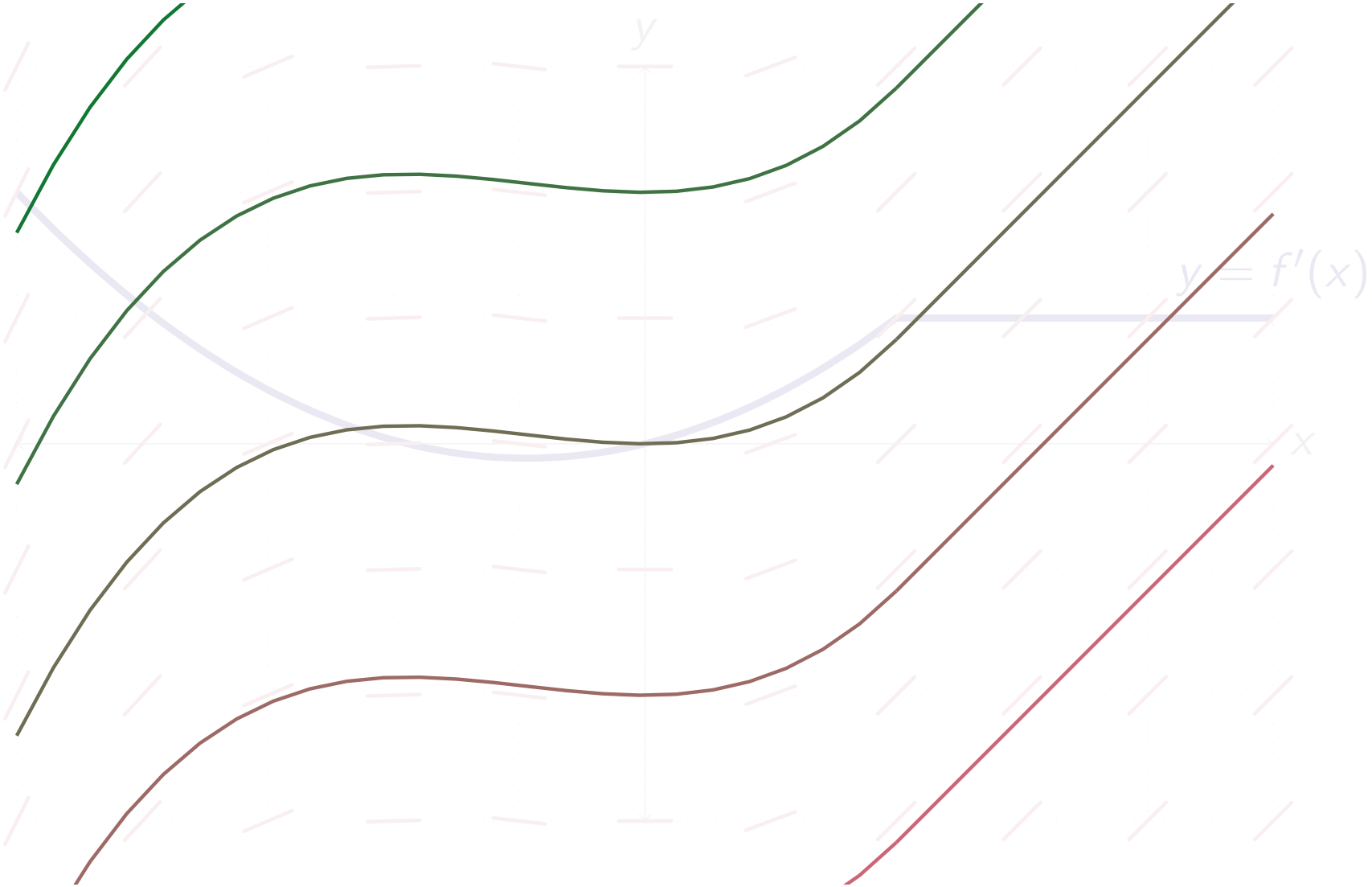

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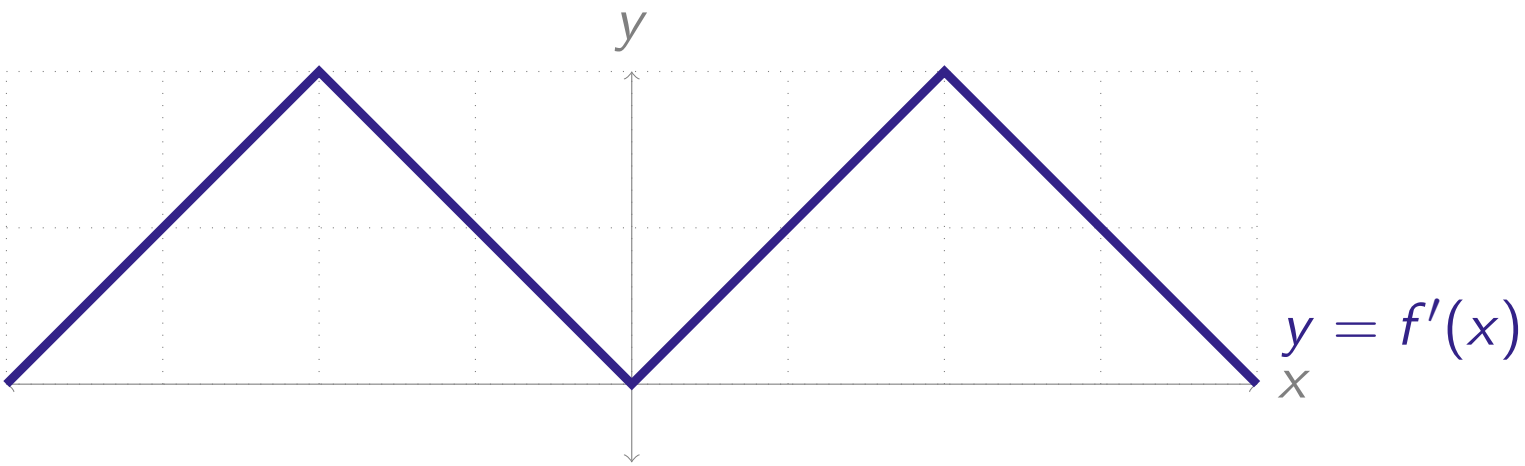

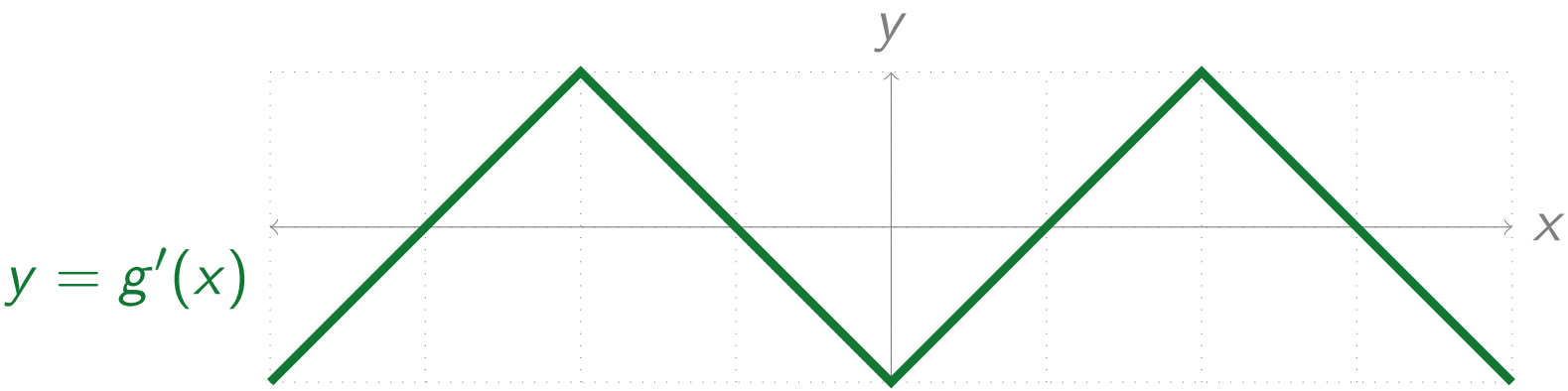

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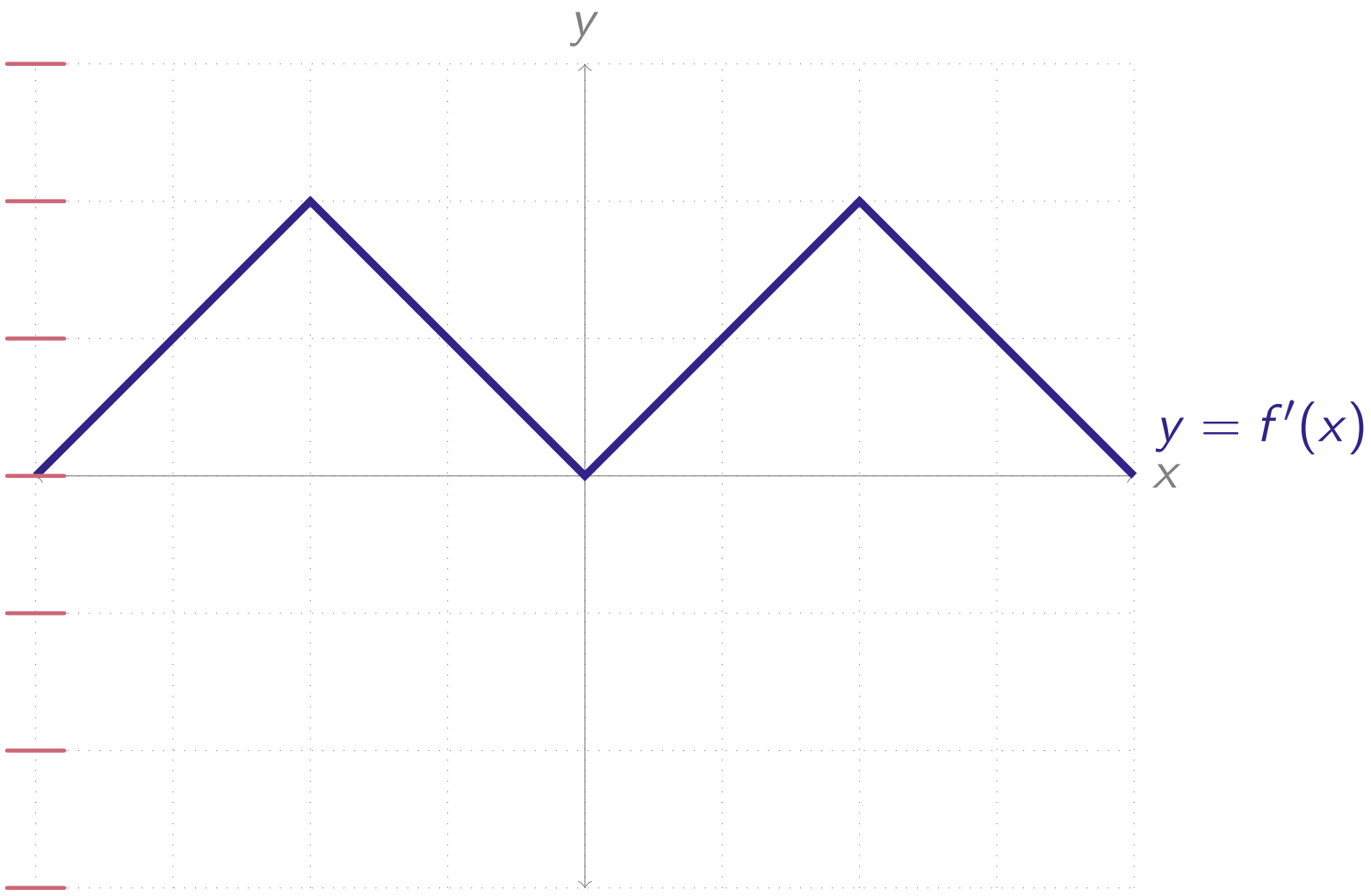

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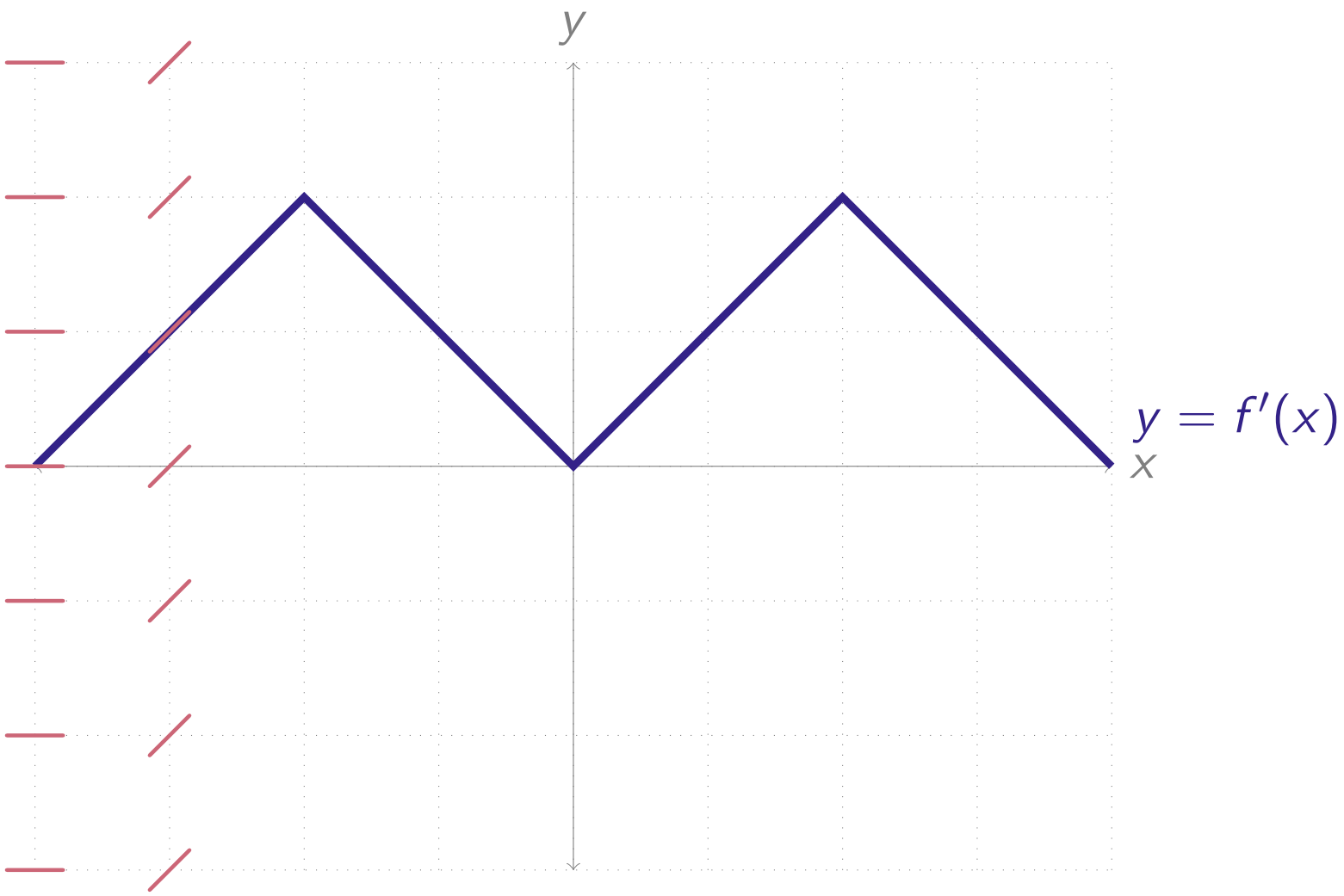

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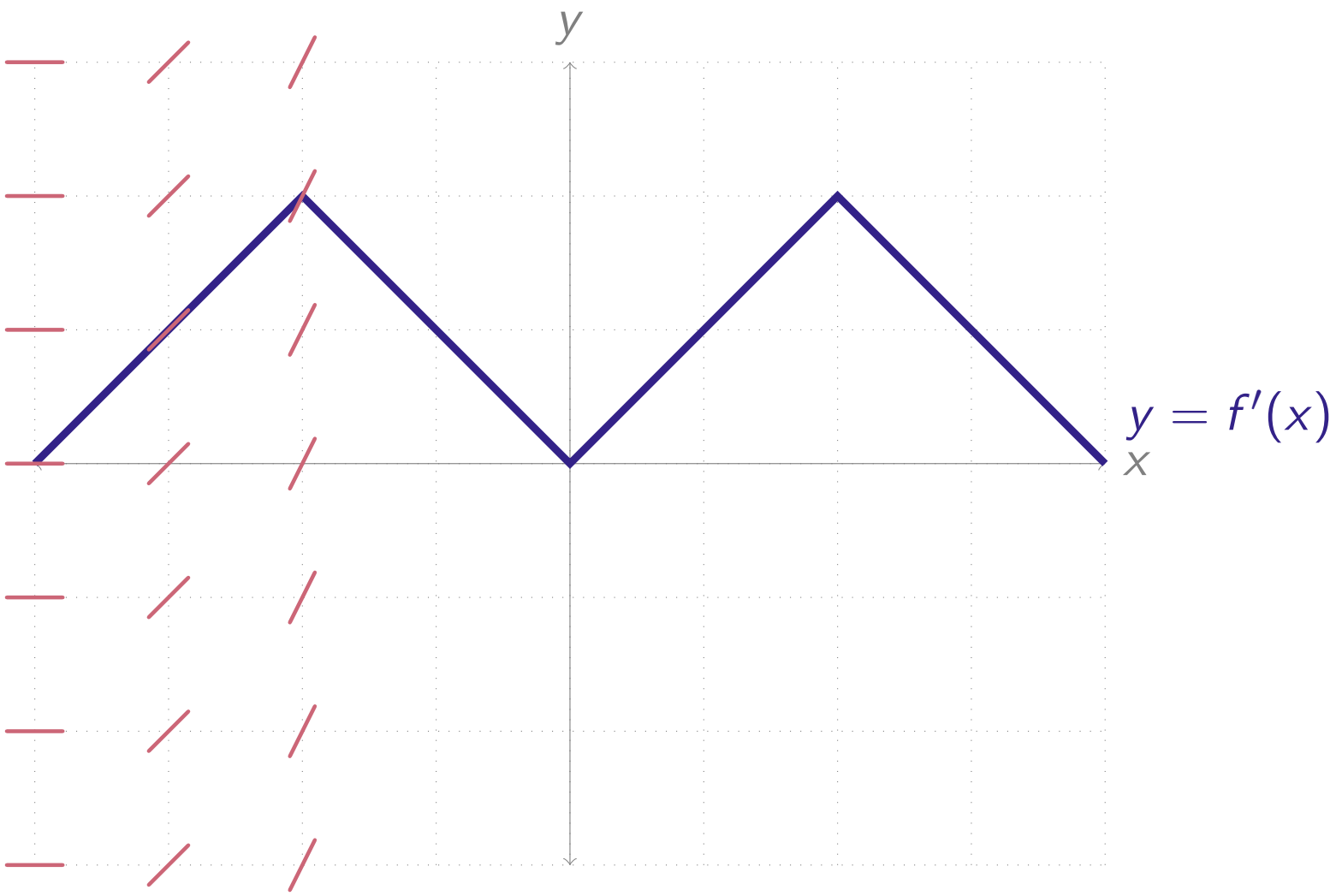

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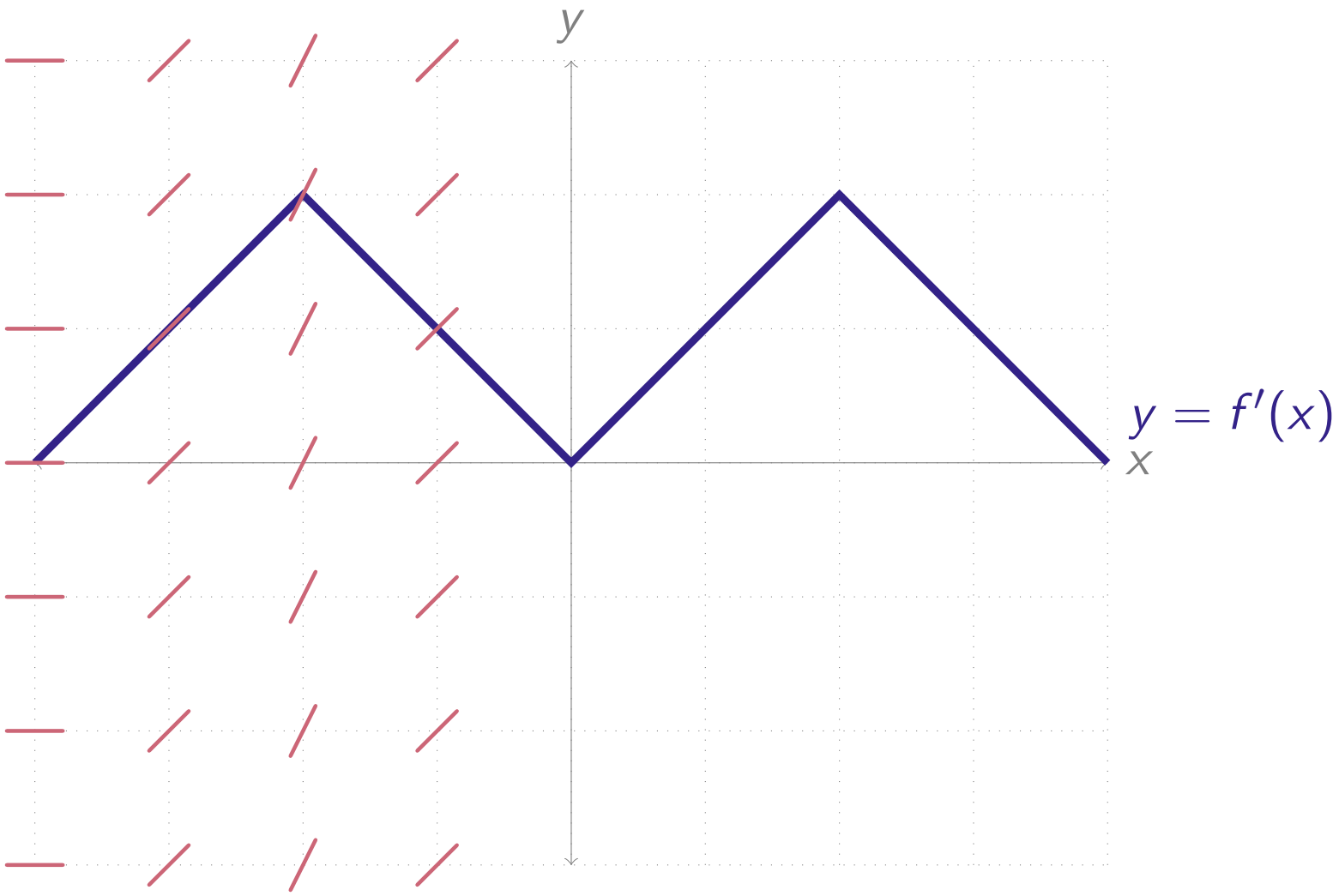

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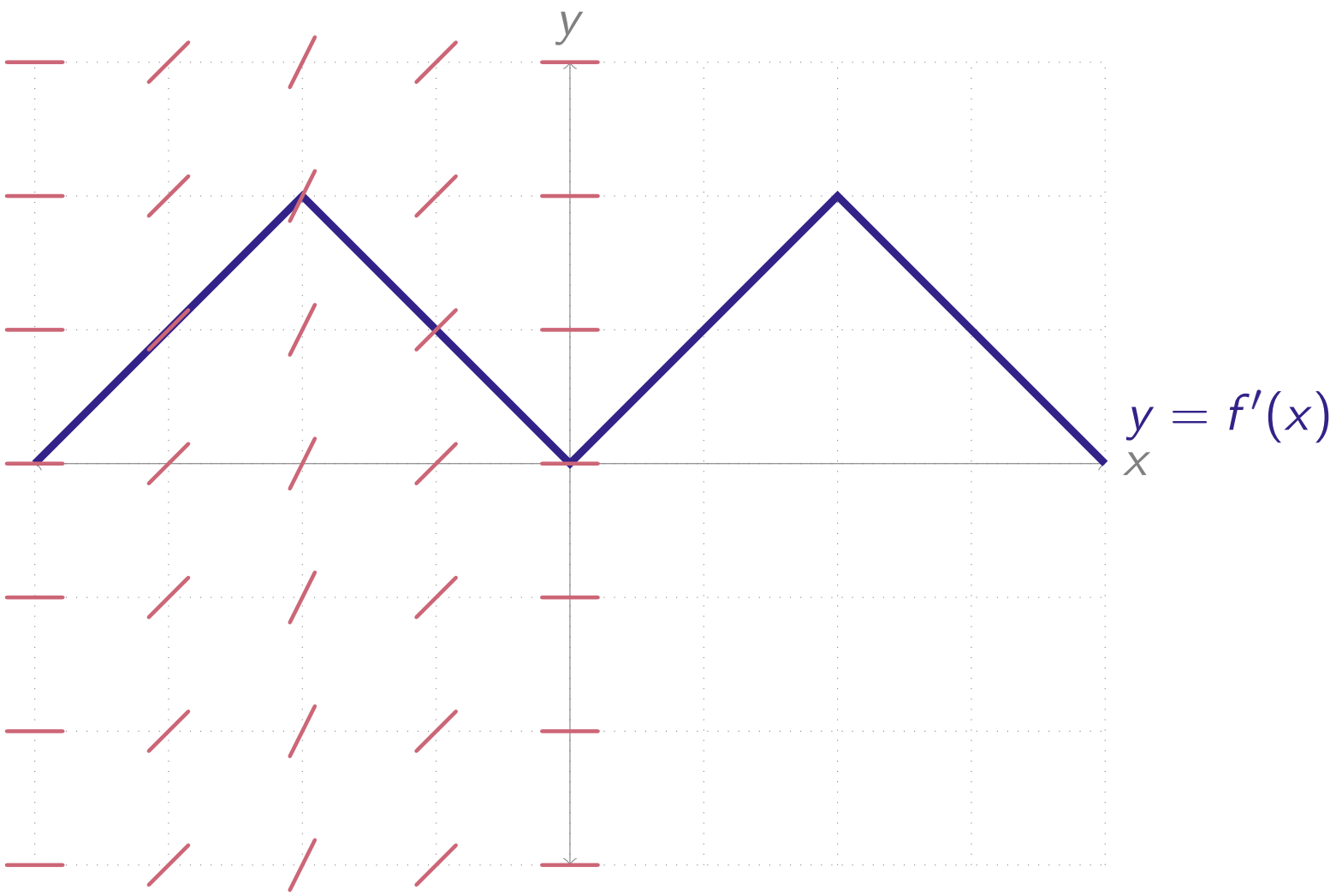

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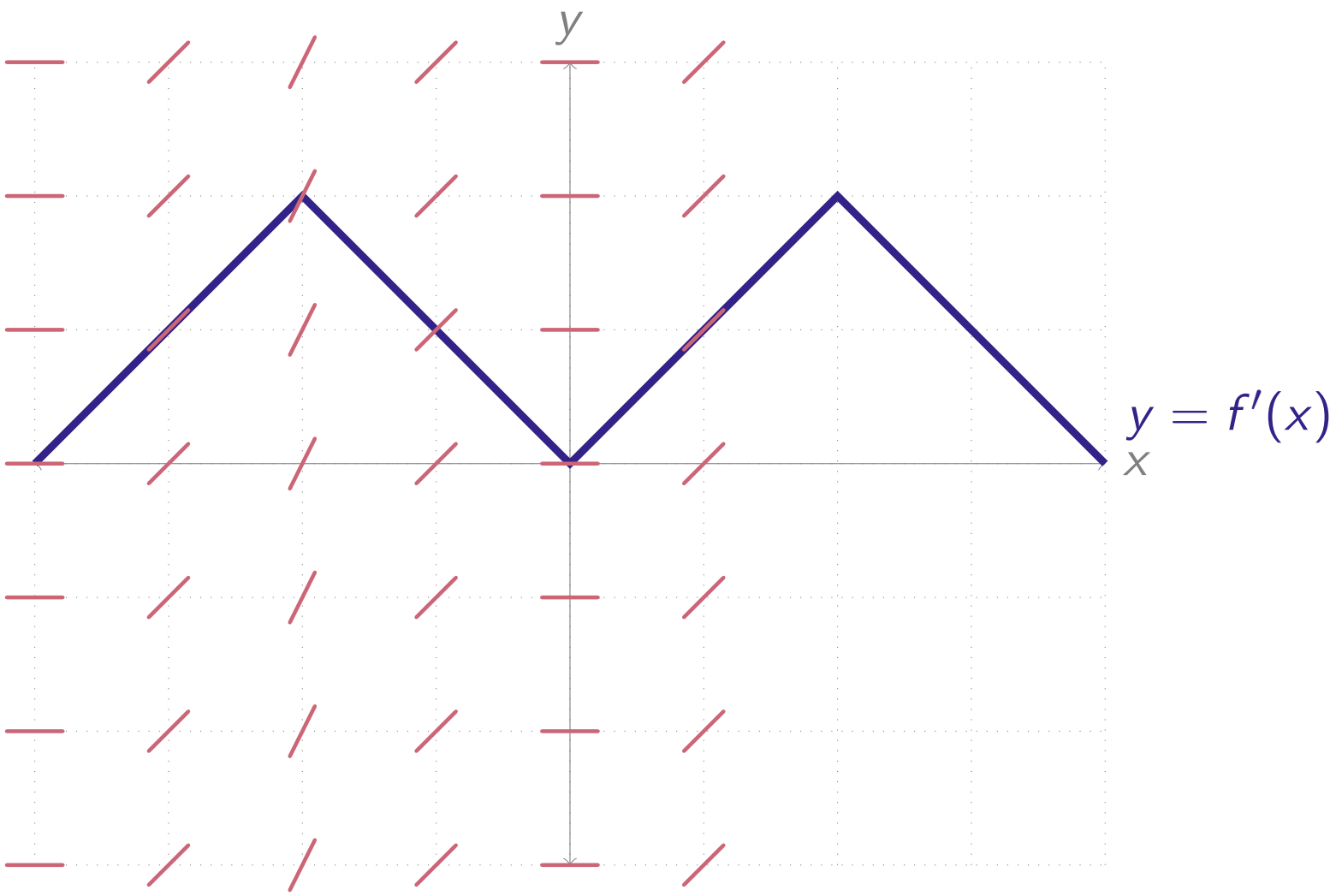

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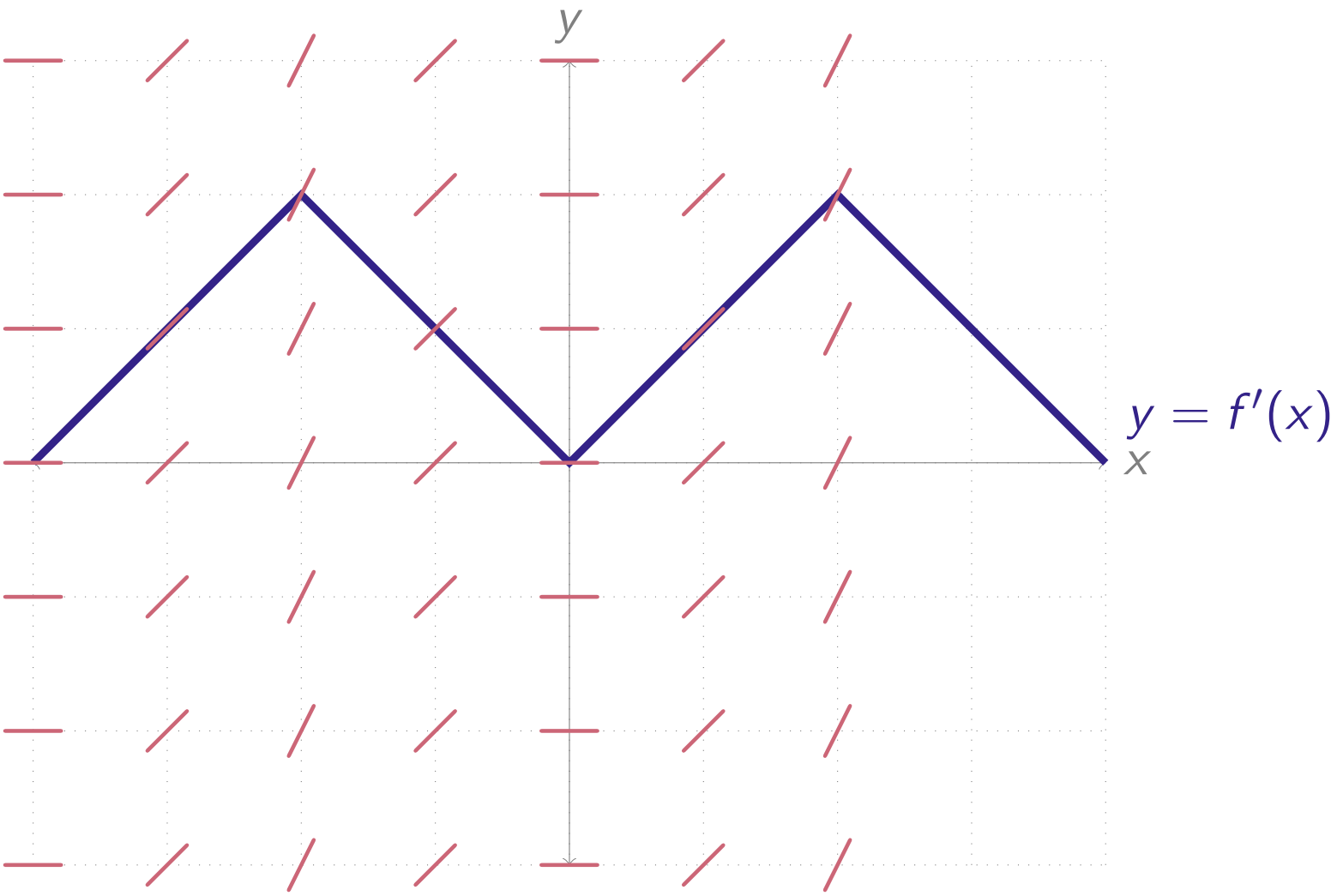

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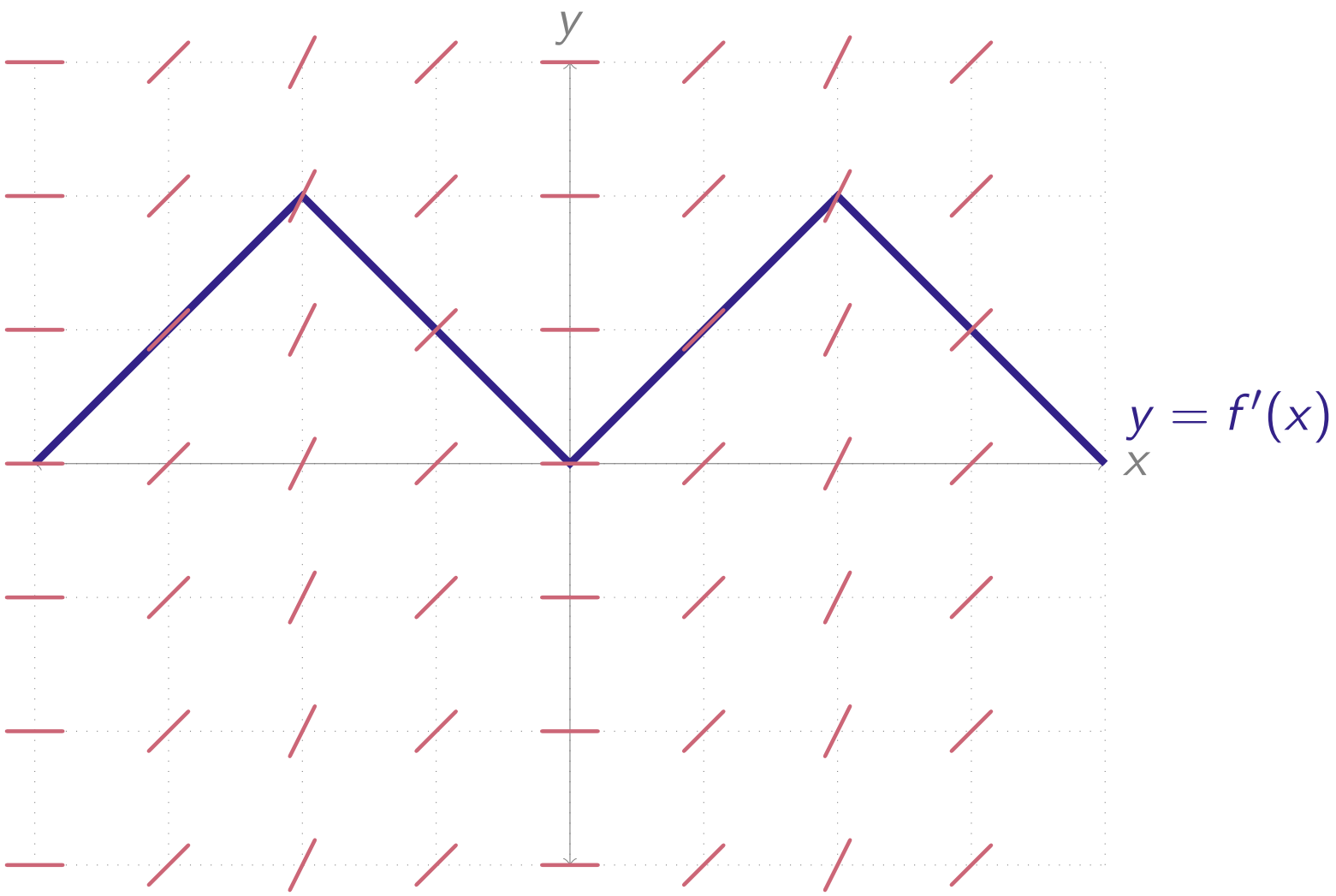

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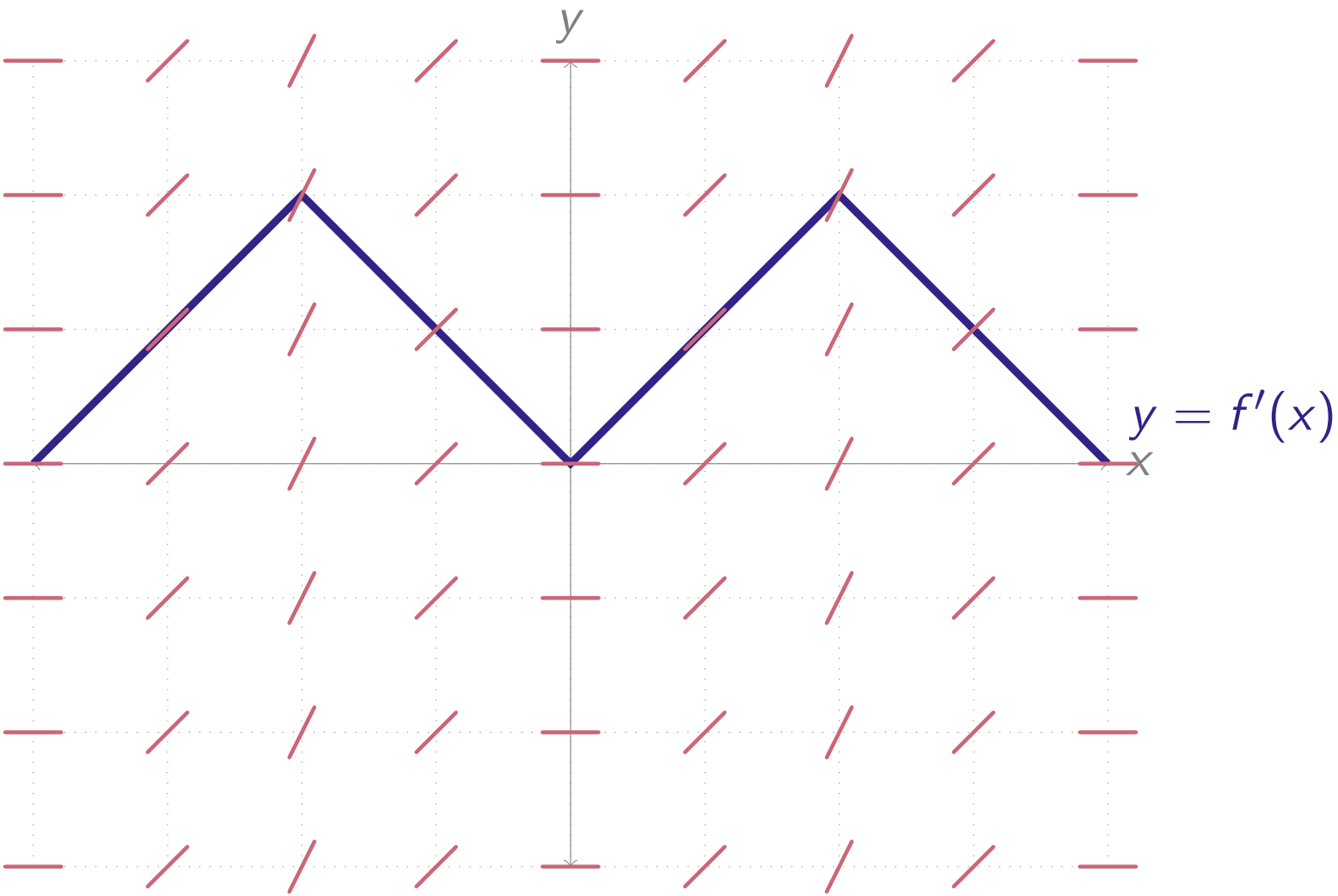

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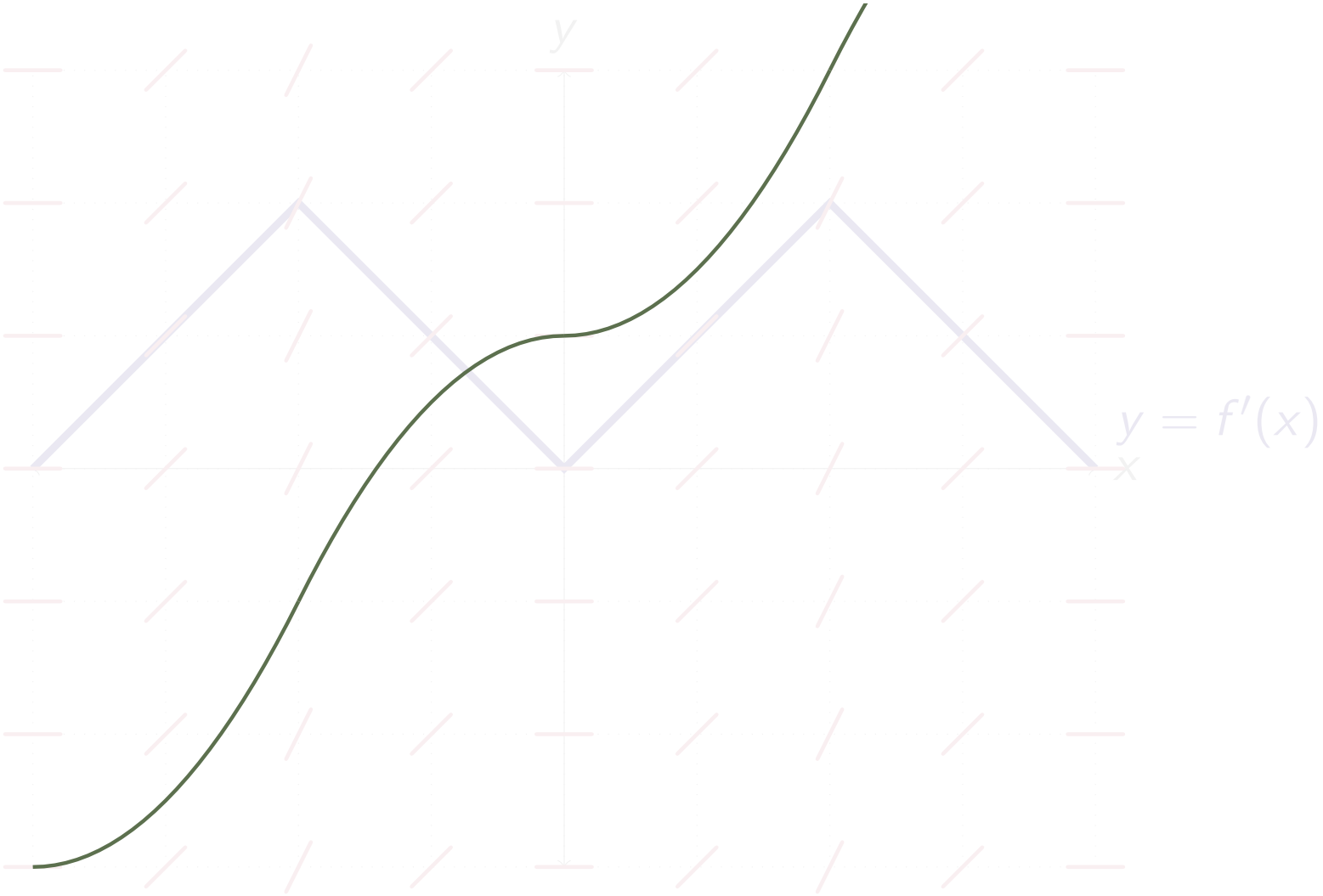

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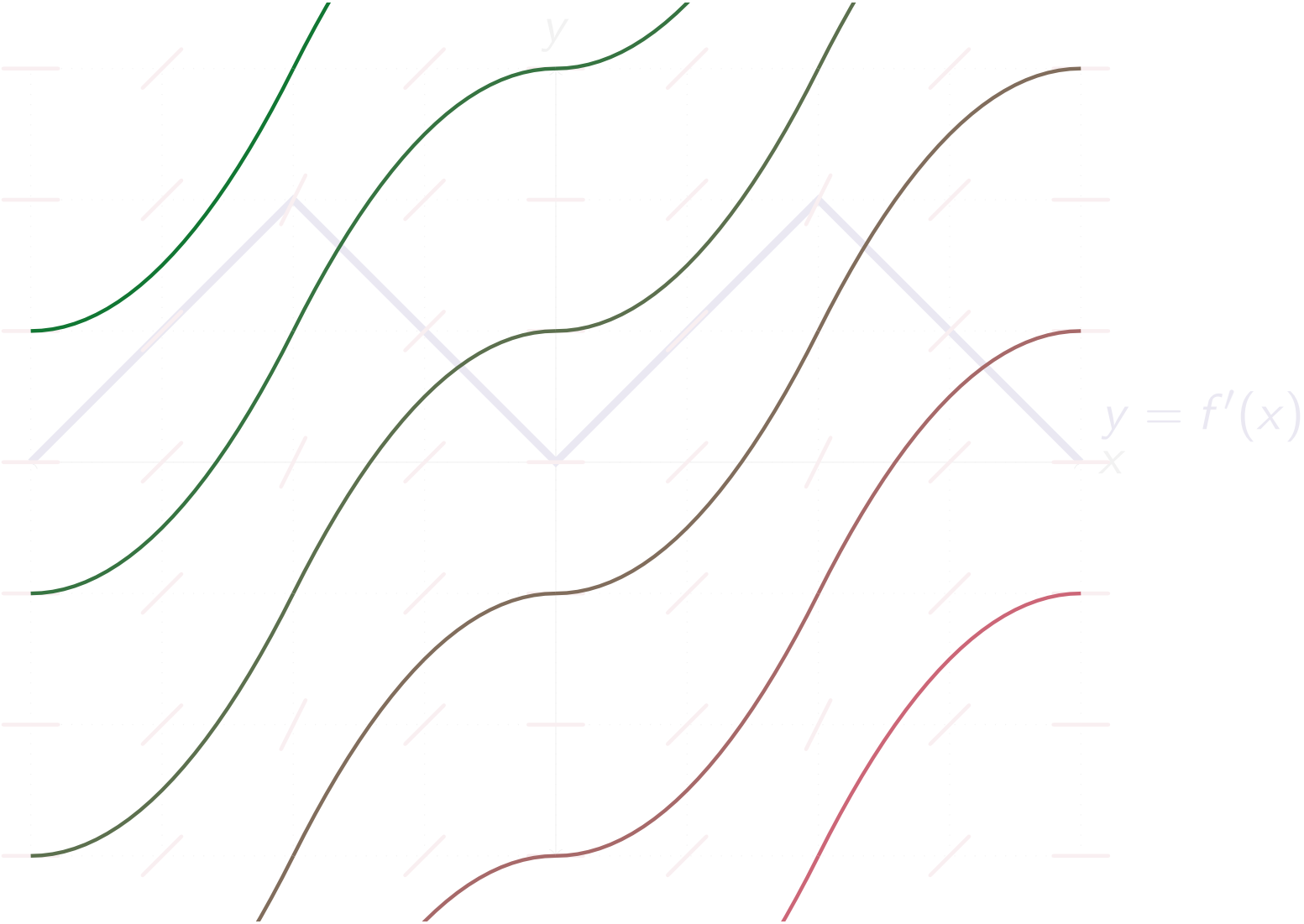

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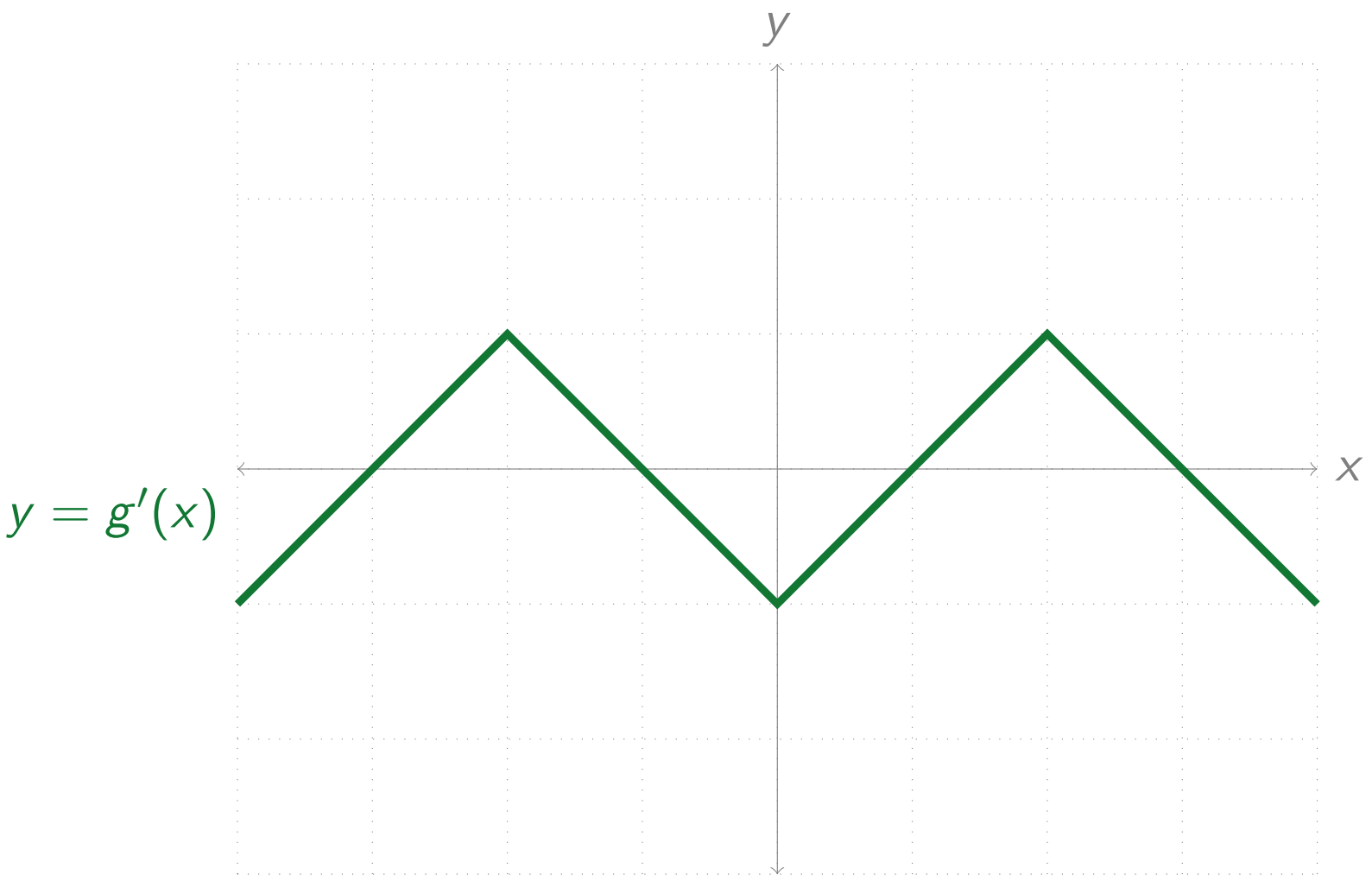

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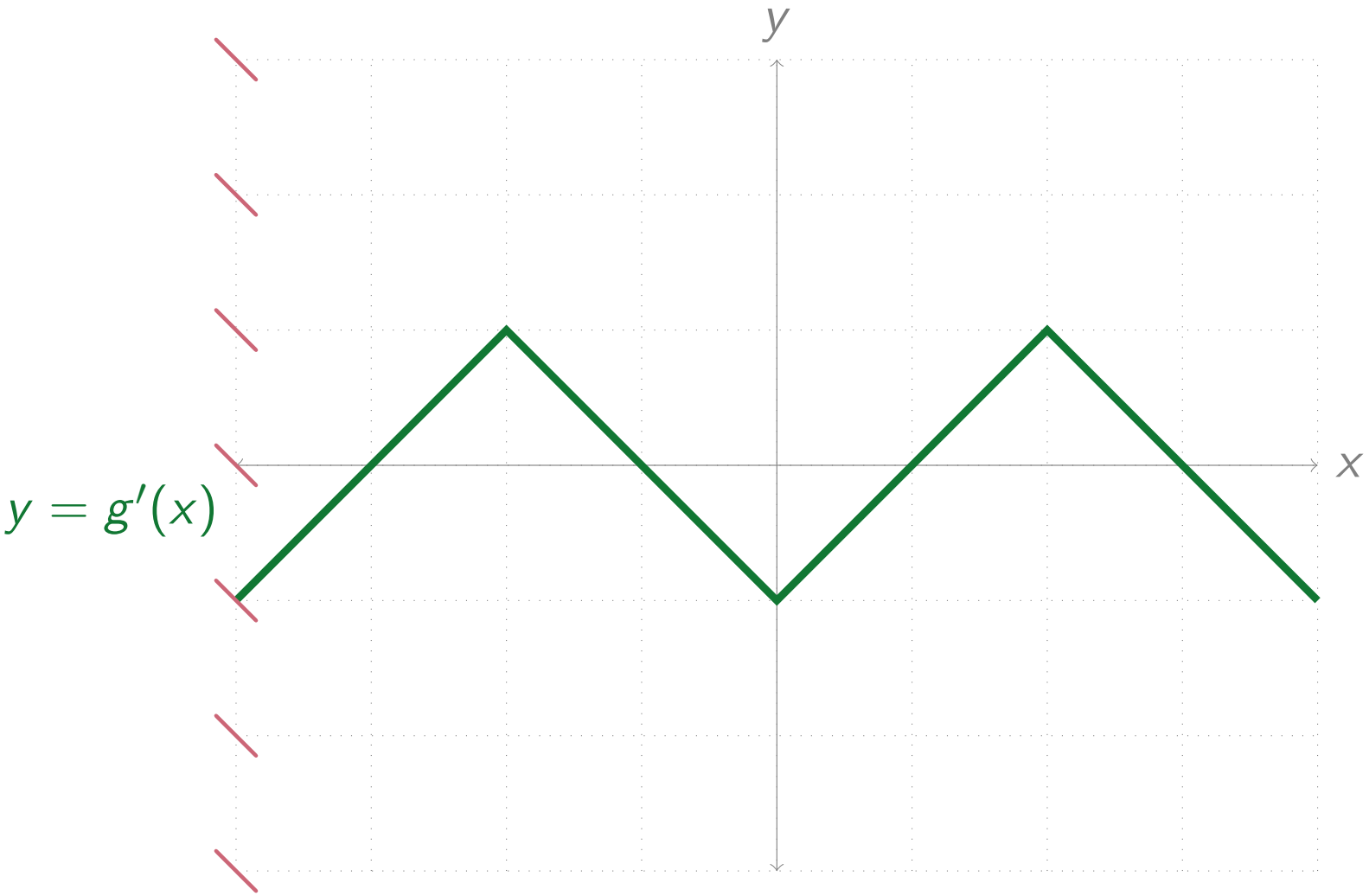

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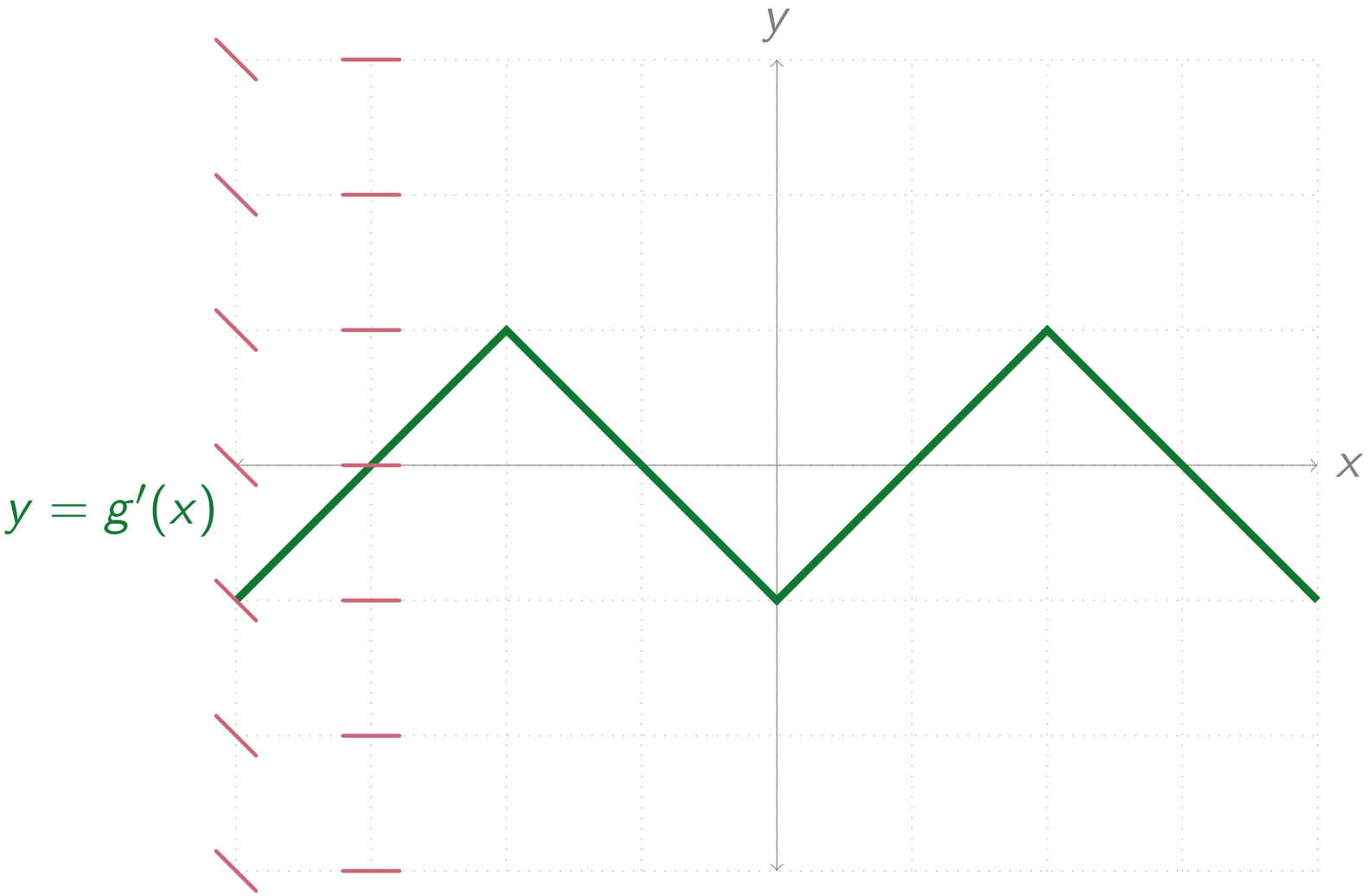

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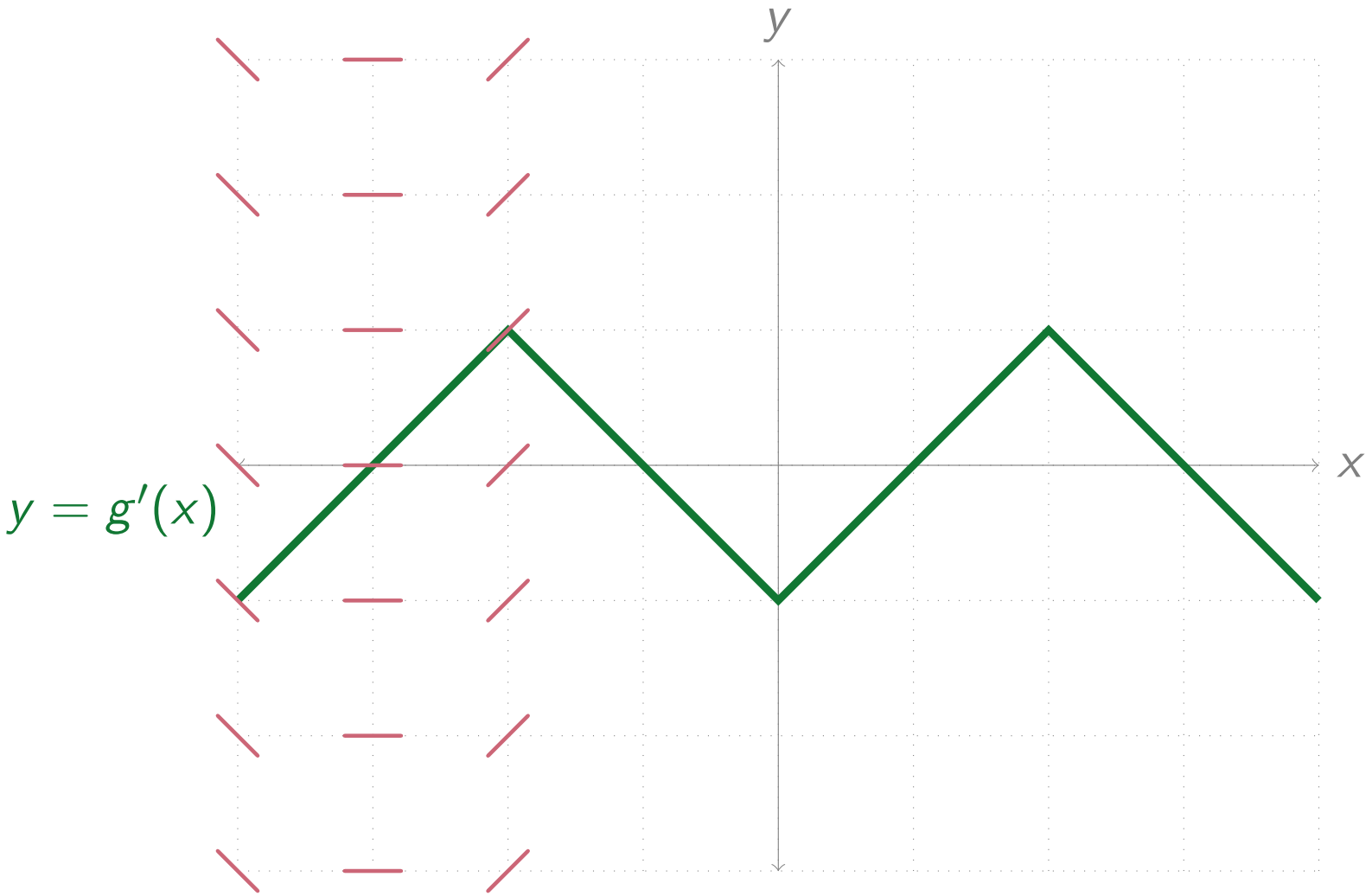

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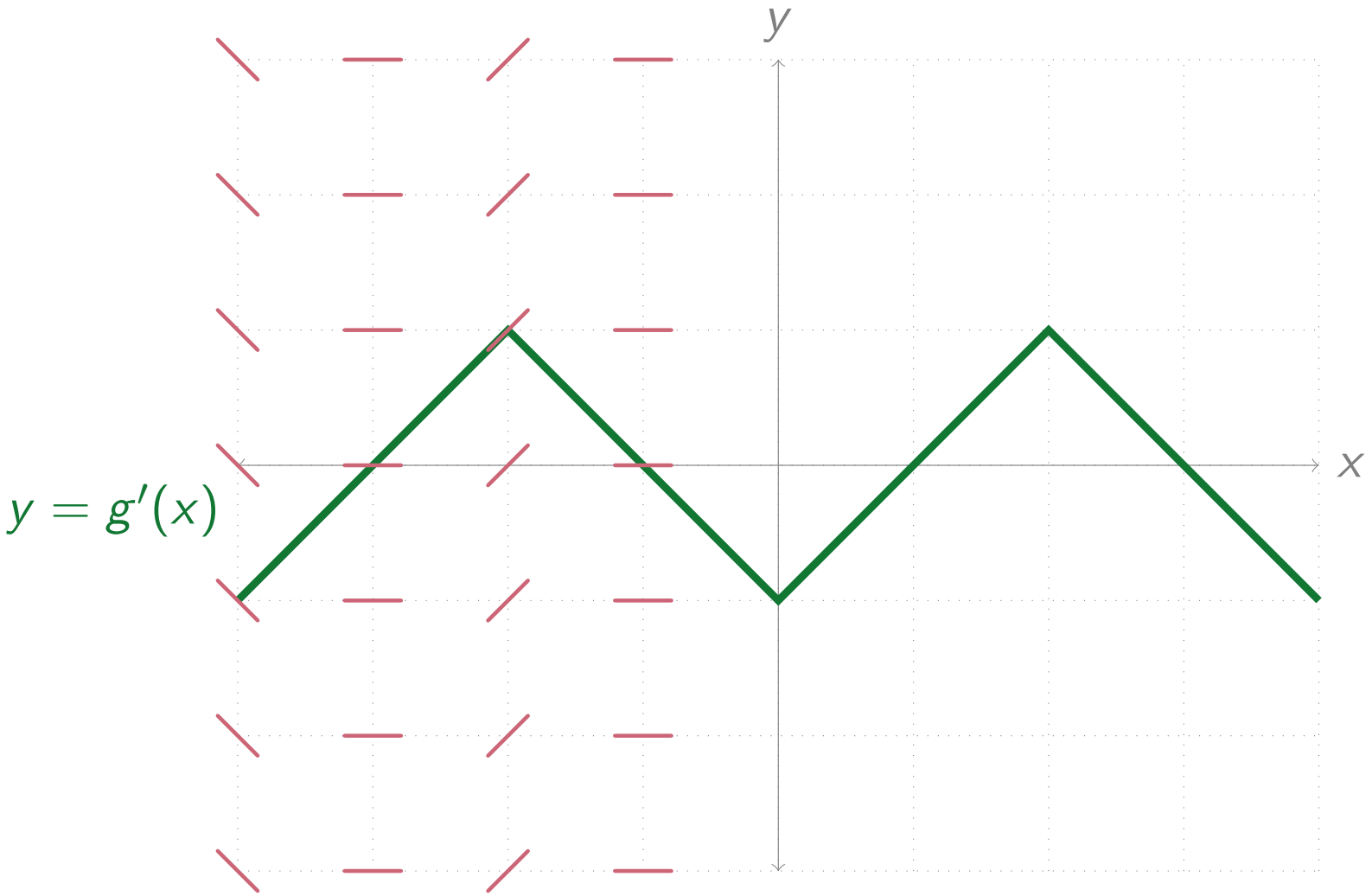

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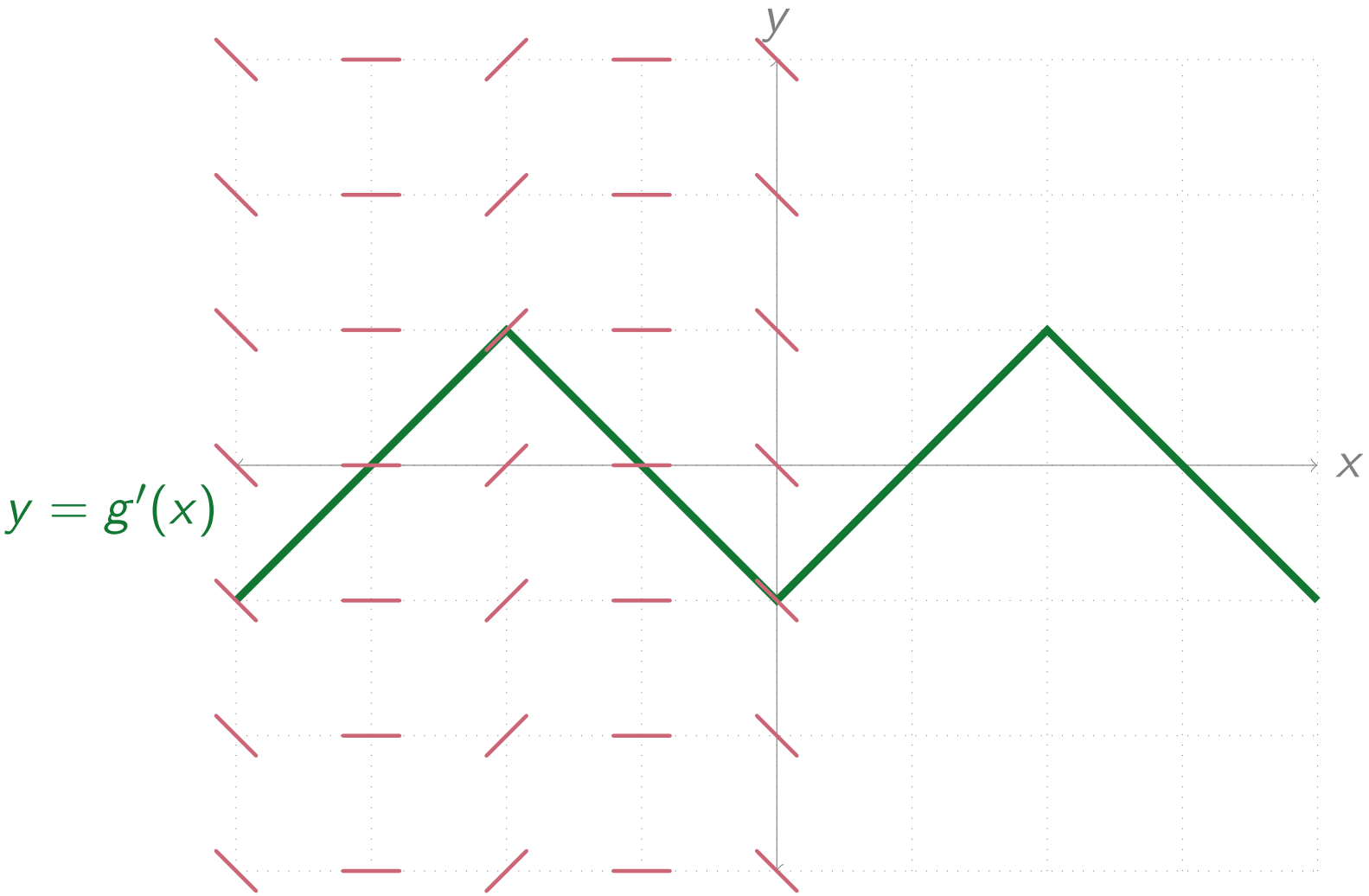

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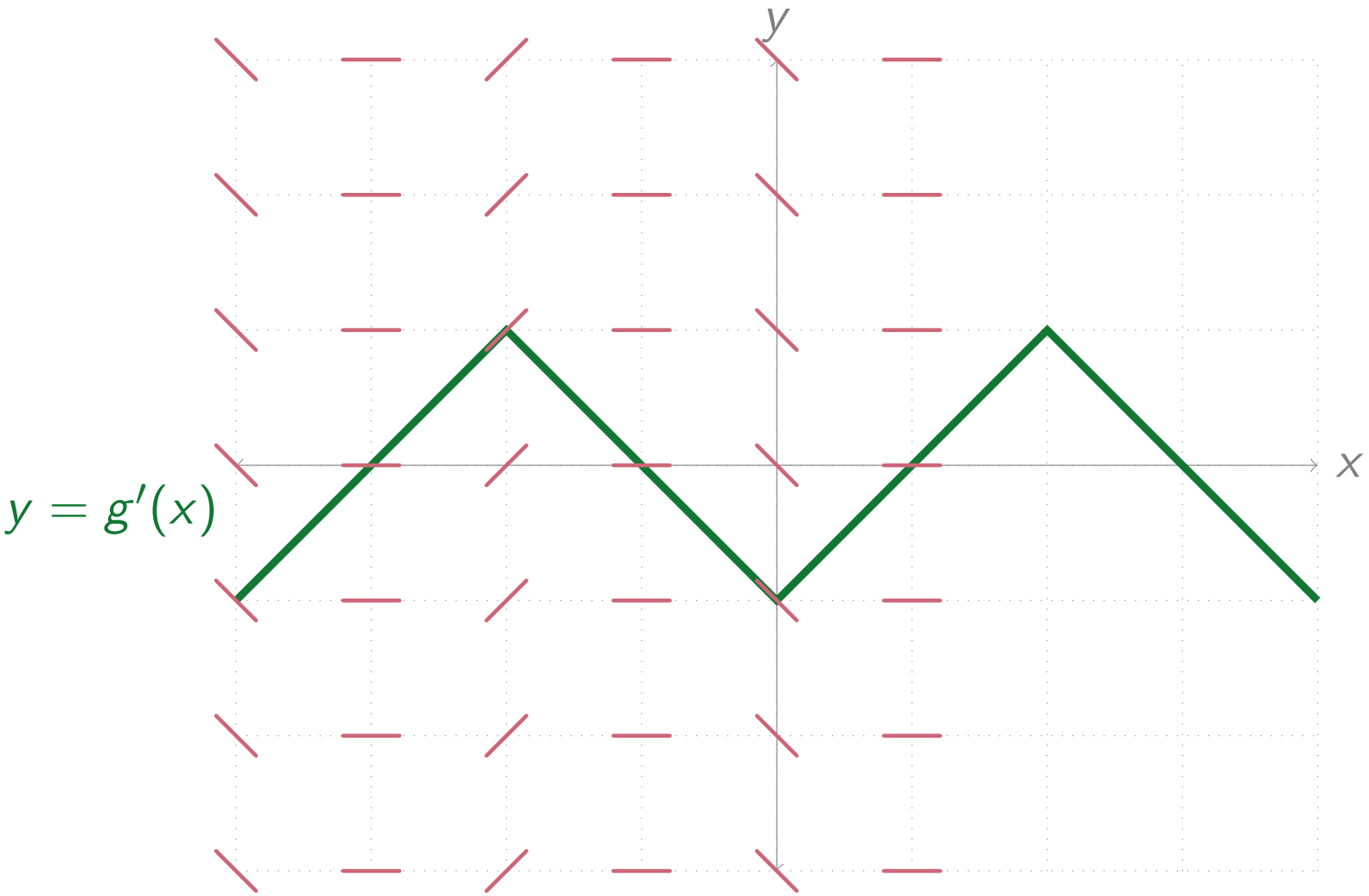

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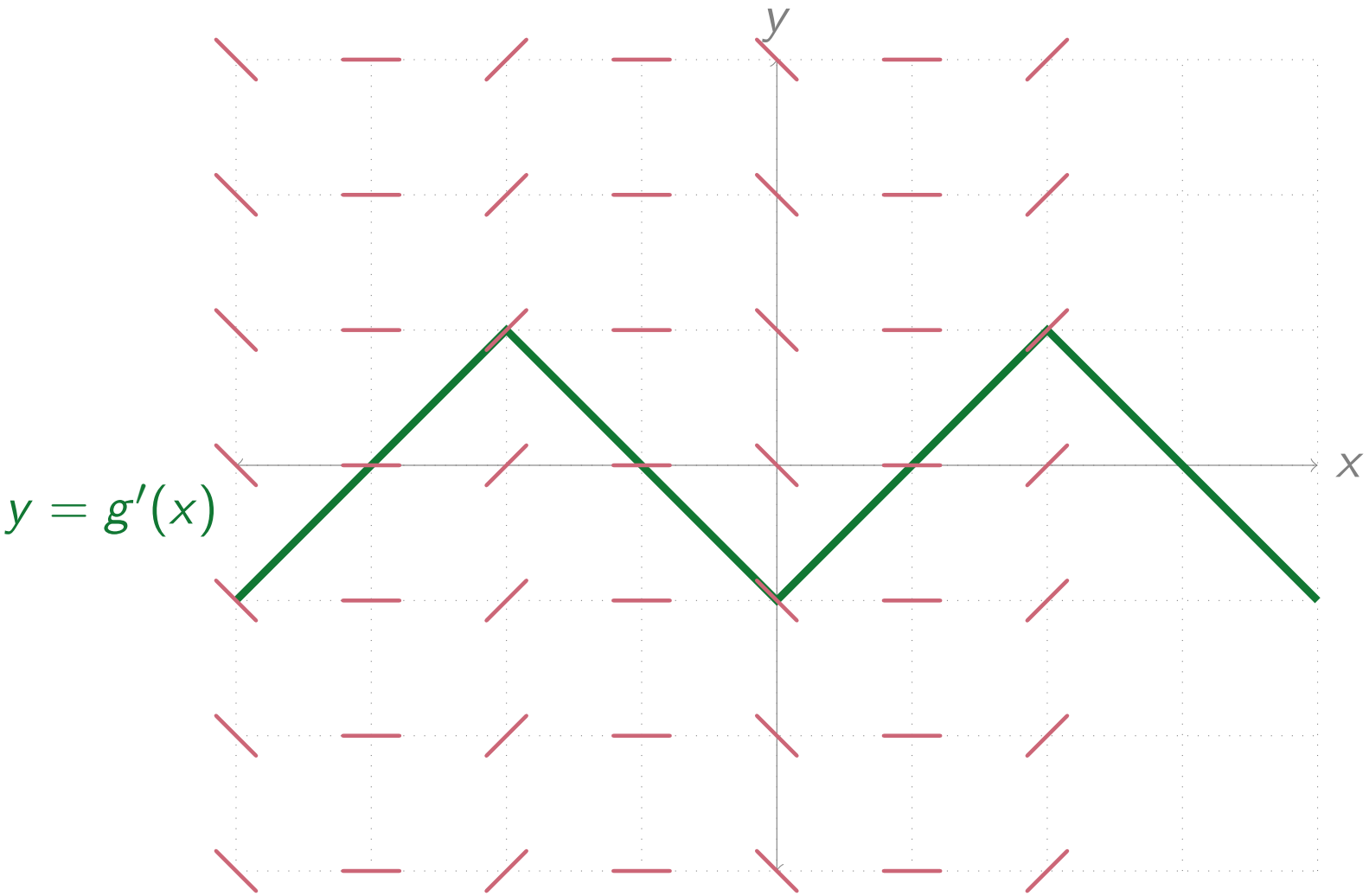

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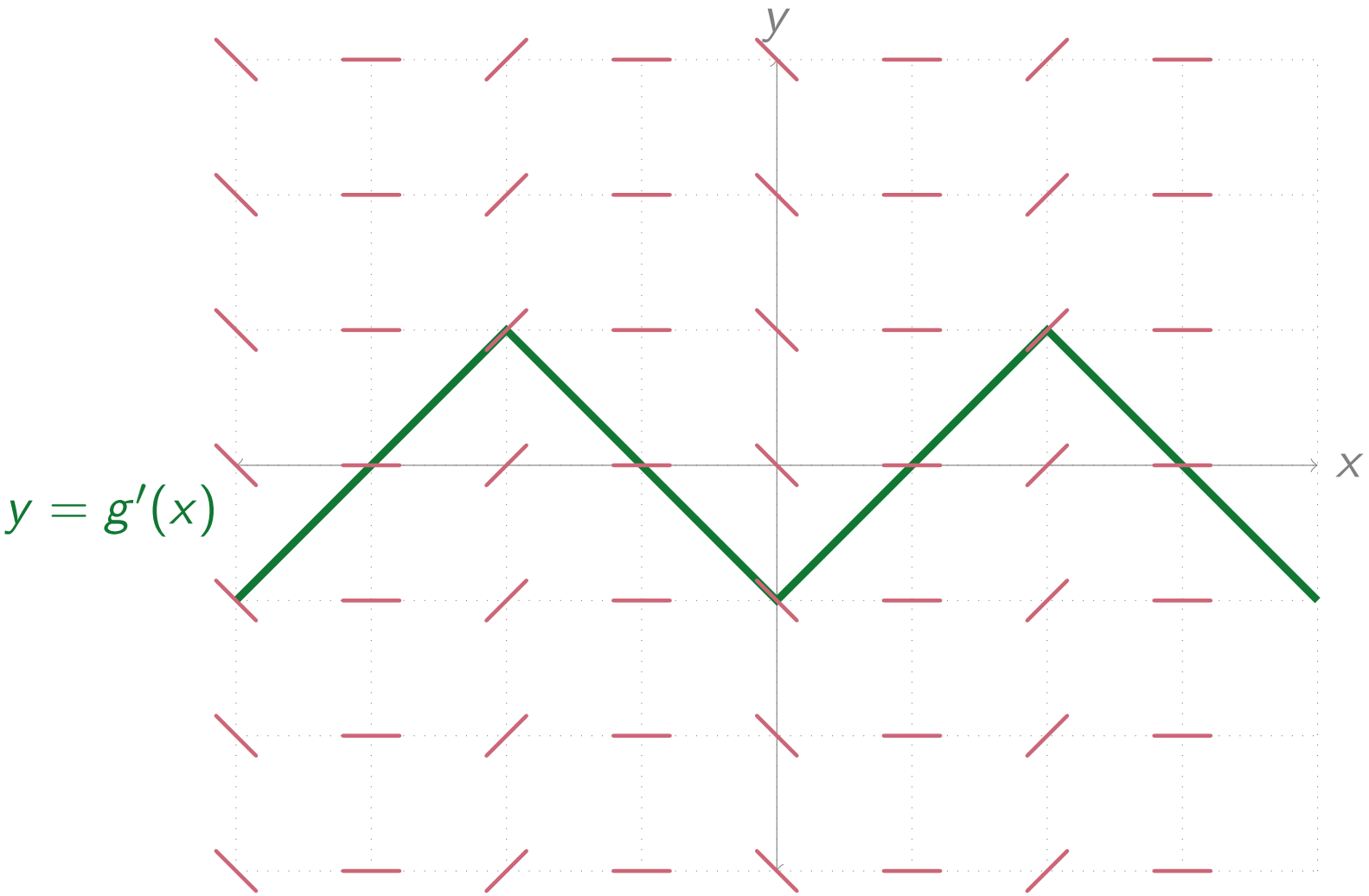

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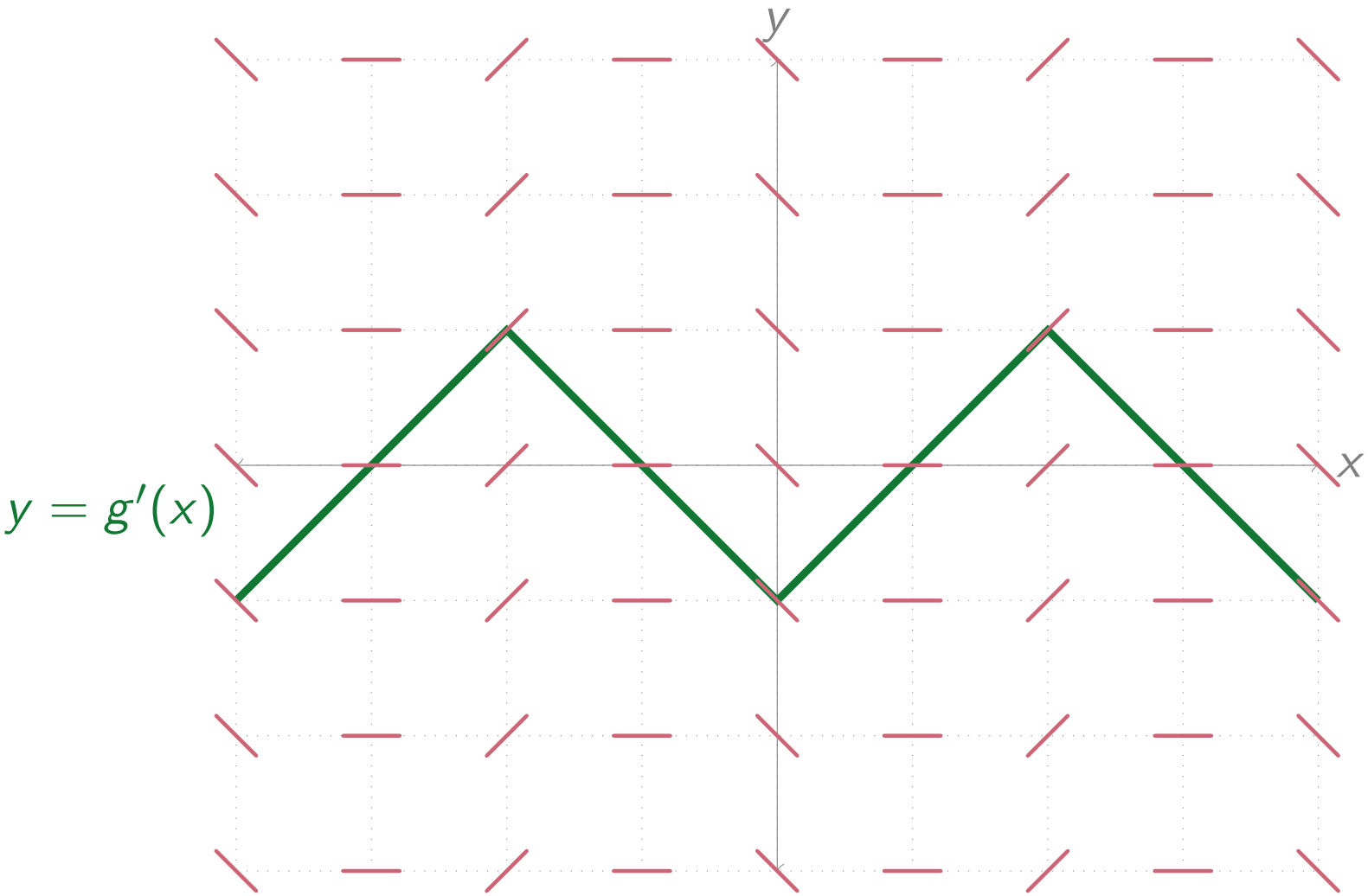

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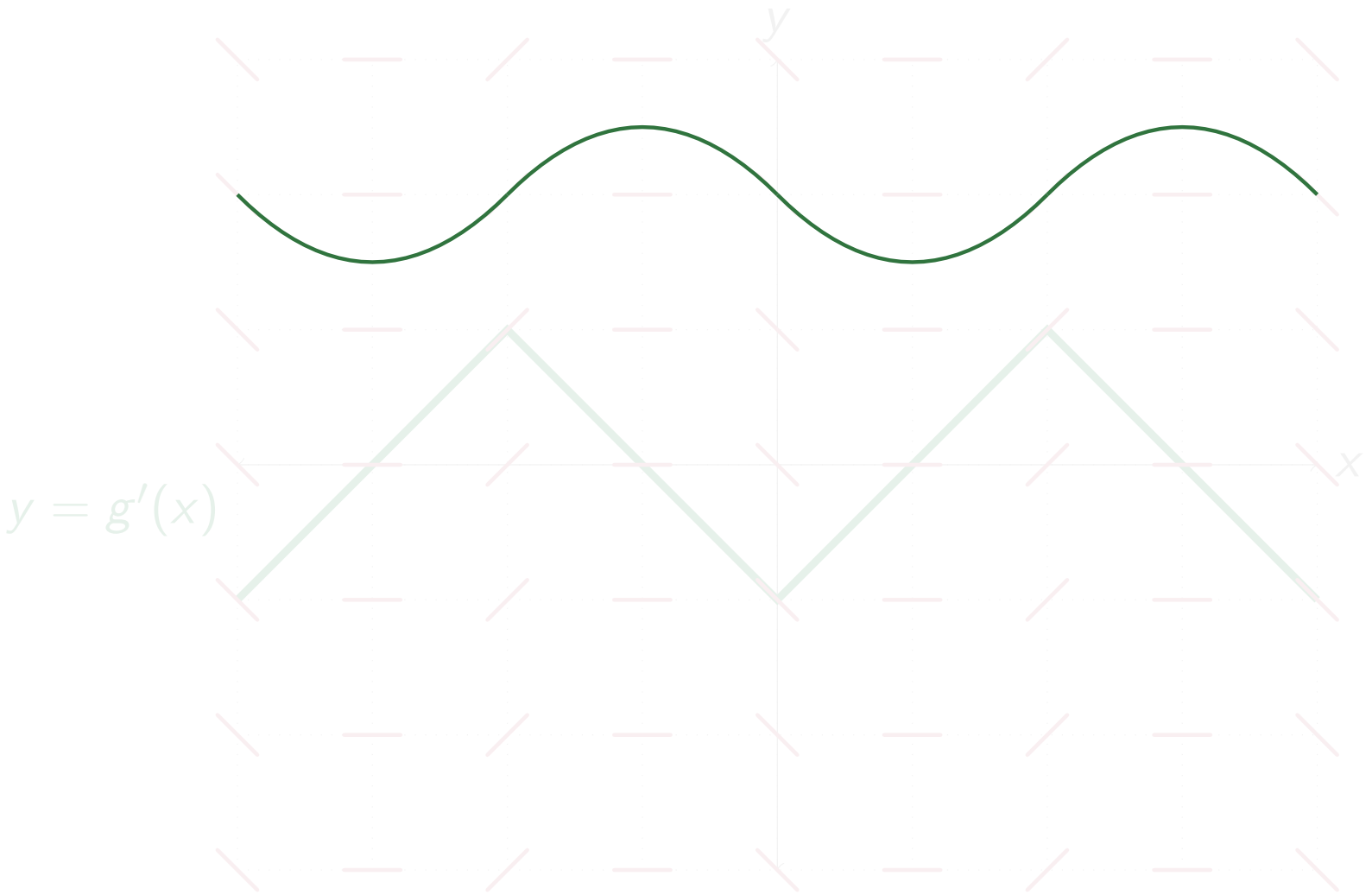

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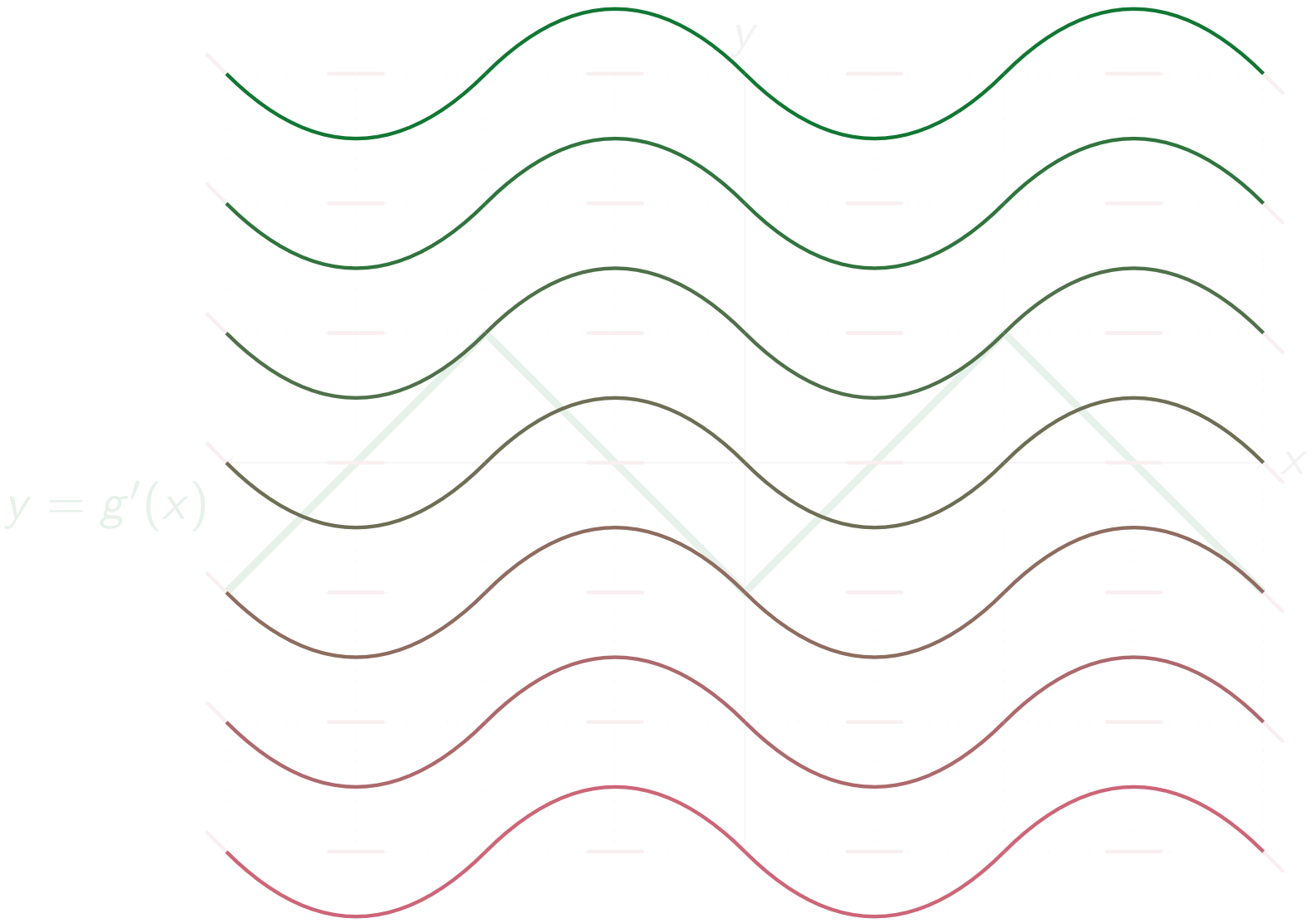

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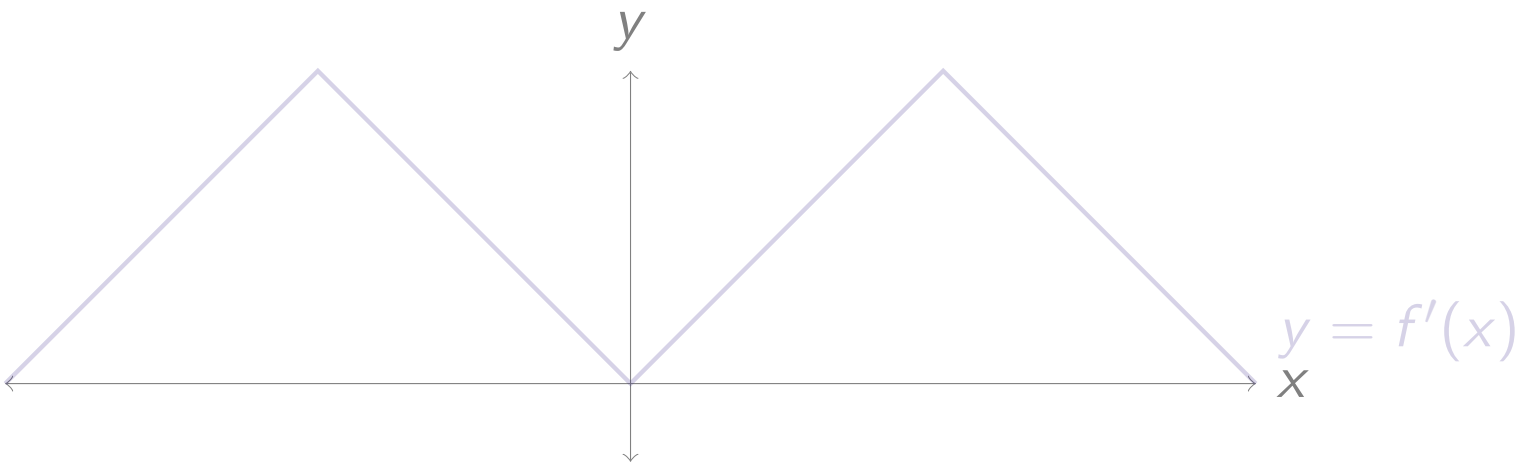

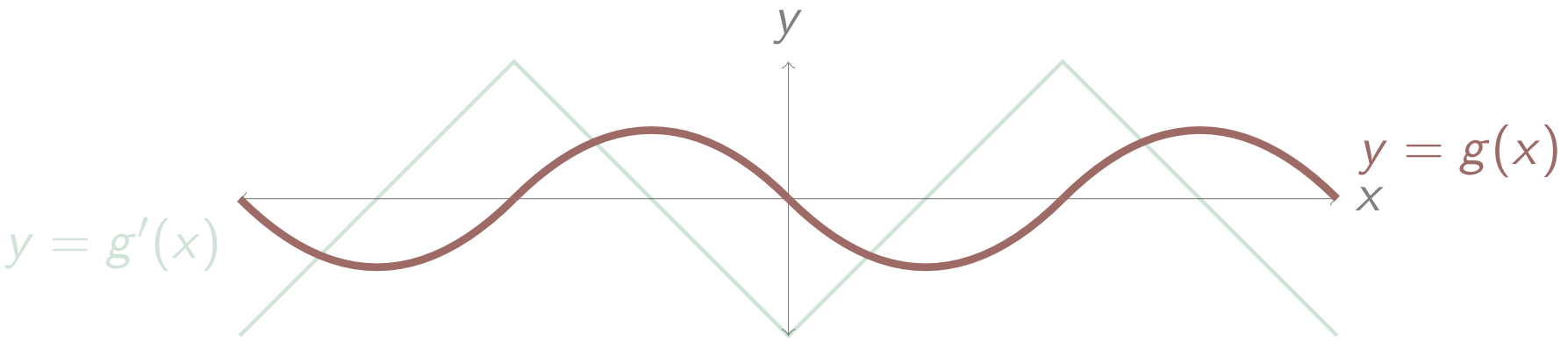

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