Welcome to Math 102, Calculus for the Life Sciences

Hi! I'm **Dr. Elyse Yeager.**
You can call me Elyse, or Dr. Yeager.

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229F Mathematics building
Office hours: Mon 11:30am-1:00pm; Wed 1:30-3:00pm;
other times by appointment.
Office hours start next week.

Basically all of information about this course is on our webpage:

`wiki.math.ubc.ca/mathbook/M102/`

You can also find it by Googling something like "`UBC Math 102 wiki`"

Our text is a free, online pdf.

Notes

What will we learn?

Differential calculus: study of rates of change.
How fast is a population growing; how fast is the speed of your car changing; more abstract uses.

Emphasis on life sciences interpretations.
Our book is tailored towards biological applications, but the underlying mathematics is very similar to other flavours of calculus at UBC and other universities.

Science Communication
You'll have assignments where you will have to clearly explain your reasoning.

Technology
You will also have assignments that involve using spreadsheet programs, like Excel or Google Sheet (which is free).

Problem Solving
!!!!!!!!!!!!!!!!!!

Notes

Administrative Info

Once more, with feeling:

`wiki.math.ubc.ca/mathbook/M102/`

Grade breakdown:

- 50% Final exam (don't make travel plans until it's scheduled, in October)
- 15% Midterm (Oct 26, evening)
- 15% 3 in-class quizzes (register with Access and Diversity if appropriate)
- 10% WeBWorK (online homework–first one due next Monday)
- 10% OSH (old-school homework, 6 assignments, presentation matters)

Course calendar on website.

More info later.

Notes

In Class

These notes are on the wiki.
Frame-by-frame and print versions available on our section's webpage.

Some exercises I do, some you do.
Bring something to write with and on. When you're working, talking with your neighbours is encouraged.

I'll wander around while you're working.
Sit on the aisle and I'll peek at your work; sit in the middle and I won't bother you.

Asking questions is encouraged,
but they should be relevant to the topic at hand.

Notes

Building Blocks

We're going to be using a lot of examples, and in order to start off simply, most of these examples will use **power functions**.

Let's get to know power functions:

- What they are
- Shapes, relative to one another
- Behaviour near the origin and far away
- Where two power functions intersect

(None of this is calculus– quick review.)

Notes

Power Functions

Power functions:

$$f(x) = Kx^n, \qquad \text{where } n \text{ is a positive integer and } K \text{ is a nonzero constant}$$

Which are power functions?

$$f(x) = x^2$$

$$f(x) = \pi x^{102}$$

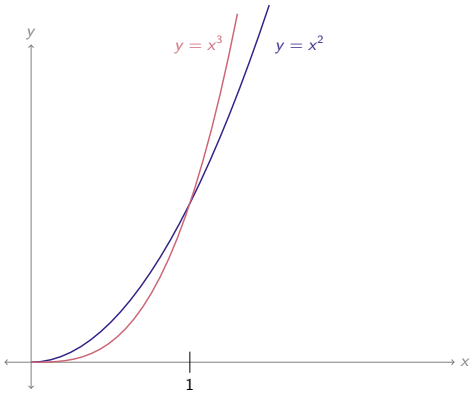
$$f(x) = \sqrt{x} = x^{1/2}$$

$$f(x) = \frac{1}{x} = x^{-1}$$

Notes

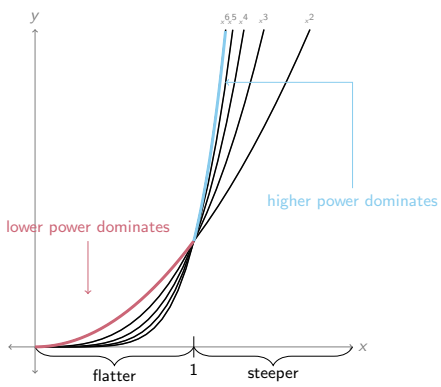
Relative Shapes of Power Functions

Familiar: $y = x^2$ What about $y = x^3$?



Notes

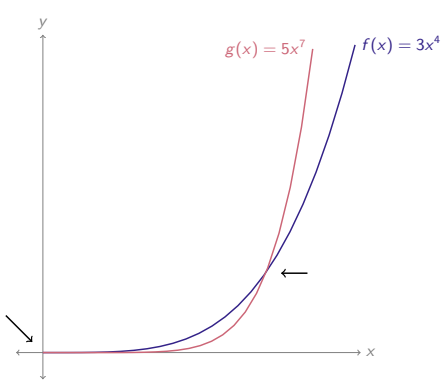
Relative Shapes of Power Functions



Notes

Intersections of Power Functions

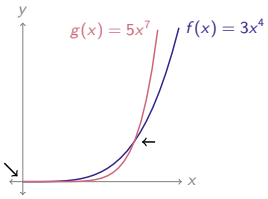
Example 1: Where do $f(x) = 3x^4$ and $g(x) = 5x^7$ intersect?



Notes

Intersections of Power Functions: solution

Solution 1: Where do $f(x) = 3x^4$ and $g(x) = 5x^7$ intersect?



$$
\begin{aligned}
5x^7 &= 3x^4 \\
5x^7 - 3x^4 &= 0 \\
x^4(5x^3 - 3) &= 0 \\
x^4 = 0 \quad &\text{OR} \quad 5x^3 - 3 = 0 \\
x = 0 \quad &\text{OR} \quad 5x^3 = 3 \\
x = 0 \quad &\text{OR} \quad x^3 = \frac{3}{5} \\
x = 0 \quad &\text{OR} \quad x = \sqrt[3]{\frac{3}{5}}
\end{aligned}
$$

Remark: finding a decimal approximation for $\sqrt[3]{\frac{3}{5}}$ is pretty technical. We'll learn methods later on.

Notes

Intersection of Power Functions

Example 2: Where do $f(x) = ax^n$ and $g(x) = bx^m$ intersect?

You may assume that a and b are positive, and n and m are positive integers with $n < m$.

Notes

Intersection of Power Functions: solution

Solution 2: Where do $f(x) = ax^n$ and $g(x) = bx^m$ intersect?

You may assume that a and b are positive, and n and m are positive integers with $n < m$.

$$
\begin{aligned}
ax^n &= bx^m \\
ax^n - bx^m &= 0 \\
x^n\left(a - bx^{m-n}\right) &= 0 \\
x^n = 0 \quad &\text{OR} \quad a - bx^{m-n} = 0 \\
x = 0 \quad &\text{OR} \quad a = bx^{m-n} \\
x = 0 \quad &\text{OR} \quad \frac{a}{b} = x^{m-n} \\
x = 0 \quad &\text{OR} \quad \underbrace{\left(\frac{a}{b}\right)^{\frac{1}{m-n}} = x}_{\text{maybe } \pm}
\end{aligned}
$$

Notes

Spherical Cells

Notes

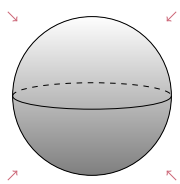
Learning goals

1. Follow and understand the derivation of a **mathematical model** (a simplified representation of a real situation) for cell nutrient absorption and consumption.
2. Develop the skill of using parameters (k_1, k_2) rather than specific numbers in mathematical expressions.
3. Understand the link between power functions in Section 1.1 and cell nutrient balance in the model.
4. Be able to verbally interpret the results of the model.

Spherical Cells - Setup

Notes

A cell absorbs nutrients through its wall, and metabolizes them inside.



Assumptions:

1. The cell is a smooth sphere.
   (Many cells are not even close to spherical–this model does not describe them well.)
2. The cell absorbs oxygen and nutrients through its surface. The larger the surface area, S, the faster the total rate of absorption. We will assume that the rate of absorption is proportional to the surface area of the cell.
3. The rate at which nutrients are consumed (i.e., used up) in metabolism is proportional the volume, V , of the cell. The bigger the volume, the more nutrients are needed to keep the cell alive.

Spherical Cells - Setup

Notes

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$$A = k_S \cdot S \quad \text{for some positive constant } k_S$$

3. The rate at which nutrients are consumed (i.e., used up) in metabolism is proportional the volume, V , of the cell. The bigger the volume, the more nutrients are needed to keep the cell alive.

$$C = k_V \cdot V \quad \text{for some positive constant } k_V$$

Constants:

S	surface area of cell	V	volume of cell
A	rate of absorption	C	rate of consumption
k_S	proportionality constant between surface area and absorption rate	k_V	proportionality constant between volume and consumption rate

Spherical Cells - Setup

$A = k_S \cdot S$ for some positive constant k_S

$C = k_V \cdot V$ for some positive constant k_V

Constants:

$S = 4\pi r^2$	surface area of cell	$V = \frac{4}{3}\pi r^3$	volume of cell
A	rate of absorption	C	rate of consumption
k_S	proportionality constant between surface area and absorption rate	k_V	proportionality constant between volume and consumption rate
r	radius of cell		

In order for the cell to not "starve," it needs to take in nutrients at least as fast as it can use them. That is, it must have:

$$\begin{aligned} A &\geq C \\ k_S \cdot S &\geq k_V \cdot V \\ k_S \cdot \left(4\pi r^2\right) &\geq k_V \left(\tfrac{4}{3}\pi r^3\right) \\ 3\left(\tfrac{k_S}{k_V}\right) r^2 &\geq r^3 \\ 3\left(\tfrac{k_S}{k_V}\right) &\geq r \end{aligned}$$

Notes

How Applicable is the Model?

Conclusion

$$r \leq 3\left(\frac{k_S}{k_V}\right)$$

If a cell fits our model, it can't be too big, or it will starve.

- Most cells are not too big: less than 1 mm in diameter.
- Notable exceptions don't fit the model:
  - Neurons are not spherical; they can get huge.

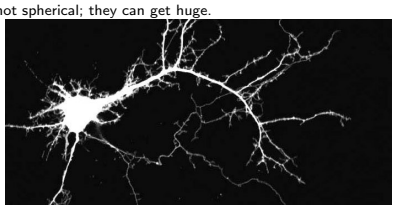
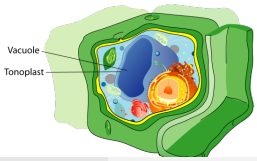


Photo credit: Laura Andreae

  - Large plant cells may have a lot of water, and a vacuole to store nutrients



Notes

Related Results: Bergmann's Rule

Roughly speaking, warm-blooded animals in cold climates are bigger than their relatives in warm climates.

↑ colder

White-tailed Deer, Canada
45-125 kg

Wood Bison, avg 840 kg

Polar bear, 350-700 kg

Snowman, 10-200 kg

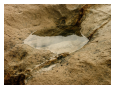
Arctic Hare, 2.5-7 kg

↓ warmer

Florida Keys and Tropics
35-50 kg

Plains bison, avg 730 kg

Brown bear, 80-600 kg



Tropical snowman, 0-15 kg

Rabbit, 0.5-2 kg

Canada deer: Douglas Sprott Other deer: Paul T Brown Wood Bison: balu Plains Bison: Rocky Mountain Outlook Polar bear: William Pitcher Brown bear: Tambako The Jaguar Snowman: Thomas Cook Arctic Hare: Steve Sayles Rabbit: WikiMedia

Obviously, it's more complicated than this:
one of several factors, many exceptions, possibly not true at all.

Notes

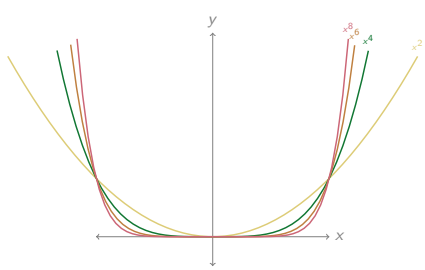
Graph Sketching: Learning Goals

- Identify even and odd functions graphically/algebraically
- Sketch simply polynomials like $y = ax^n + bx^m$
- Sketch simple rational functions like $y = \dfrac{Ax^n}{b + x^m}$
- Gain intuition about the behaviours of simple functions

Notes

Even and Odd Power Functions

Power functions with even powers:

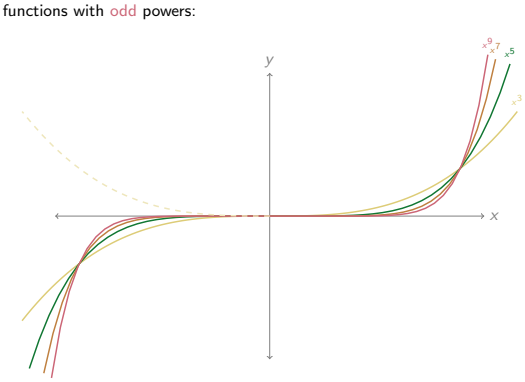


Symmetry: mirrored about y-axis

Notes

Even and Odd Power Functions

Power functions with odd powers:

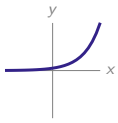


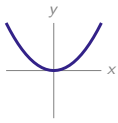
Symmetry: Mirrored about **both** axes.

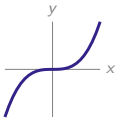
Notes

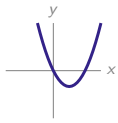
Even and Odd Power Functions

Example 3: Even, Odd, or Neither?

A:


B:


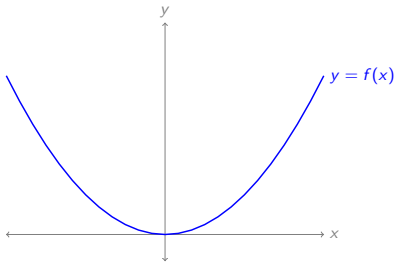
C:


D:


Notes

Algebraic Properties

Even function:



If $f(3) = 7$, then $f(-3) = 7$

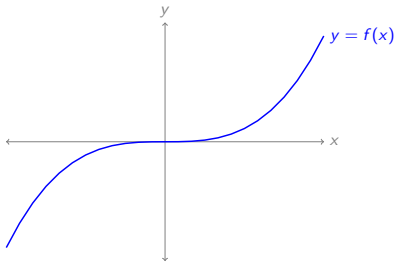
Definition

A function $f(x)$ is **even** if $f(x) = f(-x)$ for all x in its domain.

Notes

Algebraic Properties

Odd function:



If $f(3) = 7$, then $f(-3) = -7$

Definition

A function $f(x)$ is **odd** if $-f(x) = f(-x)$ for all x in its domain.

Notes

Even and Odd Functions

Notes

Definition

A function $f(x)$ is **even** if $f(x) = f(-x)$ for all x in its domain.
A function $f(x)$ is **odd** if $-f(x) = f(-x)$ for all x in its domain.

Example 4: Show that the following functions are even, odd, or neither.

- $f(x) = x^4 - x^2$
- $g(x) = x^3 + \frac{1}{x}$
- $h(x) = x + x^2$

A Quick Note about Approximations

Notes

There are about 7 billion people on the planet. One person is born. What is a good approximation for the number of people on the planet now?

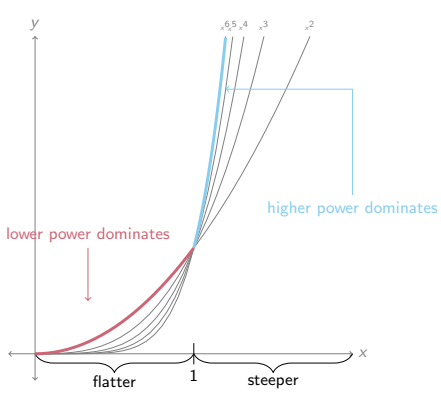
Example 5: Partner A has way more money in the bank thank Partner B.

They combine their accounts. What is the more reasonable approximation for their combined account balance?

(a) Partner A's original balance
(b) Partner B's original balance

Using Approximations to sketch Two-Term Polynomials

Notes

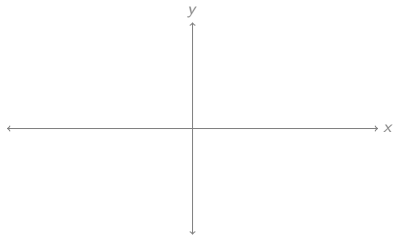


Using Approximations to sketch Two-Term Polynomials

$$f(x) = 3x + x^2$$
$$f(x) = 3x + x^2$$

$y = 3x$: straight line, pointing up
smaller power dominates near origin

$y = x^2$: parabola, pointing up
larger power dominates elsewhere



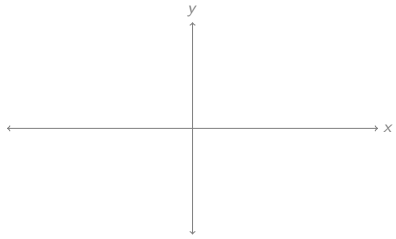
Notes

Using Approximations to sketch Two-Term Polynomials

$$f(x) = x^2 - x^4$$
$$f(x) = x^2 - x^4$$

$y = x^2$: parabola, pointing up
smaller power dominates near origin

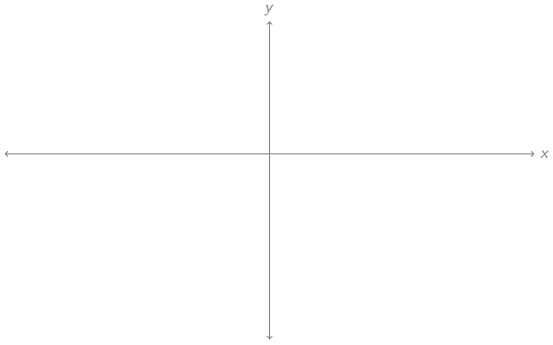
$y = -x^4$: pointing down
larger power dominates elsewhere



Notes

Now You!

$$f(x) = x^3 - x^2$$



Notes

More About Approximations

Notes

Example 6: You buy one million grains of rice, and each grain of rice costs $\$\frac{1}{1,000}$.

(That is, each grain costs one-tenth of a cent.)

What is a good approximation for the amount of money you spent?

A. About \$1,000,000

B. About $\$\frac{1}{1,000}$

C. Both of these approximations are terrible.

The method we're using for approximating ("choose the bigger one") works pretty well for addition and subtraction, but not so much for multiplication.

Simple Rational Functions

Notes

Definition

A **rational function** is the quotient of two polynomials, $\frac{\text{poly}}{\text{other poly}}$

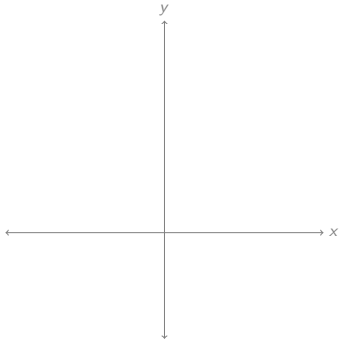
$$f(x) = \frac{Ax^n}{B + x^n}$$

We can use our approximation method on $B + x^n$.

- $f(0) =$
- When x is very close to zero, $B + x^n \approx$, so $f(x) \approx$
- When x is very far from zero, $B + x^n \approx$, so $f(x) \approx$

Notes

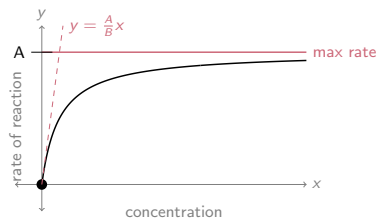
Example 7: Sketch $f(x) = \frac{5x^2}{1 + x^2}$



- $f(0) = 0$
- Near the origin, $f(x) \approx \frac{5x^2}{1} = 5x^2$
- Away from origin, $f(x) \approx \frac{5x^2}{x^2} = 5$

Enzyme-Catalyzed Reactions

The rate of reaction depends on the concentration of substrate, and properties of enzyme:



Michaelis-Menten kinetics

$$v = \frac{Ax}{B + x}$$

Notes

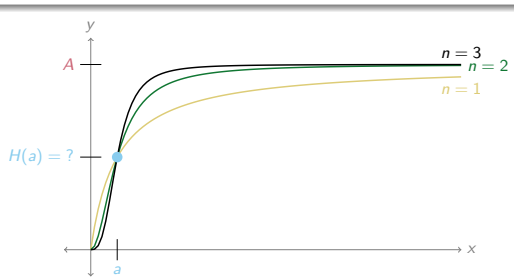
Hill Functions (Archibald Hill, 2010)

Definition

A **Hill function** has the form

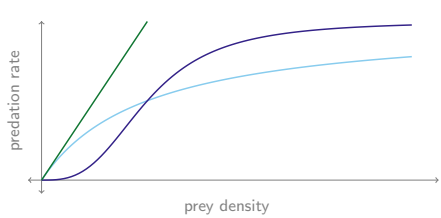
$$H(x) = \frac{Ax^n}{a^n + x^n} \quad , \qquad x \geq 0$$

for some constants A, a. We call a the **half-maximal activation level.**



Notes

Predator-Prey Interactions: Holling Predator Response



Type I: The more prey there is, the more I can eat

Type II: I get satiated and cannot keep eating more and more prey

Type III: I can hardly find the prey when the prey density is low, but I also get satiated at high prey density.

$$P(x) = Kx \qquad P(x) = \frac{Kx}{a + x} \qquad P(x) = \frac{Kx^n}{a^n + x^n}\ , \quad n \geq 2$$

Notes