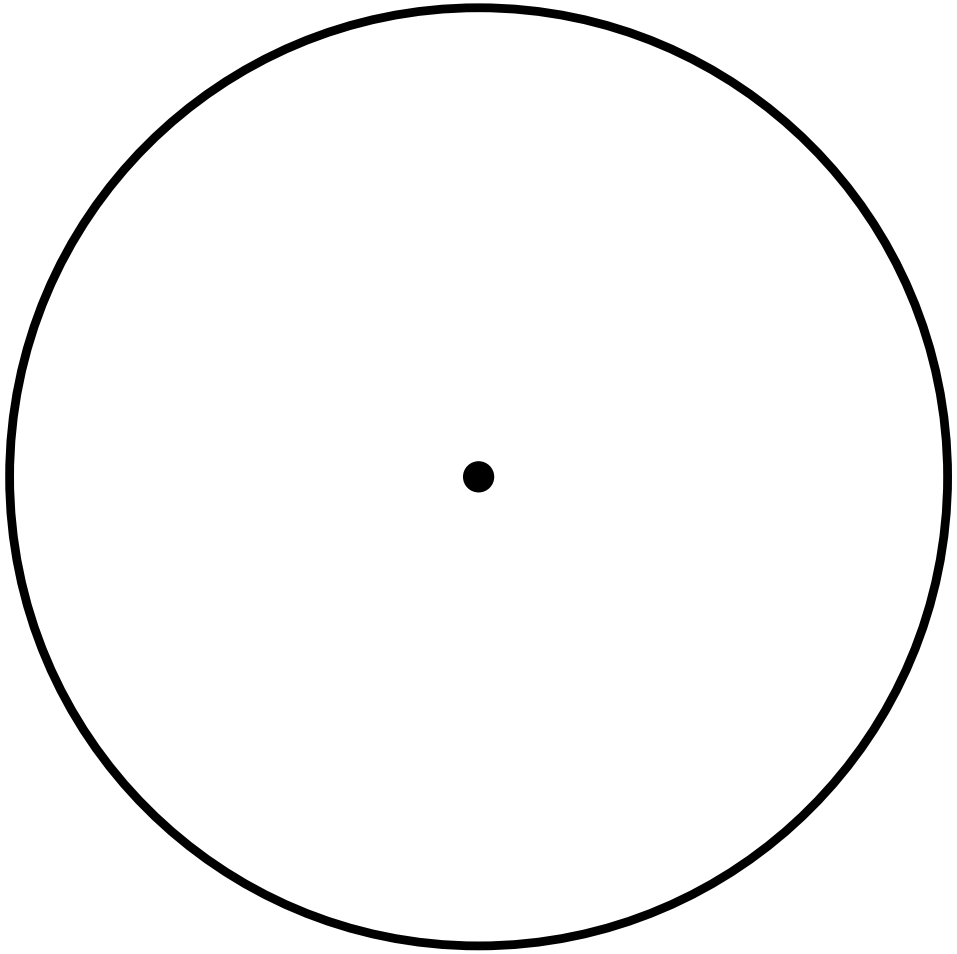
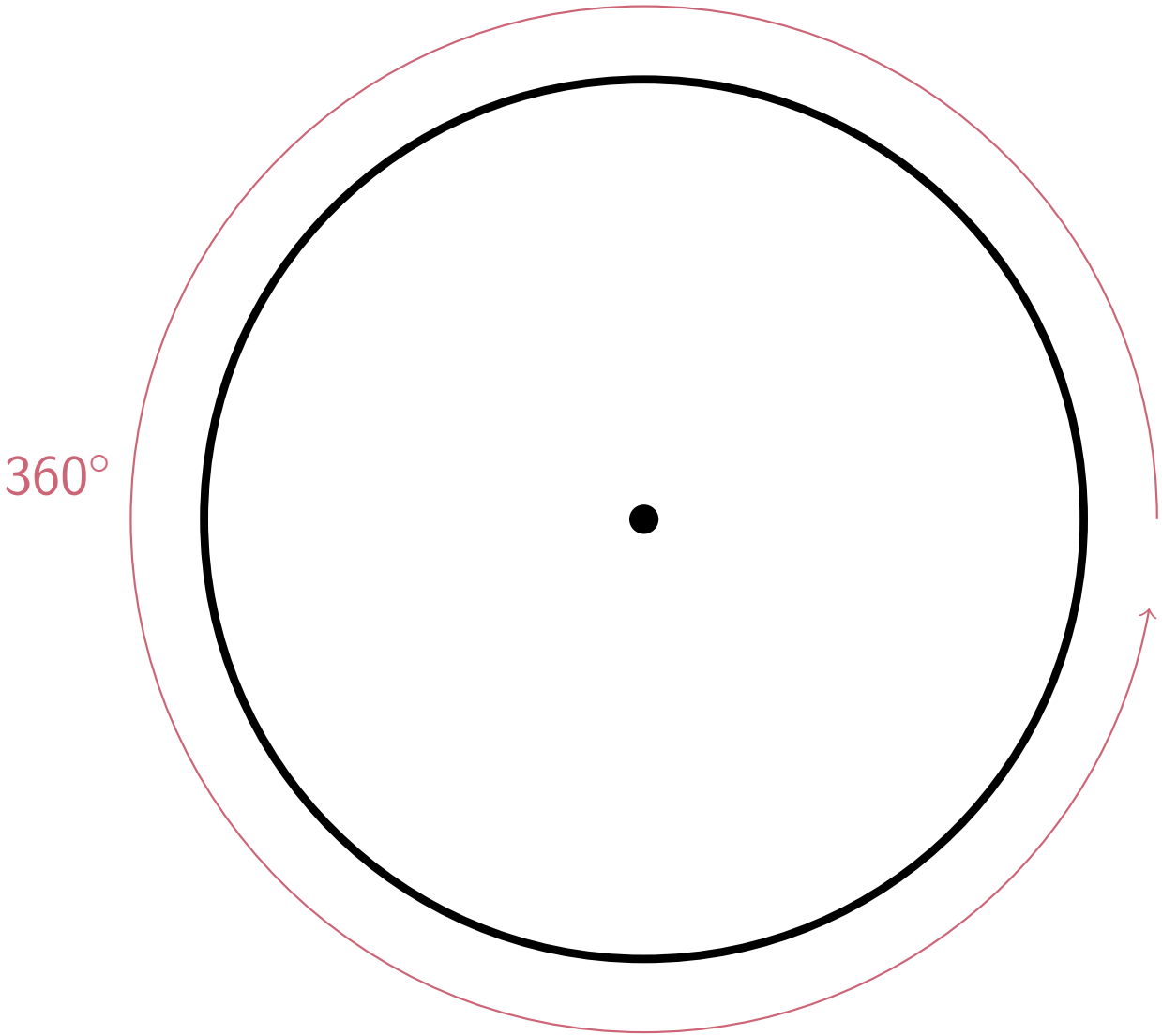
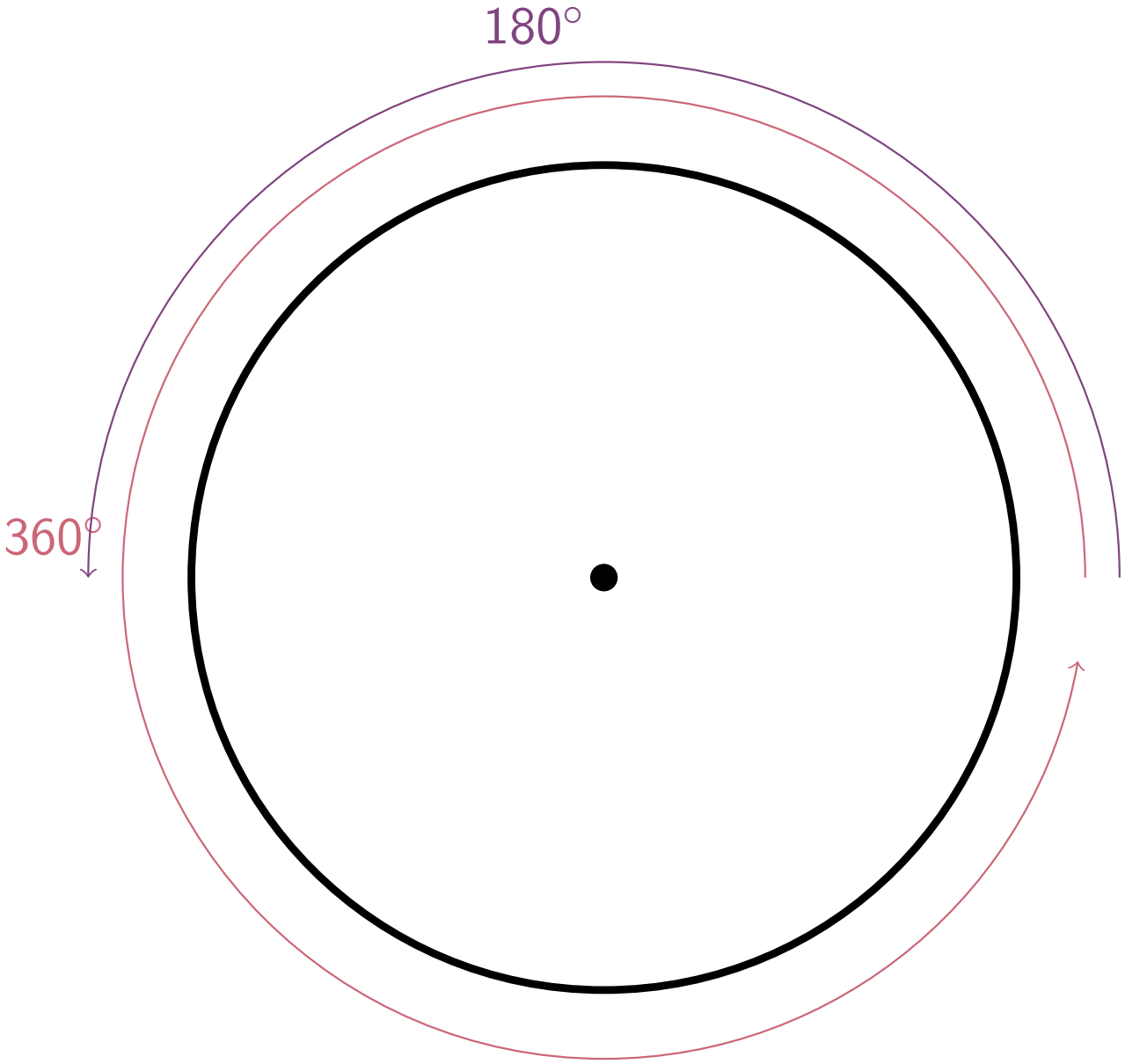
Radians



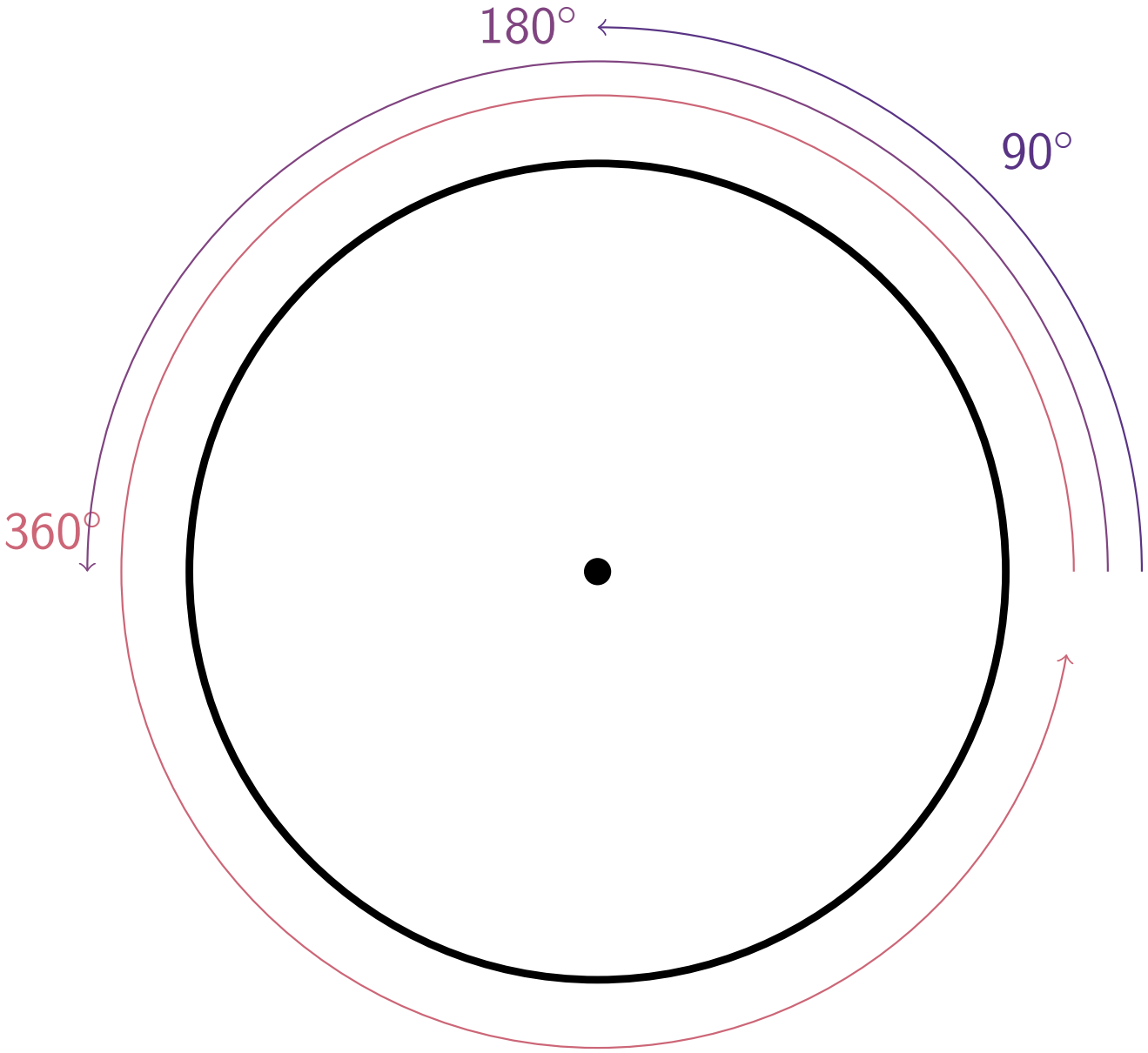
Radians



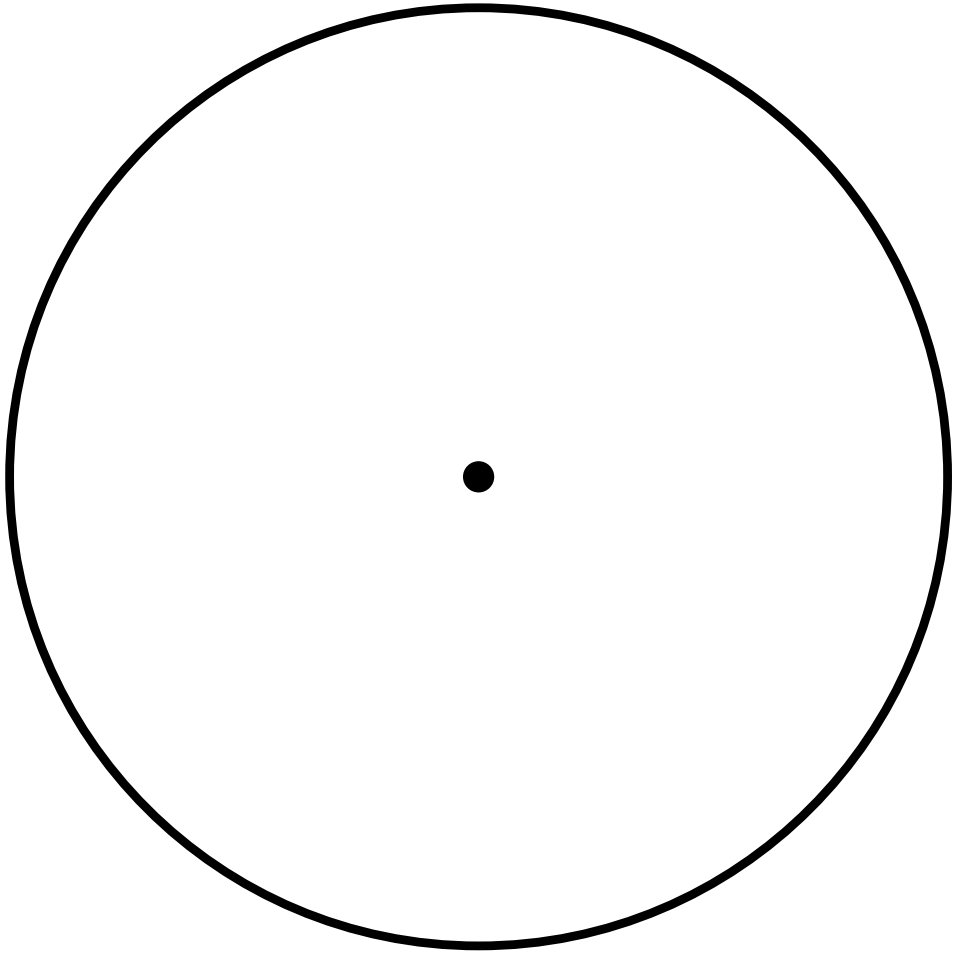
Radians



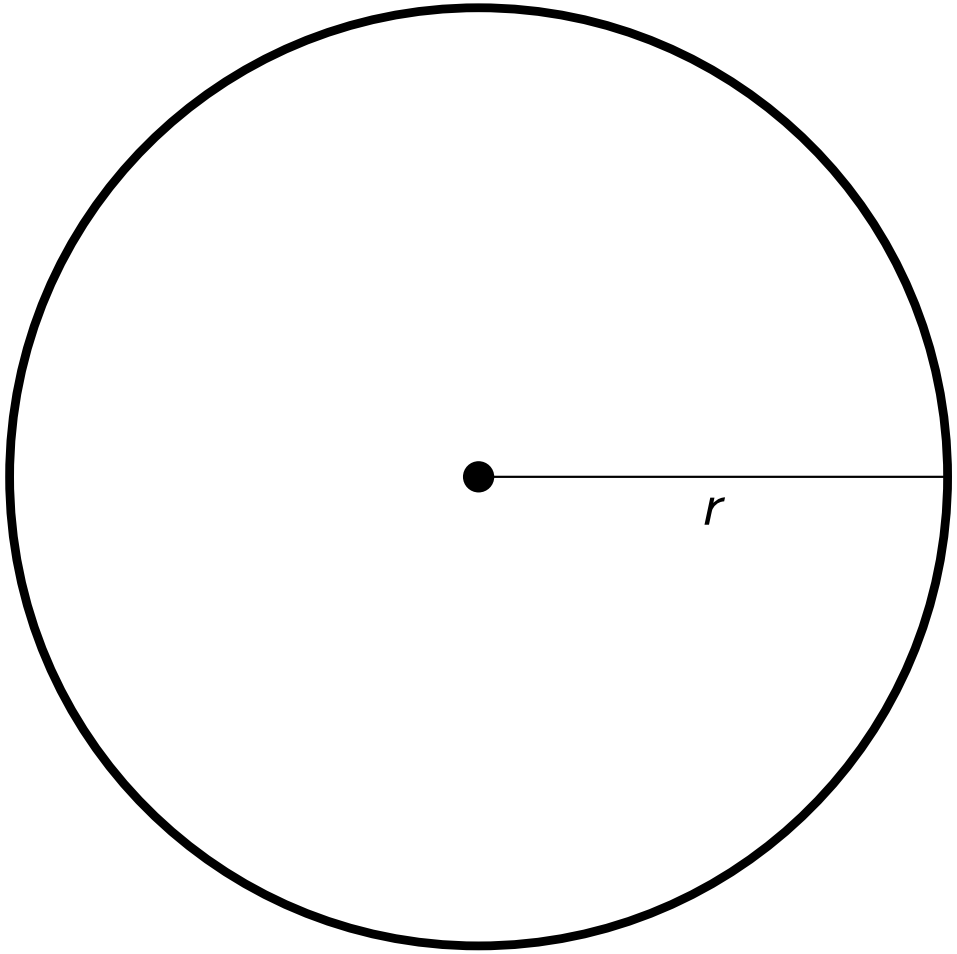
Radians



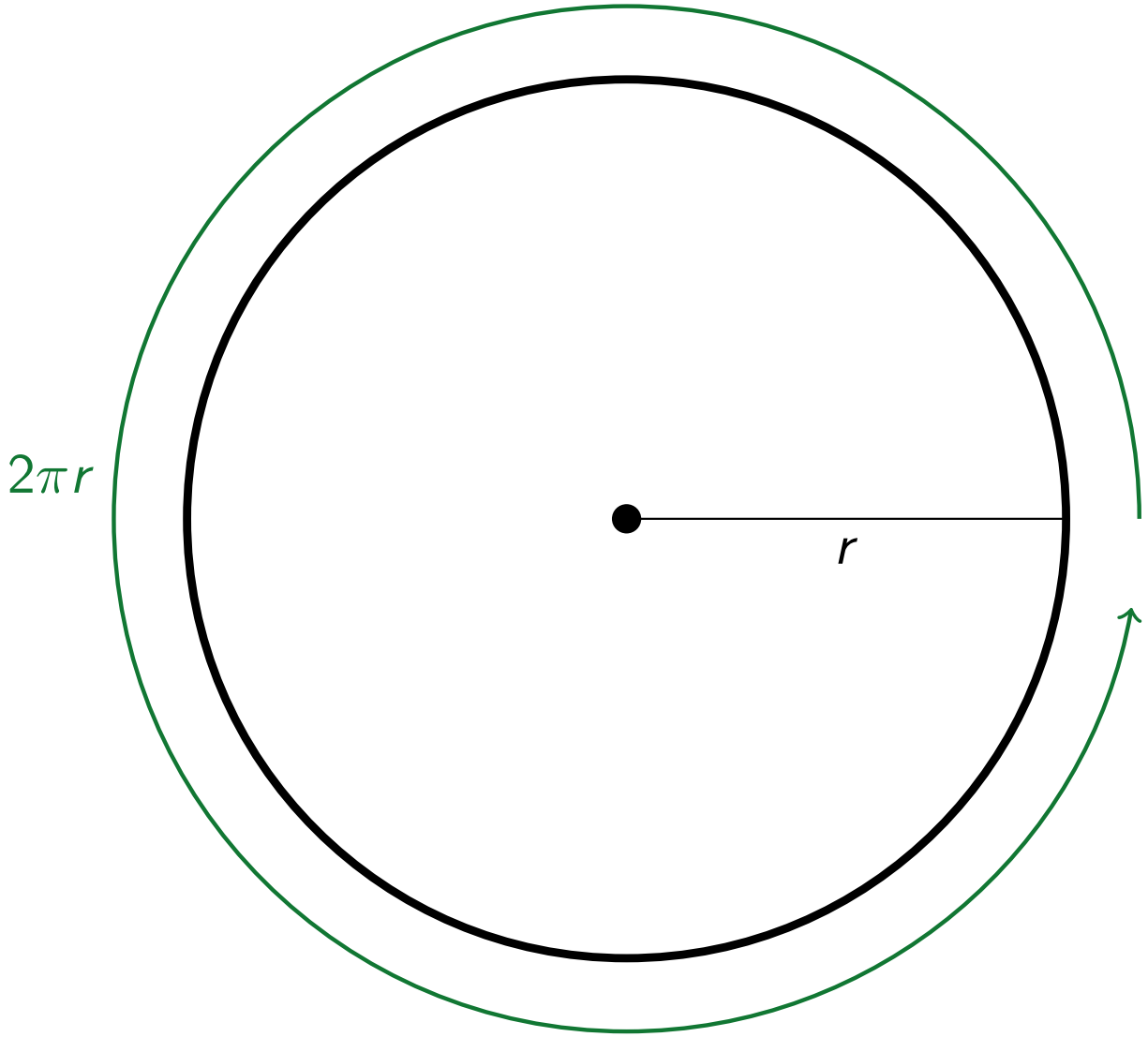
Radians



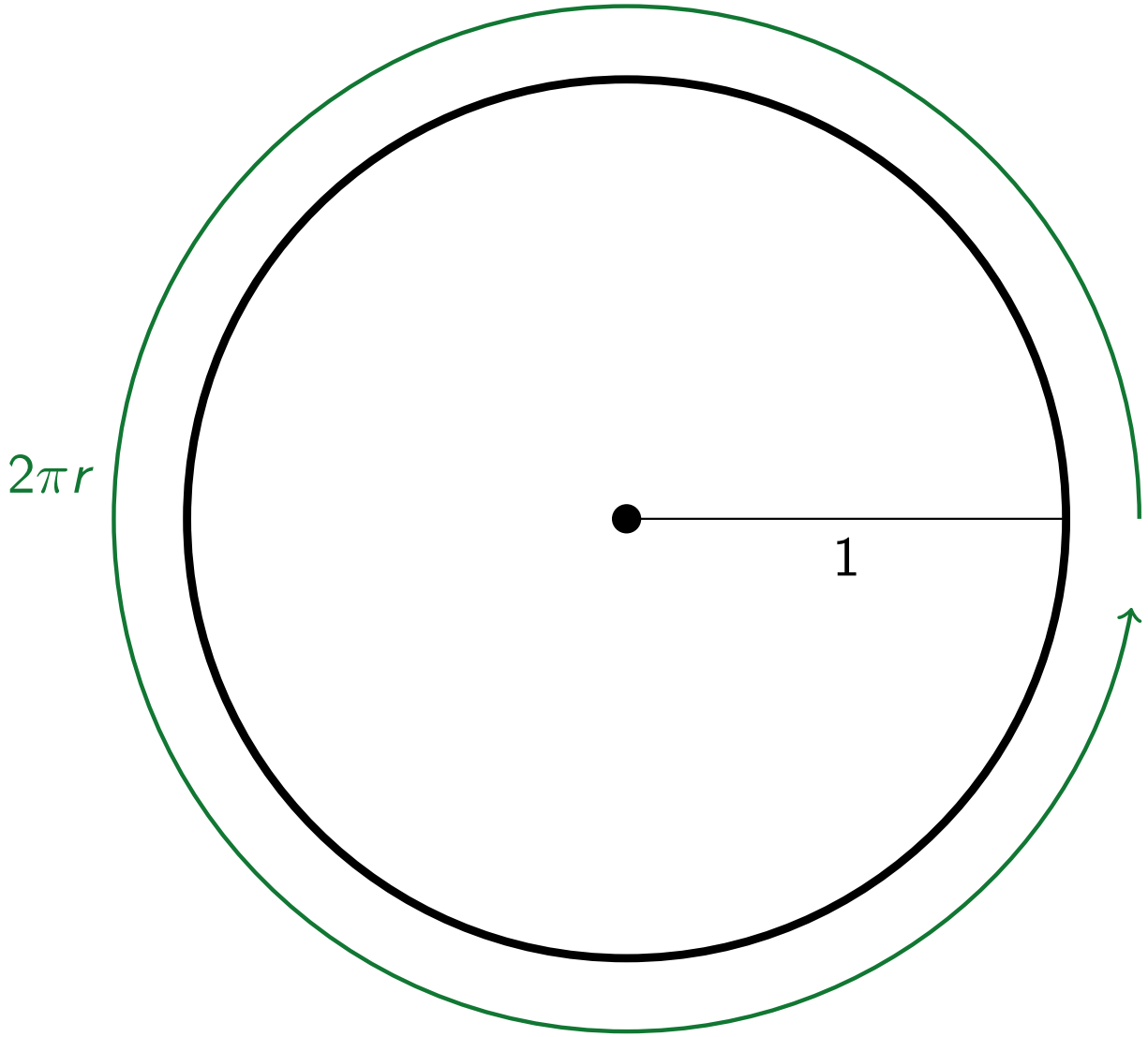
Radians



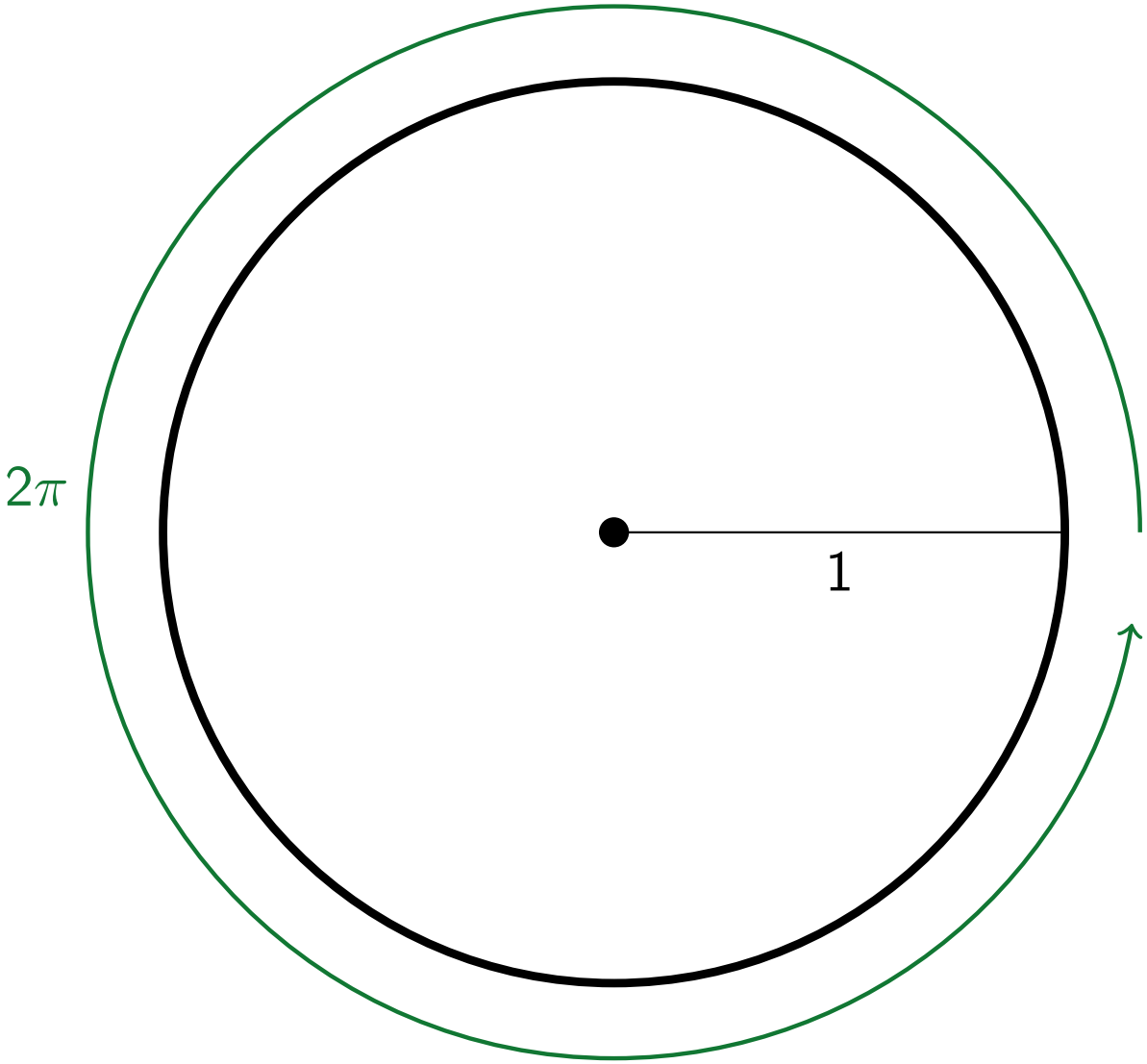
Radians



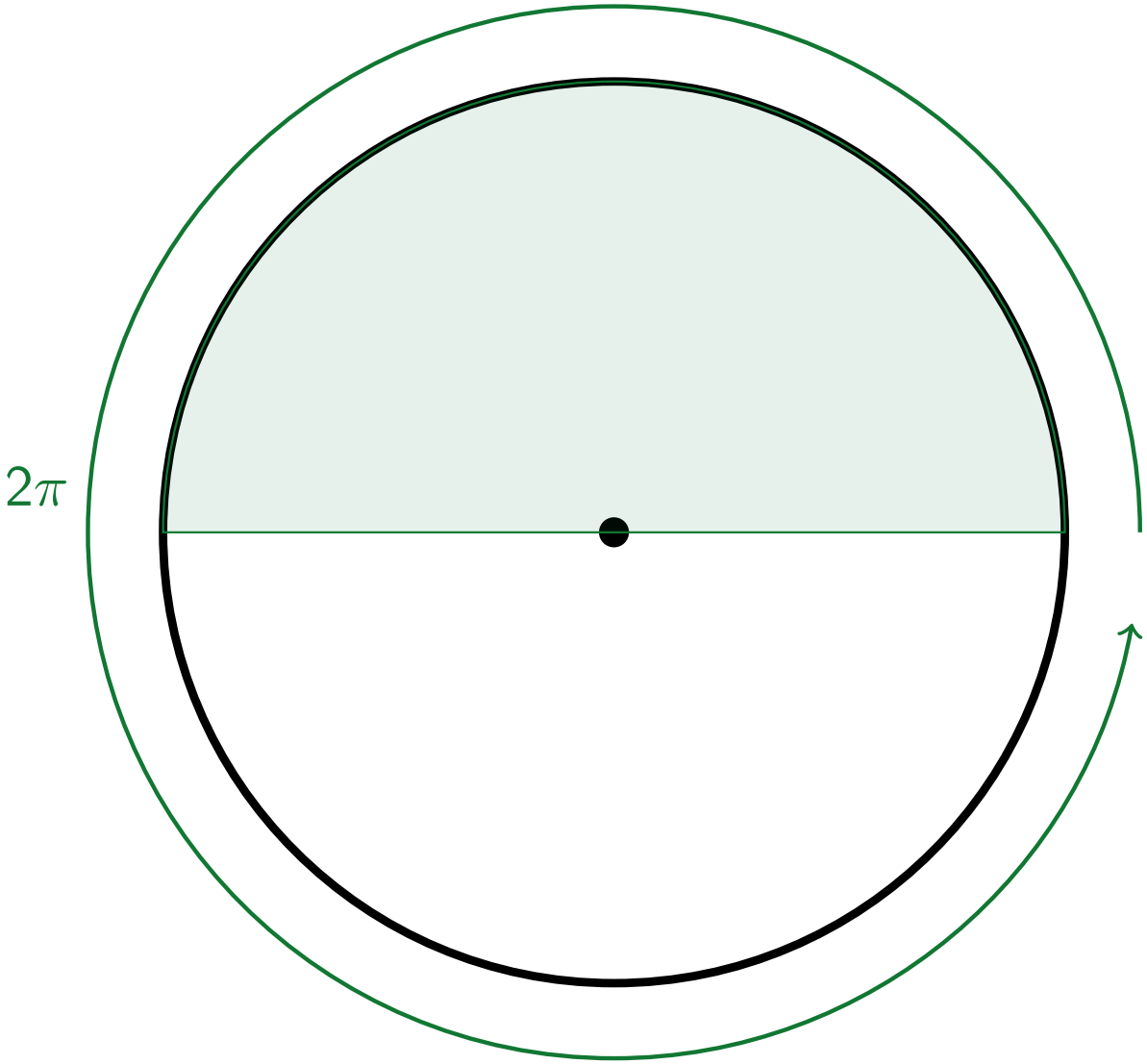
Radians



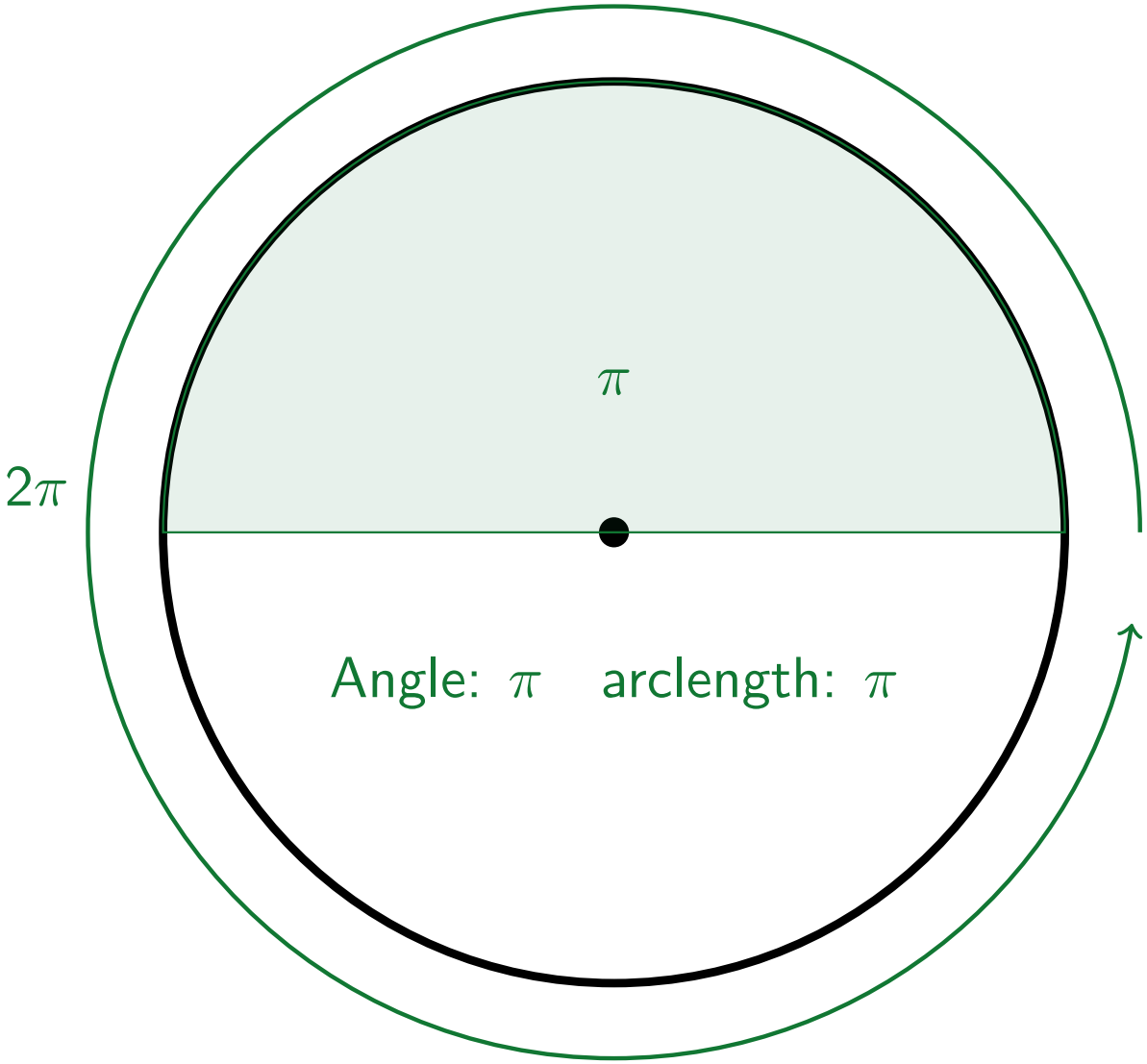
Radians



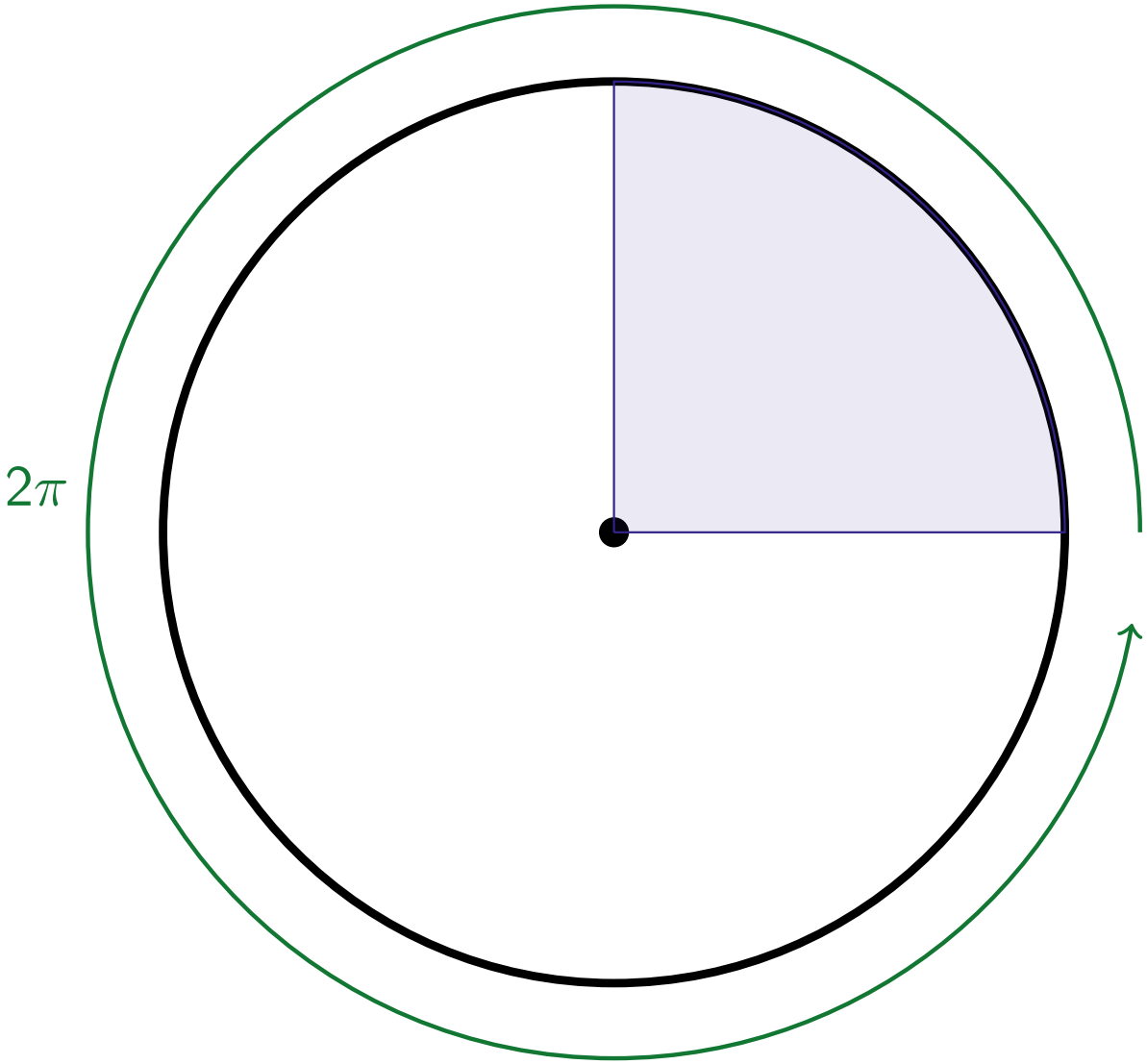
Radians



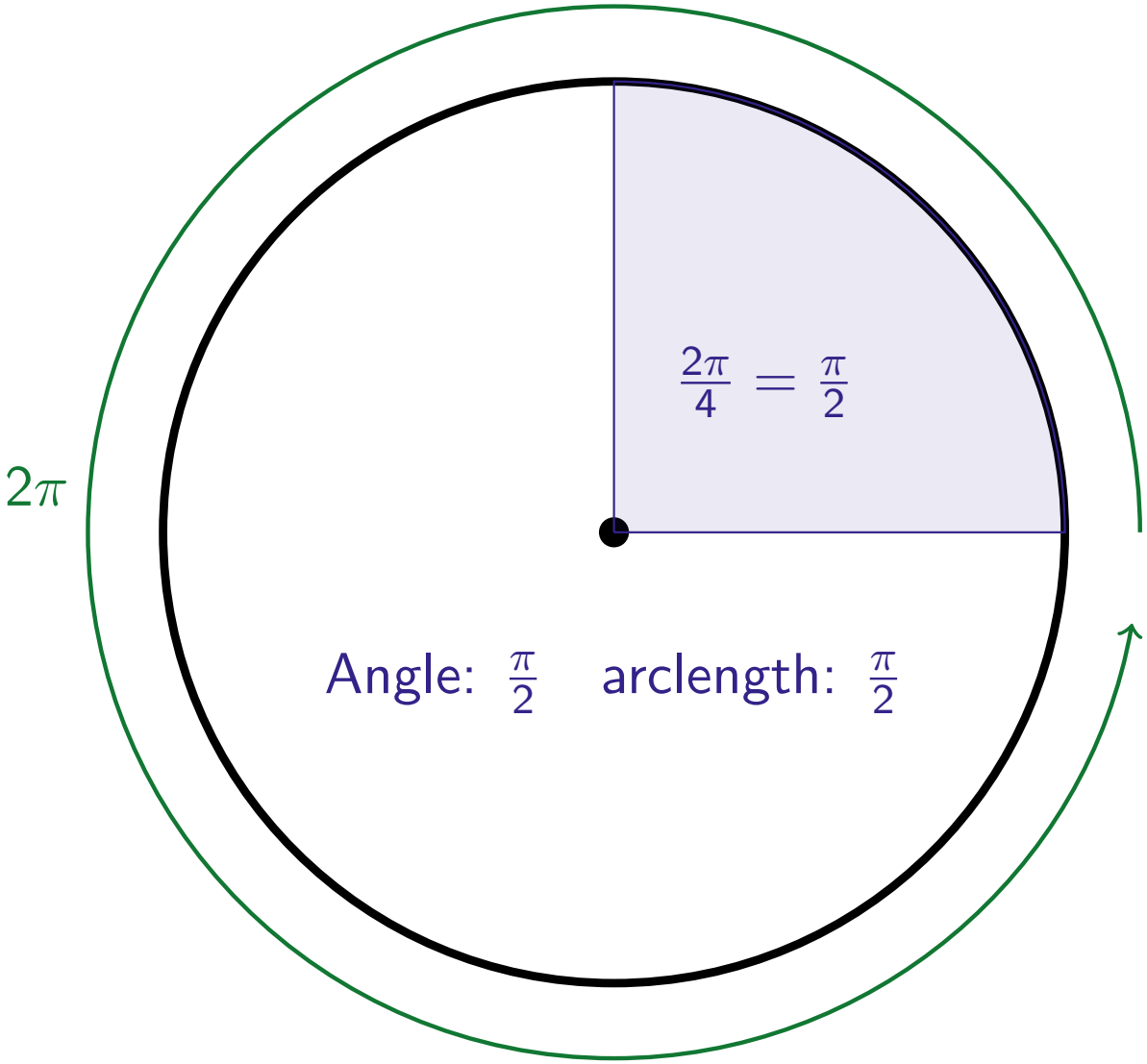
Radians



Radians

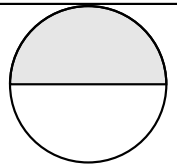
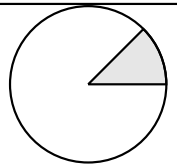
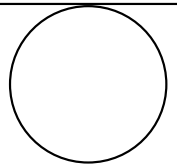
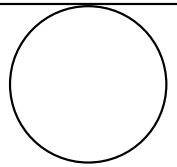


Radians



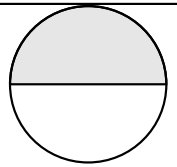
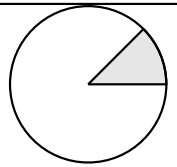
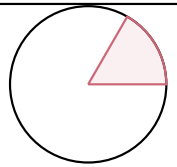
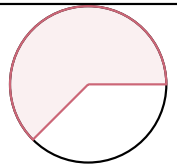
Radians and Degrees

$\dfrac{\text{radians}}{2\pi} = \dfrac{\text{degrees}}{360}$ gives the angle as a fraction of the whole circle

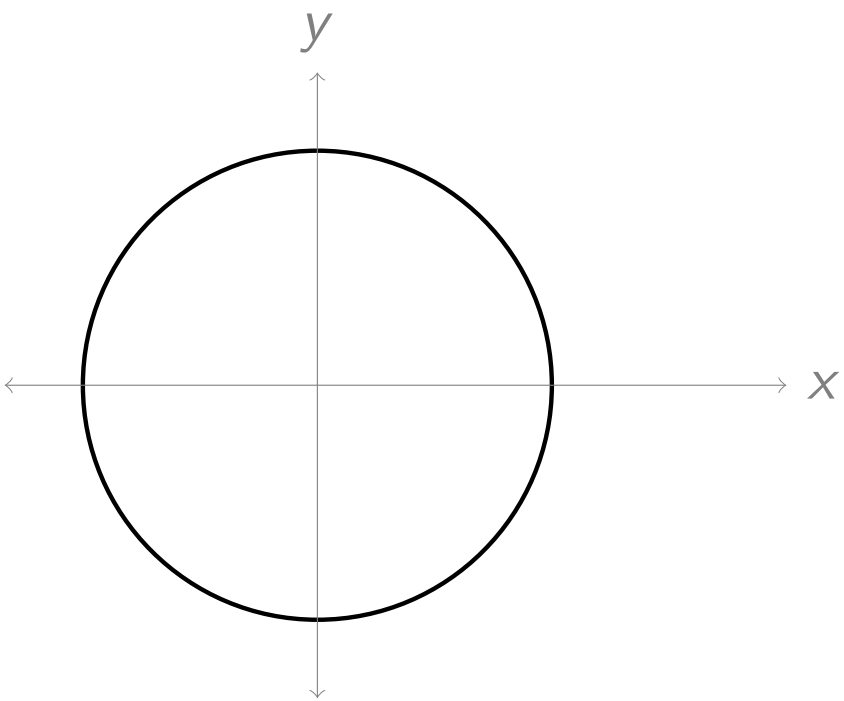
radians	**degrees**	**fraction of circle**	**sketch**
π	180	$\frac{1}{2}$	
	45	$\frac{1}{8}$	
	60		
$\frac{5\pi}{4}$			

Radians and Degrees

$\dfrac{\text{radians}}{2\pi} = \dfrac{\text{degrees}}{360}$ gives the angle as a fraction of the whole circle

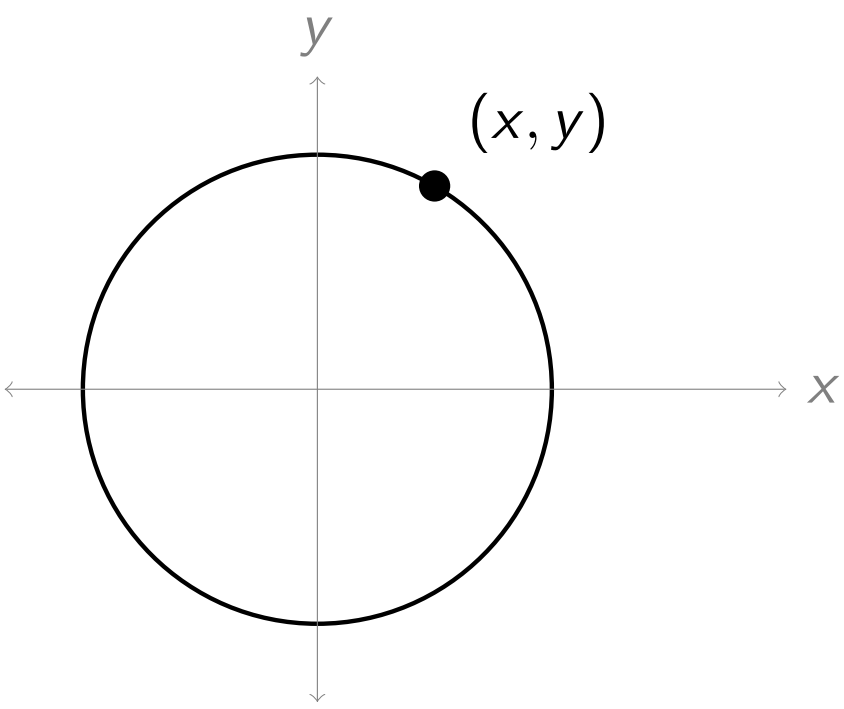
radians	**degrees**	**fraction of circle**	**sketch**
π	180	$\frac{1}{2}$	
$\frac{\pi}{4}$	45	$\frac{1}{8}$	
$\frac{\pi}{3}$	60	$\frac{1}{6}$	
$\frac{5\pi}{4}$	225	$\frac{5}{8}$	

Unit circle, sines and cosines



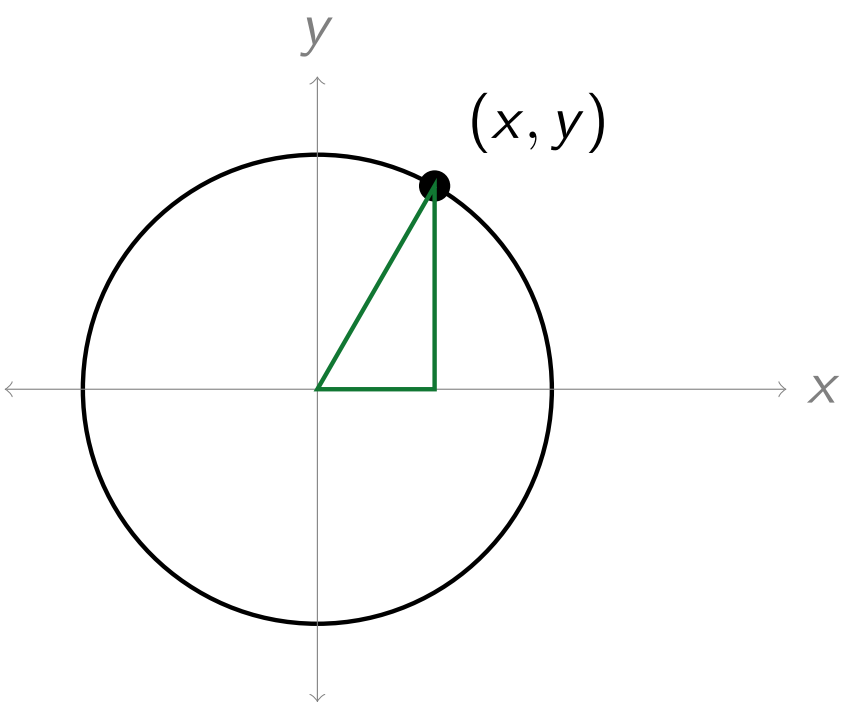
unit circle, $r = 1$

Unit circle, sines and cosines



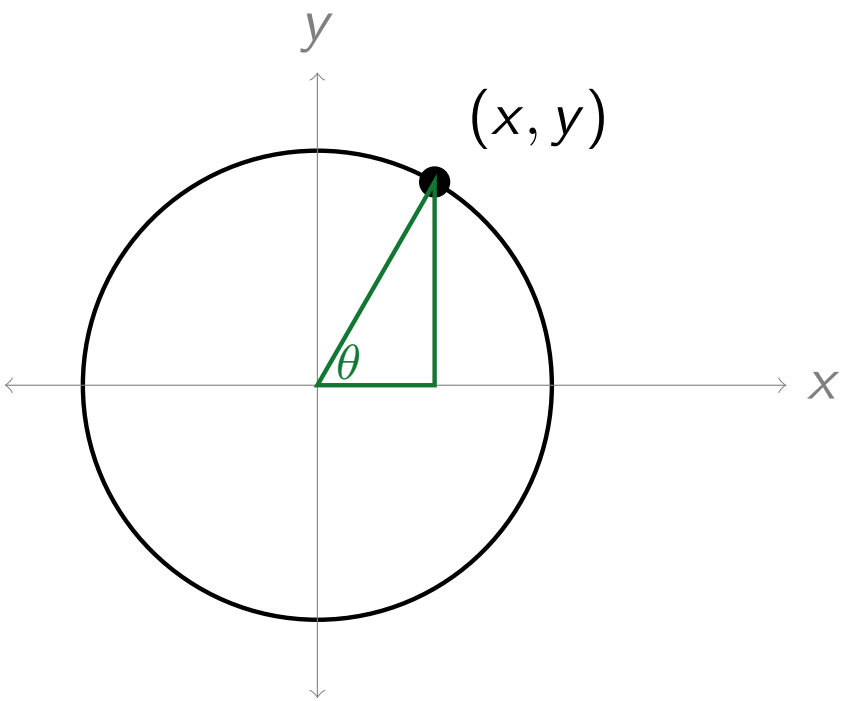
unit circle, $r = 1$

Unit circle, sines and cosines



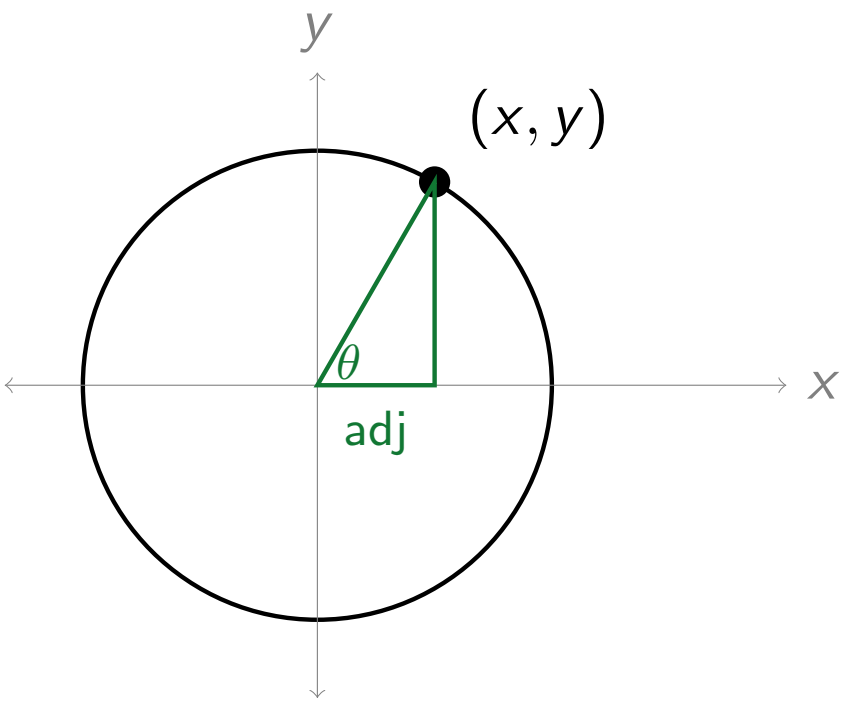
unit circle, $r = 1$

Unit circle, sines and cosines



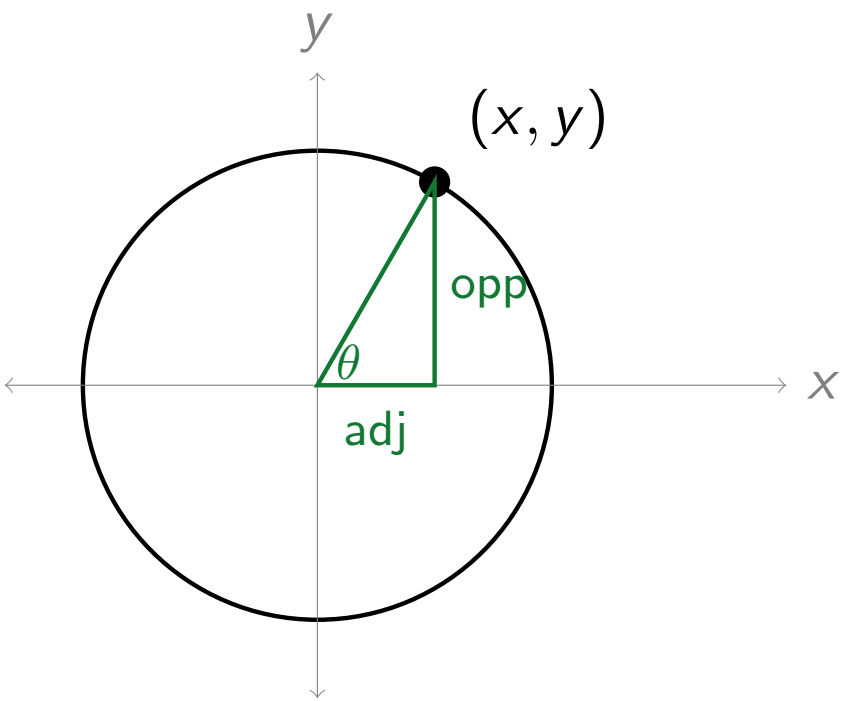
unit circle, $r = 1$

Unit circle, sines and cosines



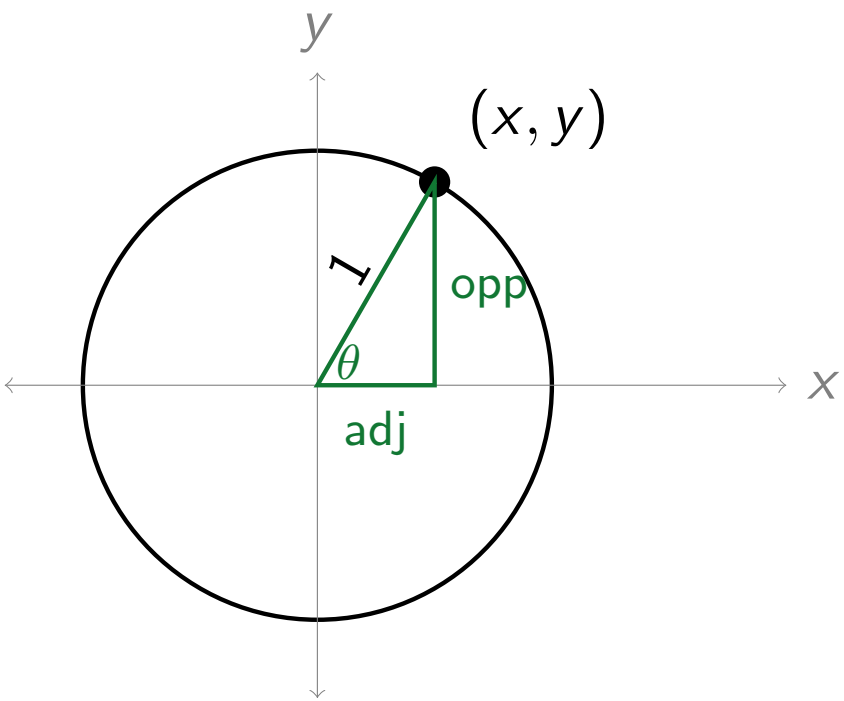
unit circle, $r = 1$

Unit circle, sines and cosines



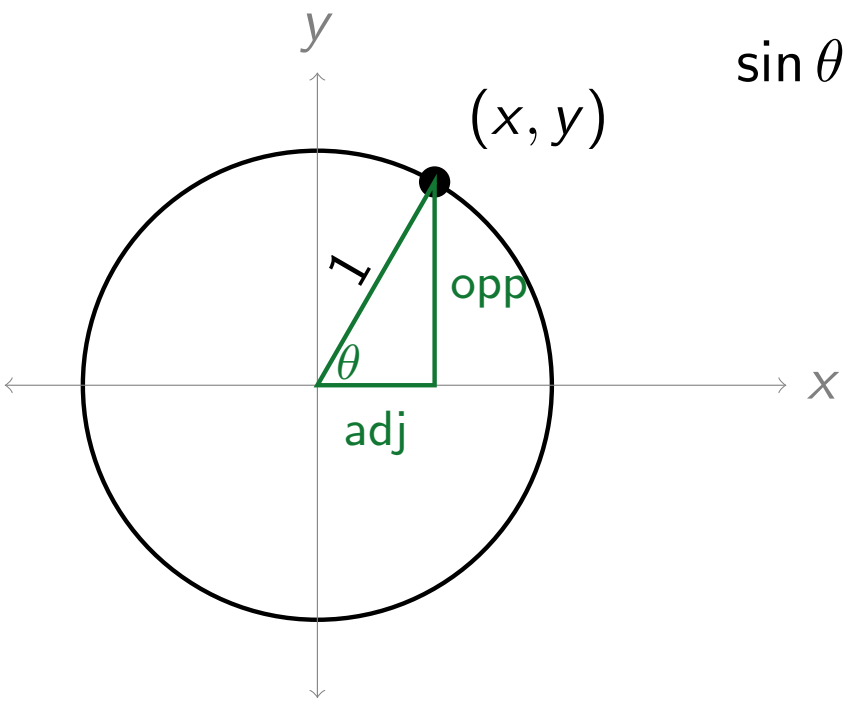
unit circle, $r = 1$

Unit circle, sines and cosines



unit circle, $r = 1$

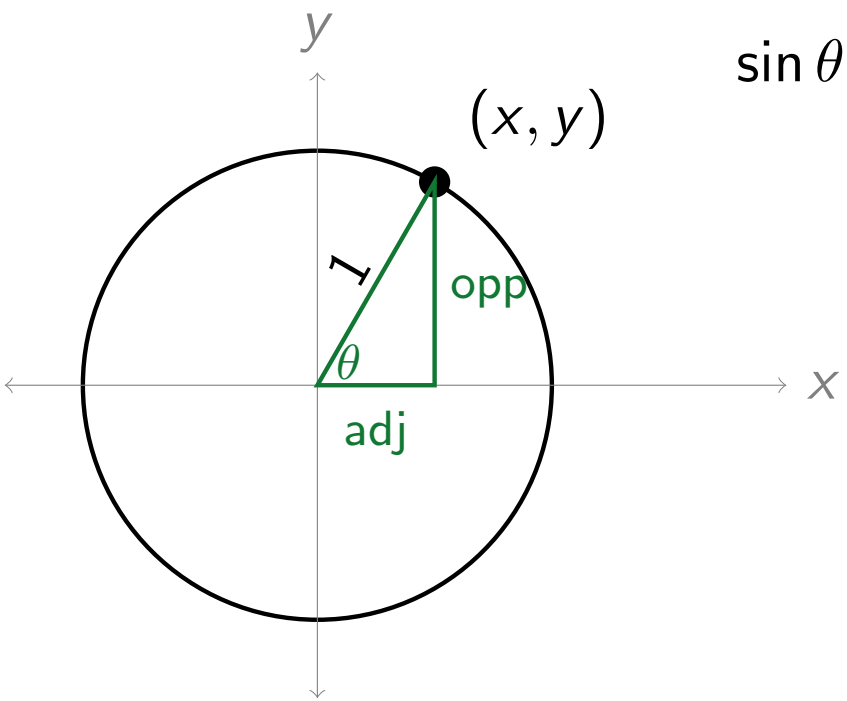
Unit circle, sines and cosines



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} =$$

unit circle, $r = 1$

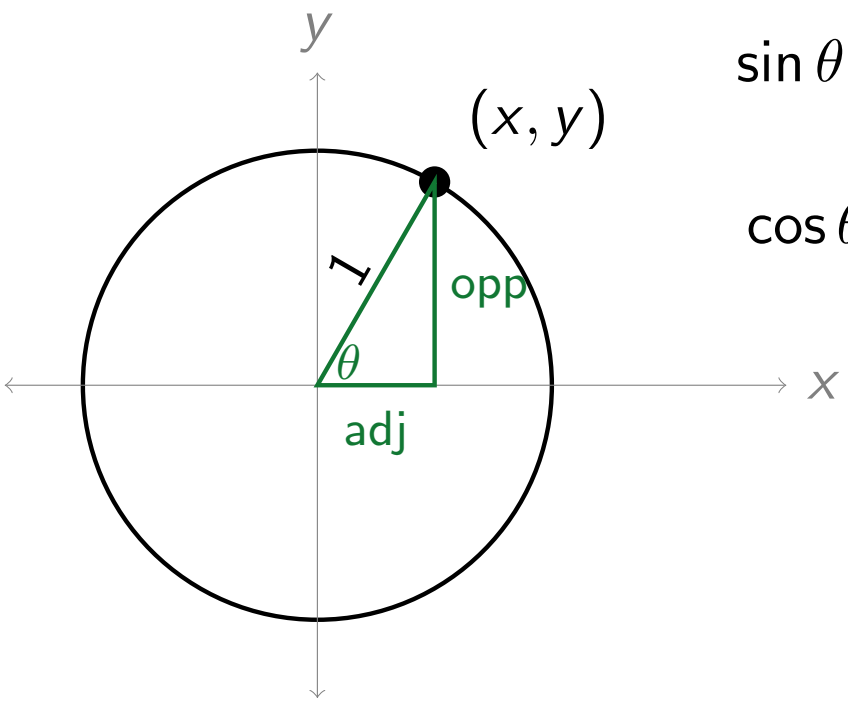
Unit circle, sines and cosines



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

unit circle, $r = 1$

Unit circle, sines and cosines

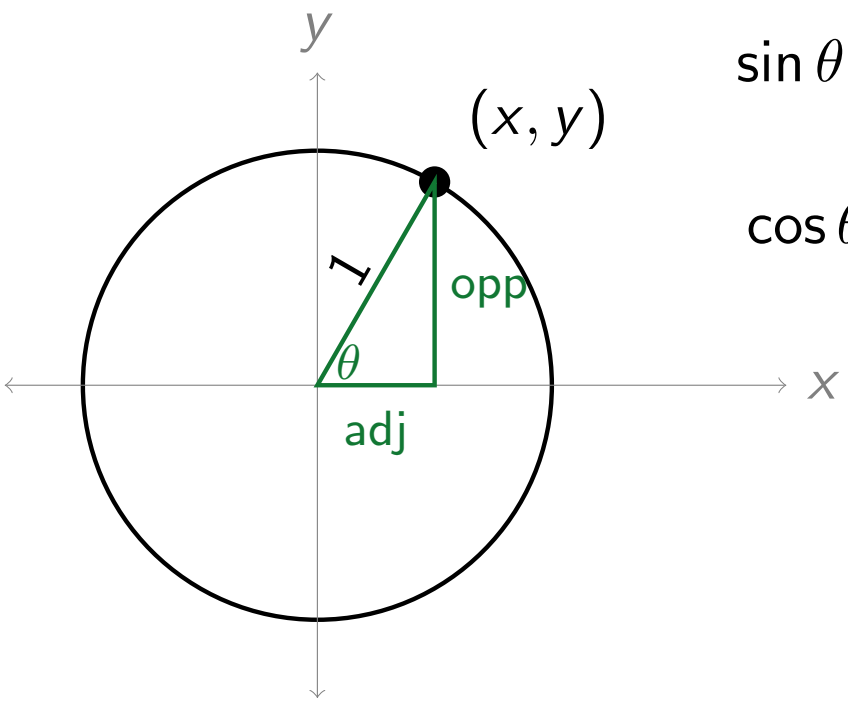


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} =$$

unit circle, $r = 1$

Unit circle, sines and cosines

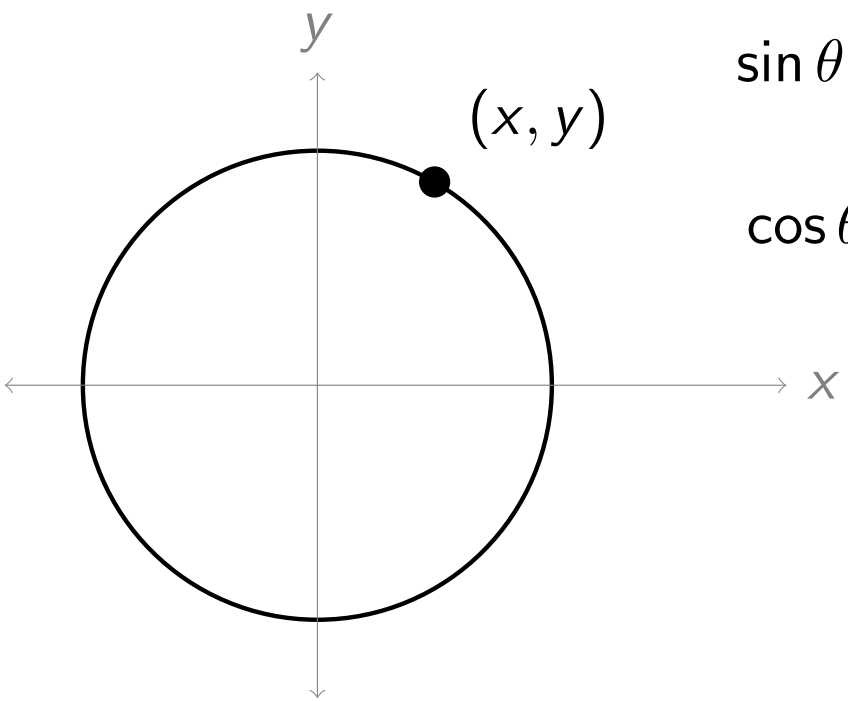


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

unit circle, $r = 1$

Unit circle, sines and cosines

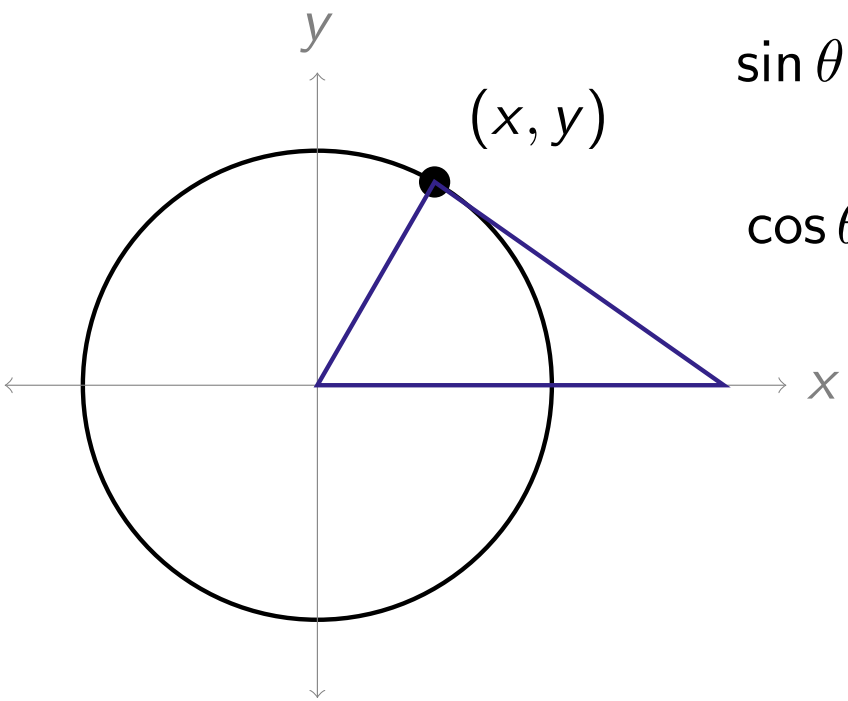


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

unit circle, $r = 1$

Unit circle, sines and cosines

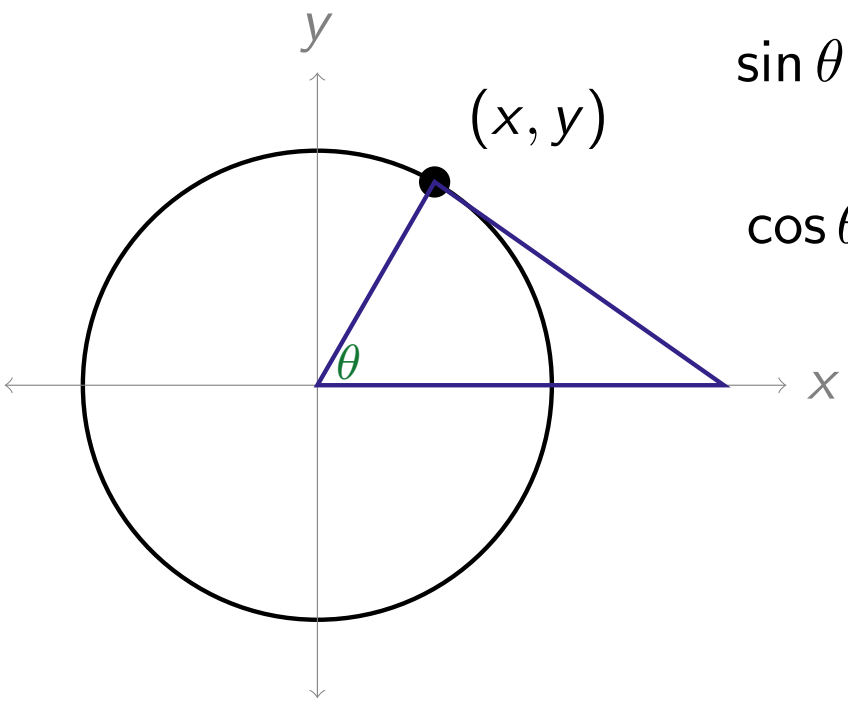


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

unit circle, $r = 1$

Unit circle, sines and cosines

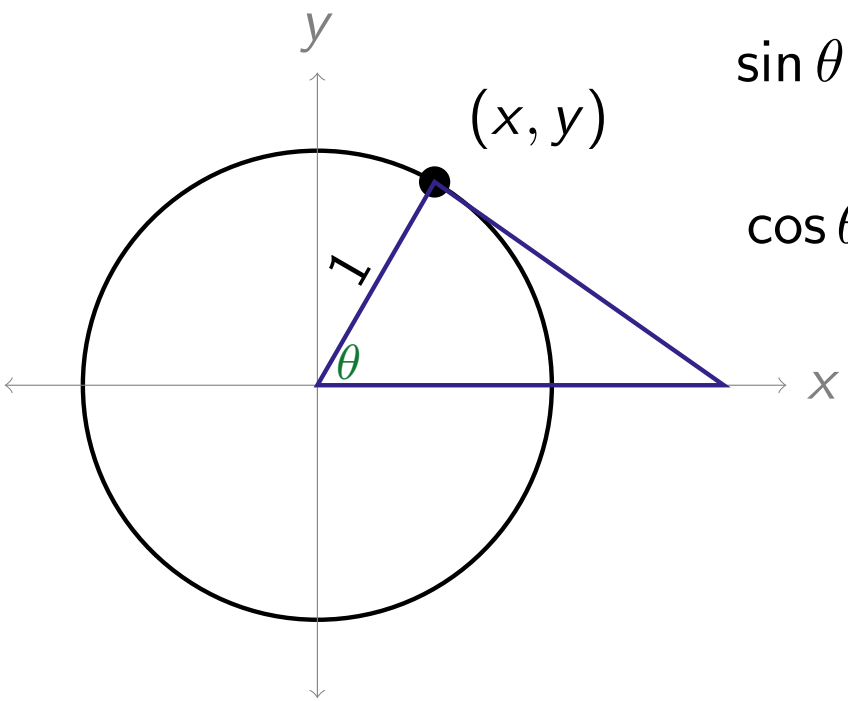


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

unit circle, $r = 1$

Unit circle, sines and cosines

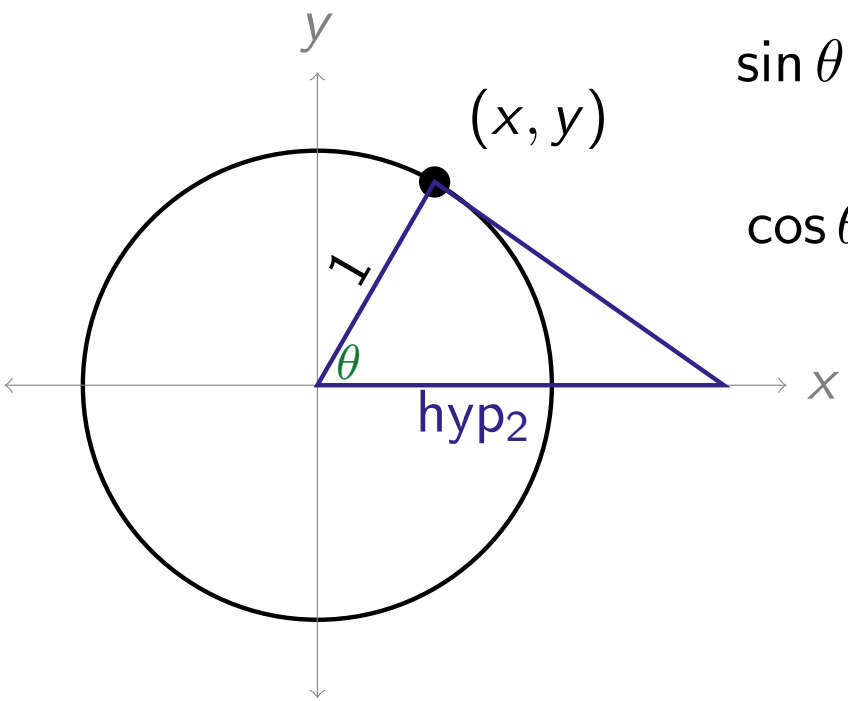


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

unit circle, $r = 1$

Unit circle, sines and cosines

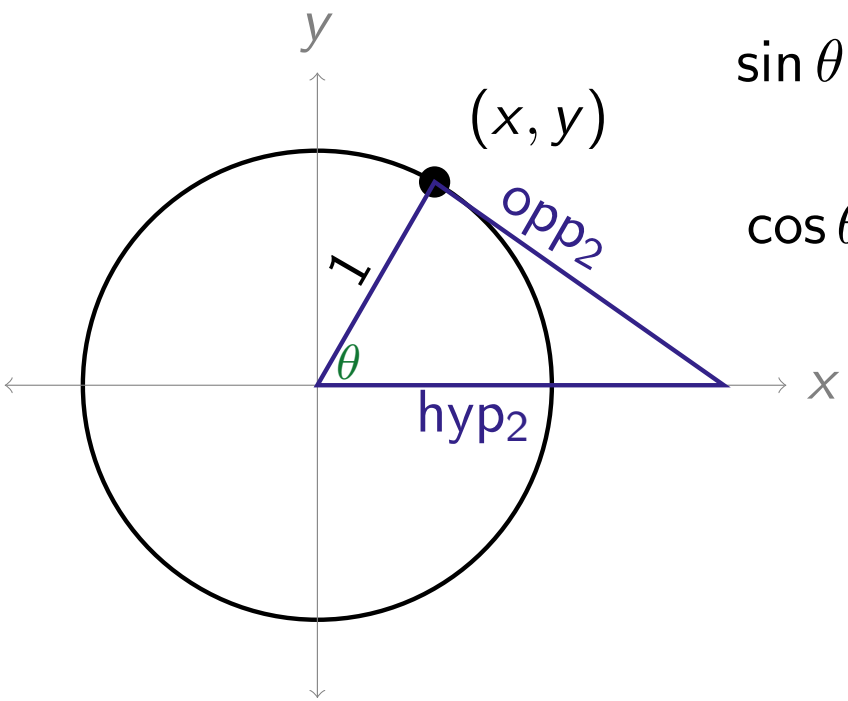


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

unit circle, $r = 1$

Unit circle, sines and cosines



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

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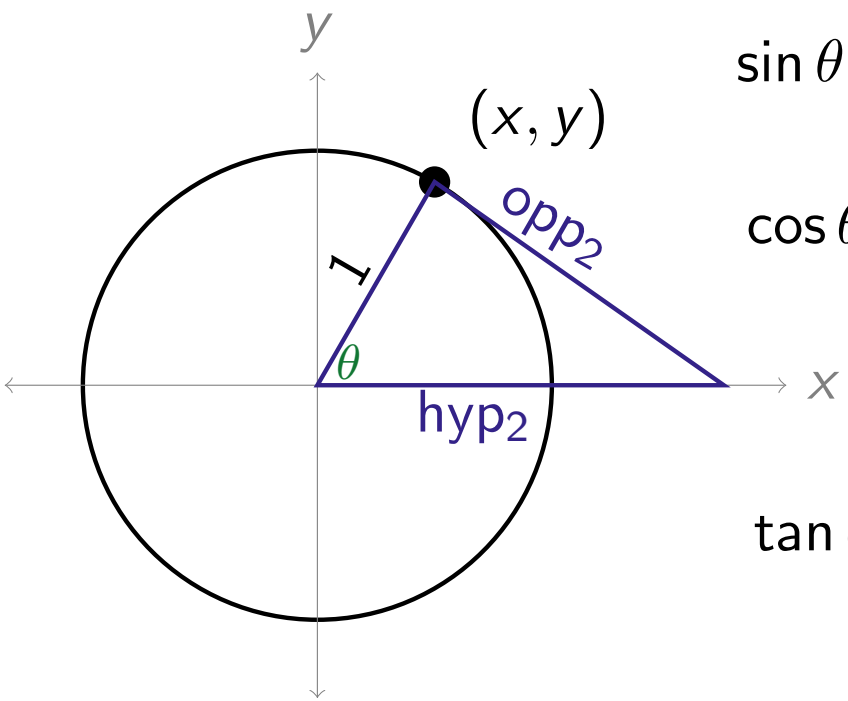
unit circle, $r = 1$

Unit circle, sines and cosines



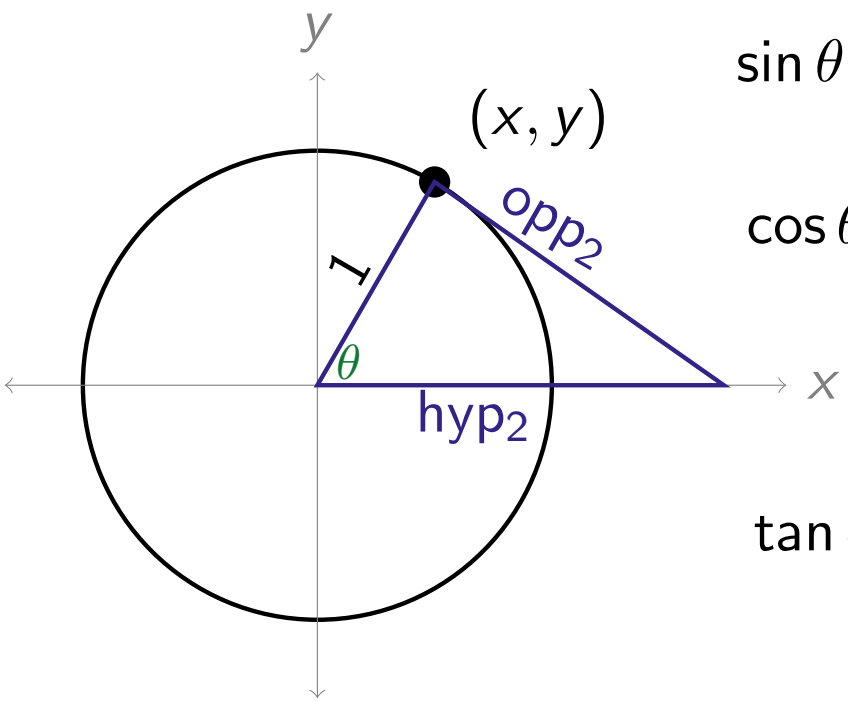
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

$$\tan\theta = \frac{\text{opp}_2}{\text{adj}_2} =$$

unit circle, $r = 1$

Unit circle, sines and cosines



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

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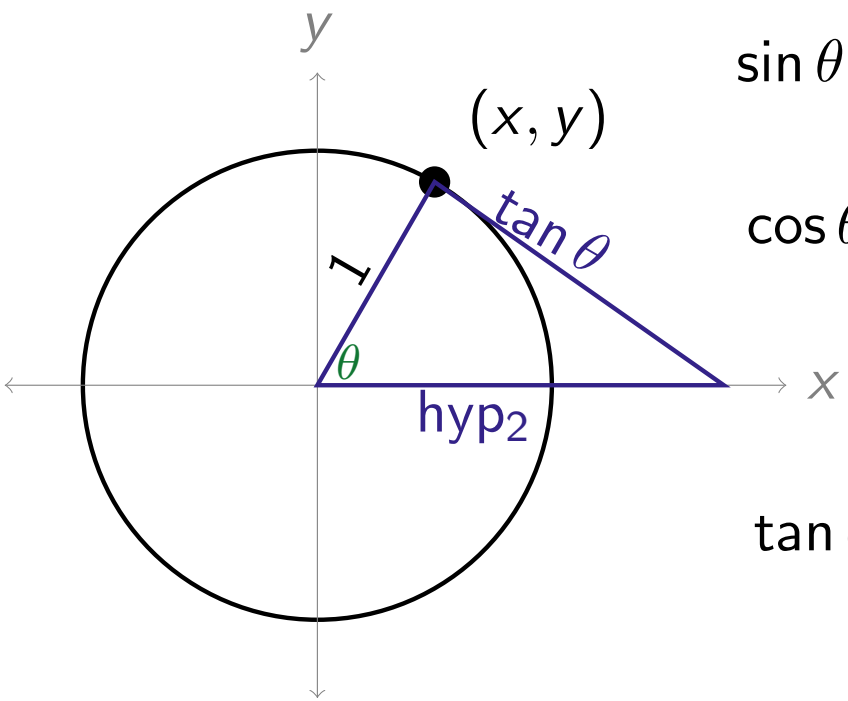
$$\tan\theta = \frac{\text{opp}_2}{\text{adj}_2} = \frac{\text{opp}_2}{1} = \text{opp}_2$$

unit circle, $r = 1$

Unit circle, sines and cosines



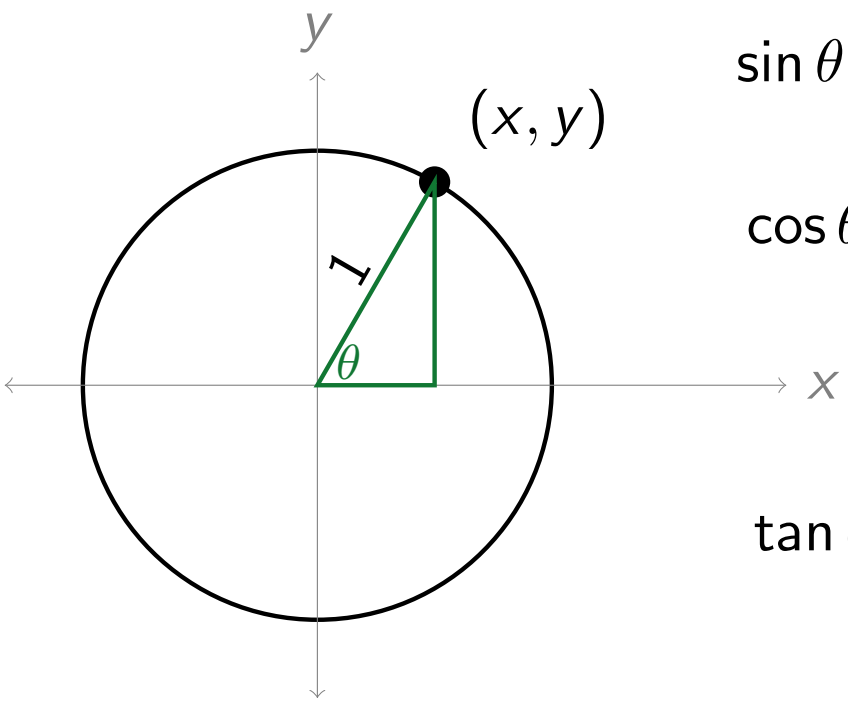
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

$$\tan\theta = \frac{\text{opp}_2}{\text{adj}_2} = \frac{\text{opp}_2}{1} = \text{opp}_2$$

unit circle, $r = 1$

Unit circle, sines and cosines



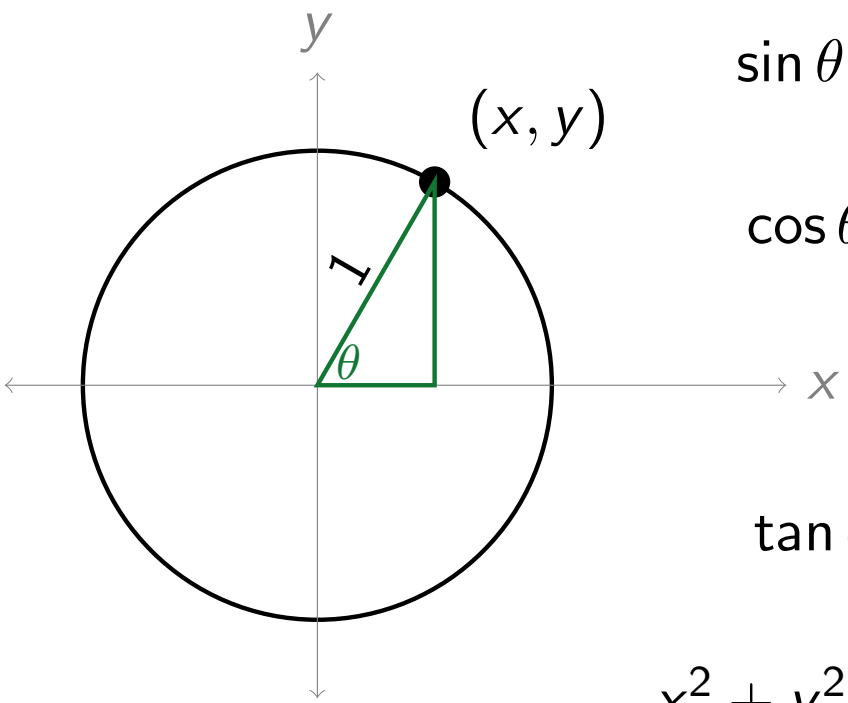
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

$$\tan\theta = \frac{\text{opp}_2}{\text{adj}_2} = \frac{\text{opp}_2}{1} = \text{opp}_2$$

unit circle, $r = 1$

Unit circle, sines and cosines



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

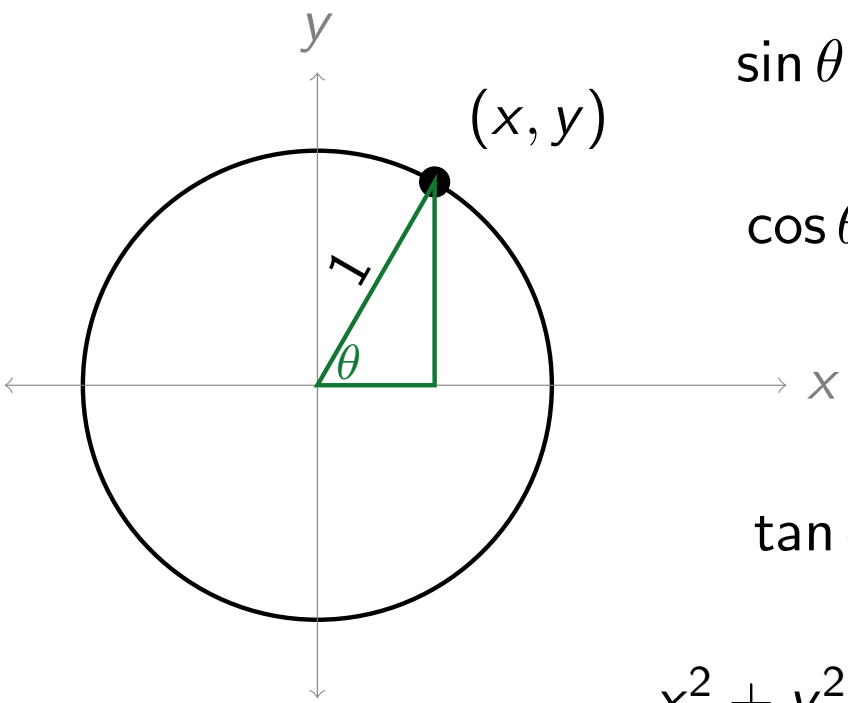
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

$$\tan\theta = \frac{\text{opp}_2}{\text{adj}_2} = \frac{\text{opp}_2}{1} = \text{opp}_2$$

$x^2 + y^2 = 1$, so

unit circle, $r = 1$

Unit circle, sines and cosines



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$$

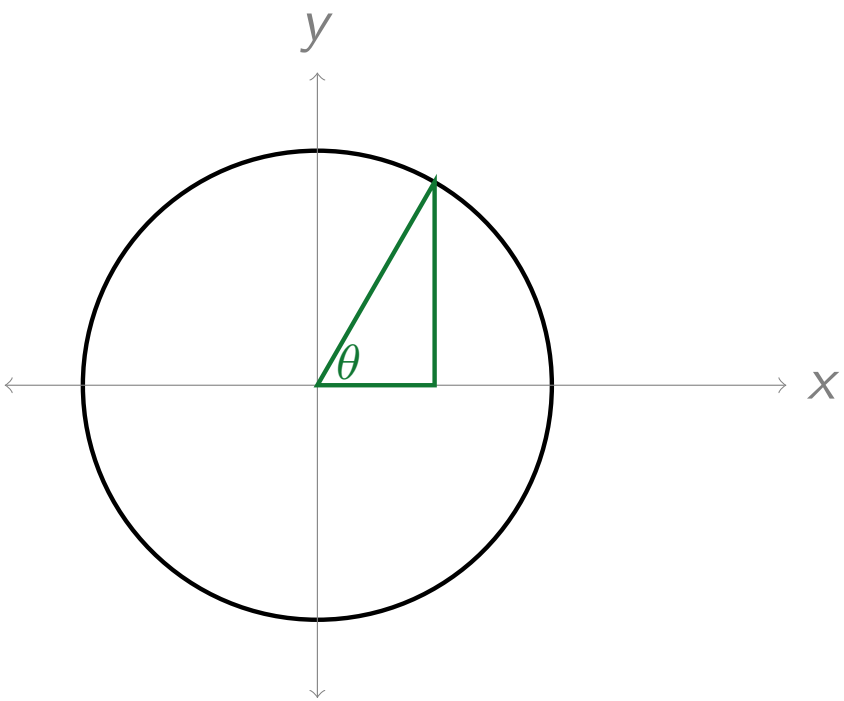
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$$

$$\tan\theta = \frac{\text{opp}_2}{\text{adj}_2} = \frac{\text{opp}_2}{1} = \text{opp}_2$$

$$x^2 + y^2 = 1, \text{ so } \sin^2\theta + \cos^2\theta = 1$$

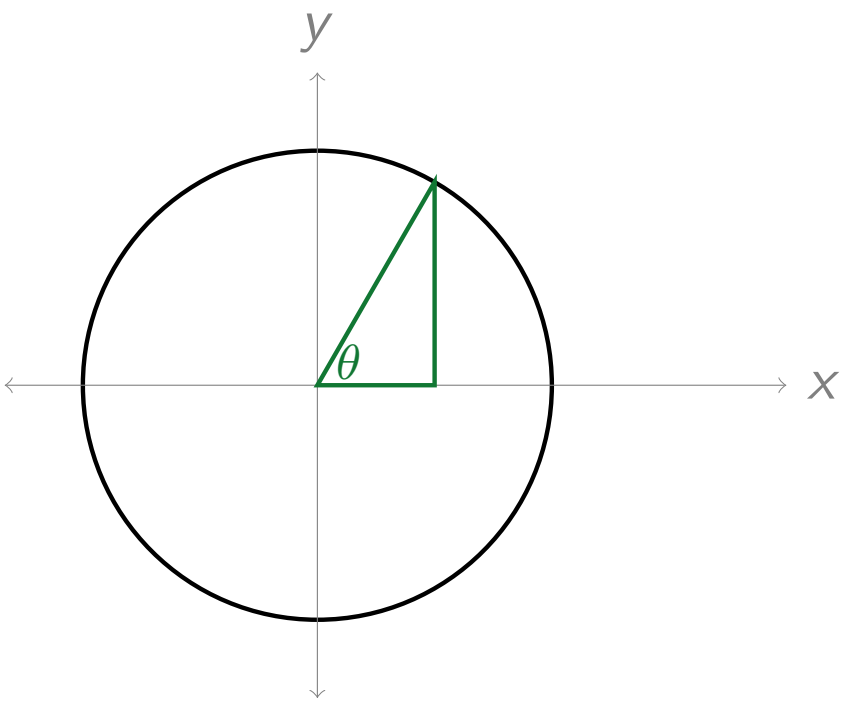
unit circle, $r = 1$

Unit circle, sines and cosines



unit circle, $r = 1$

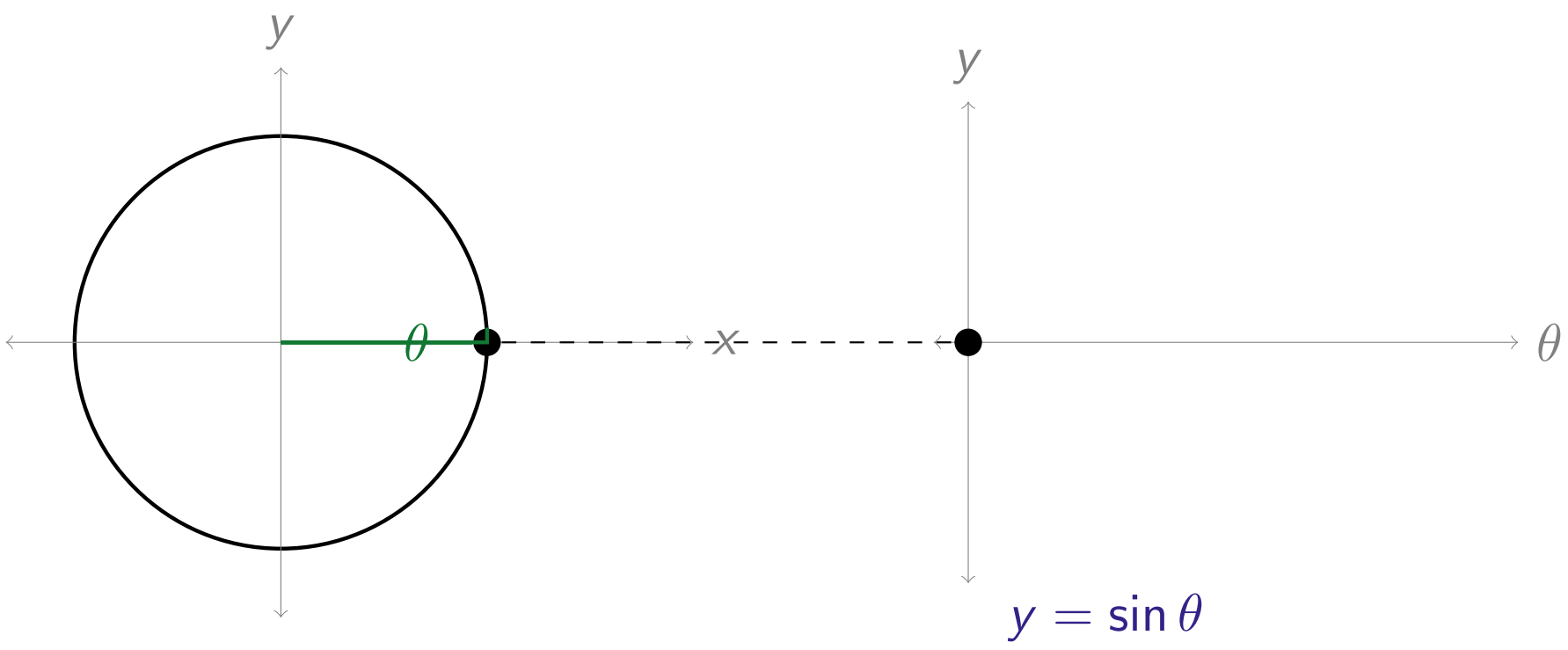
Unit circle, sines and cosines



unit circle, $r = 1$

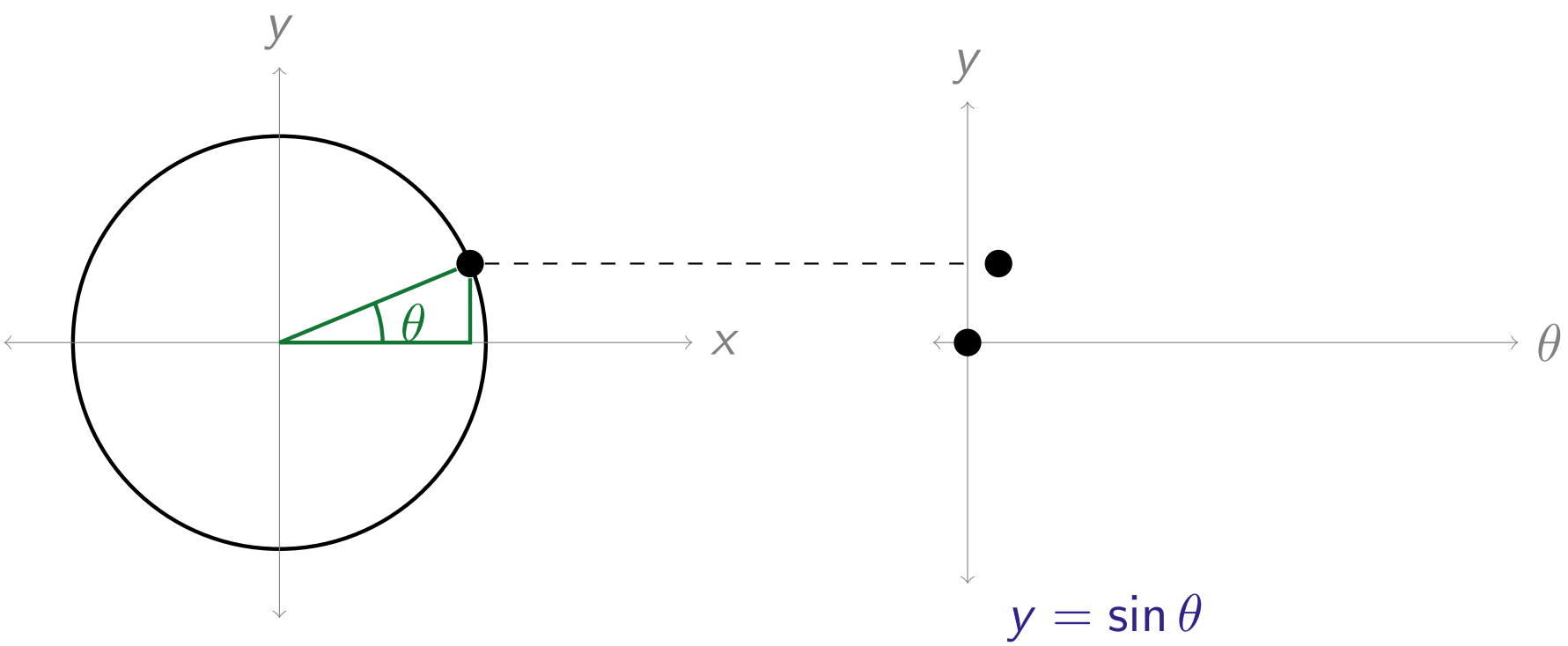
$y = \sin\theta$

Unit circle, sines and cosines



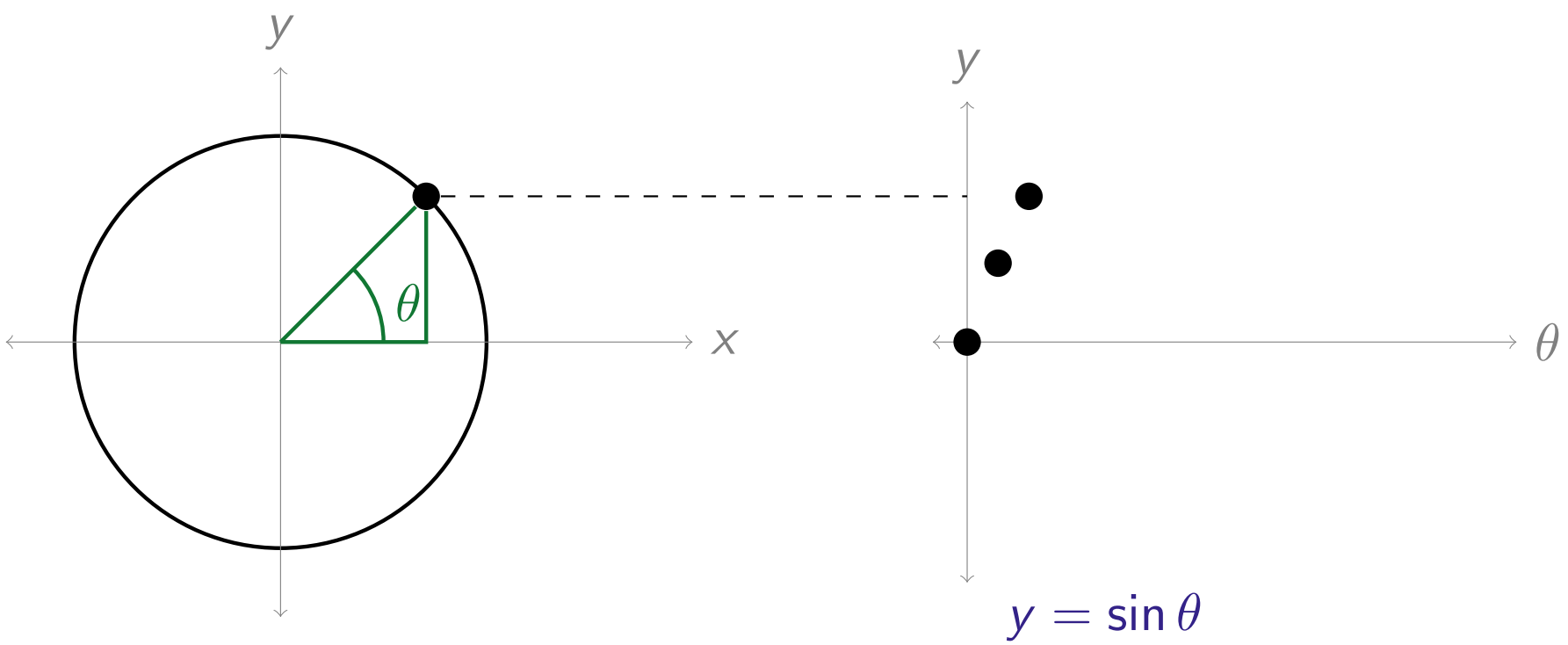
unit circle, $r = 1$

Unit circle, sines and cosines



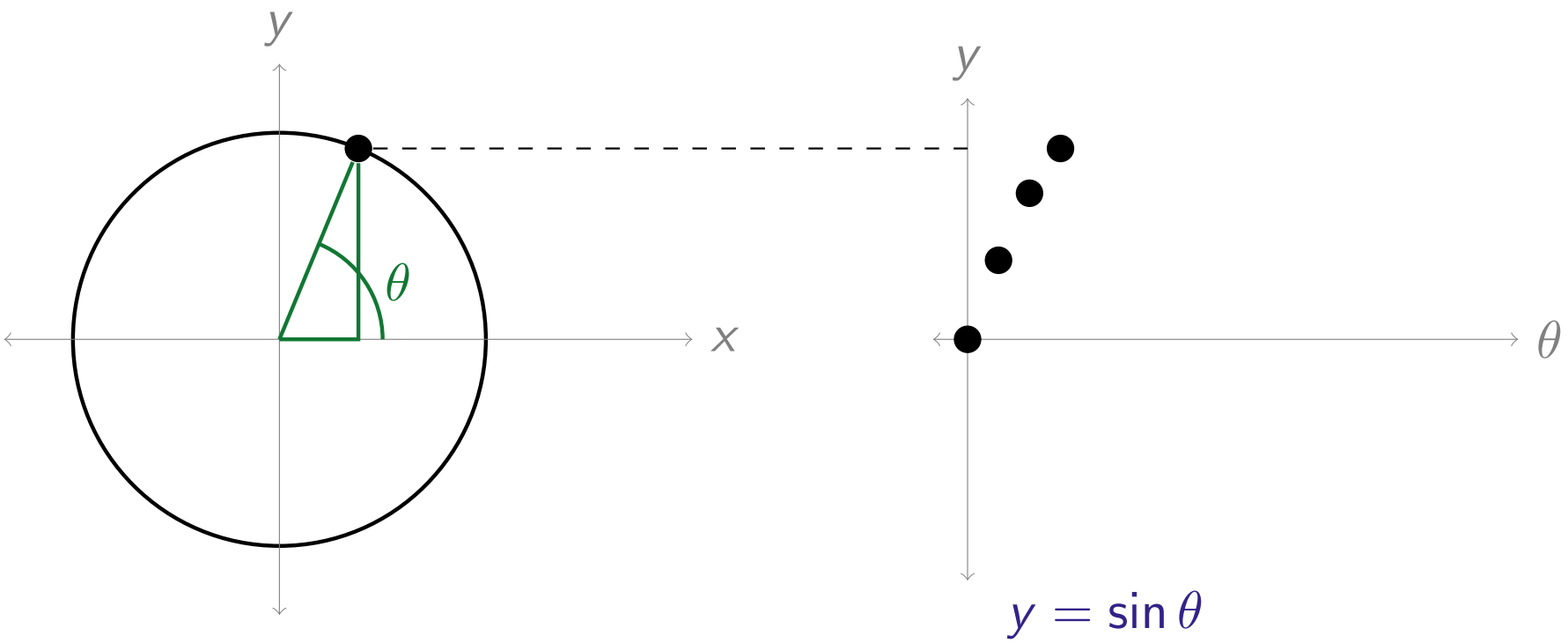
unit circle, $r = 1$

Unit circle, sines and cosines



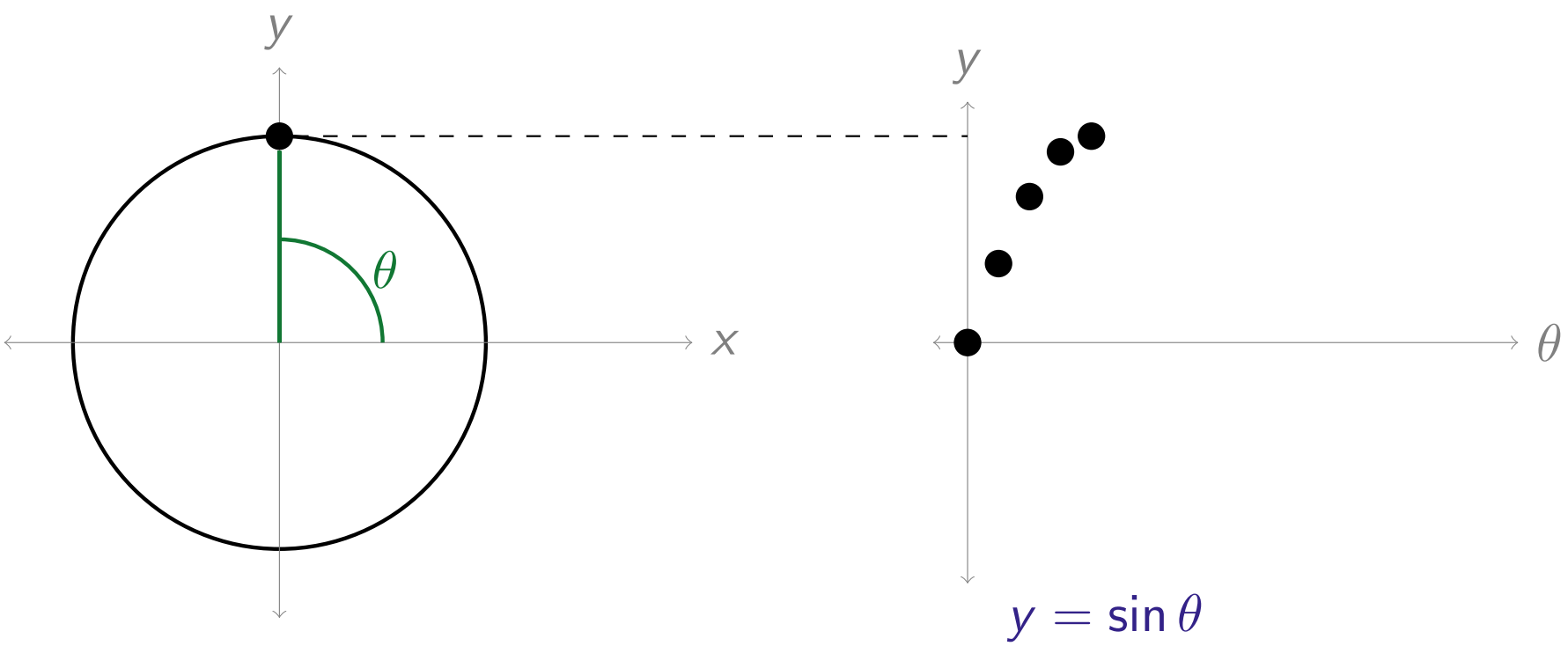
unit circle, $r = 1$

Unit circle, sines and cosines



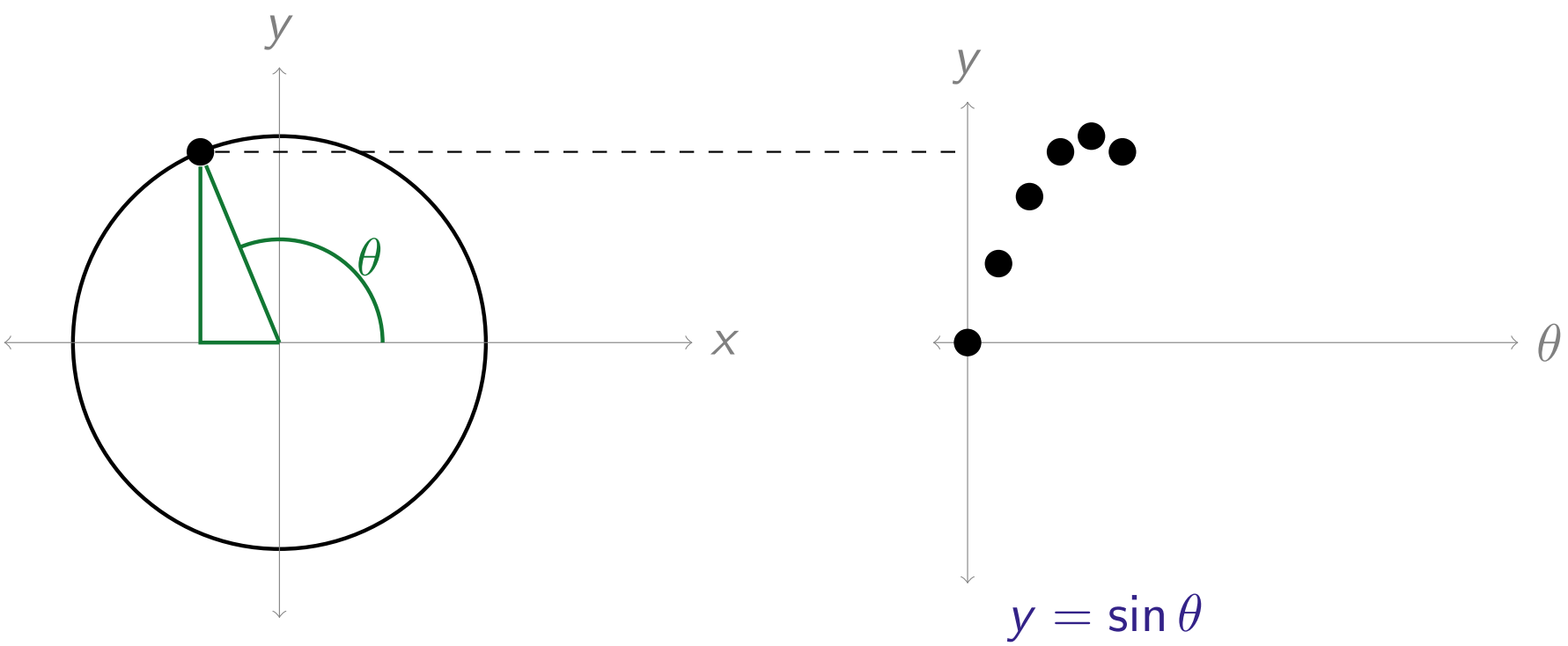
unit circle, $r = 1$

Unit circle, sines and cosines



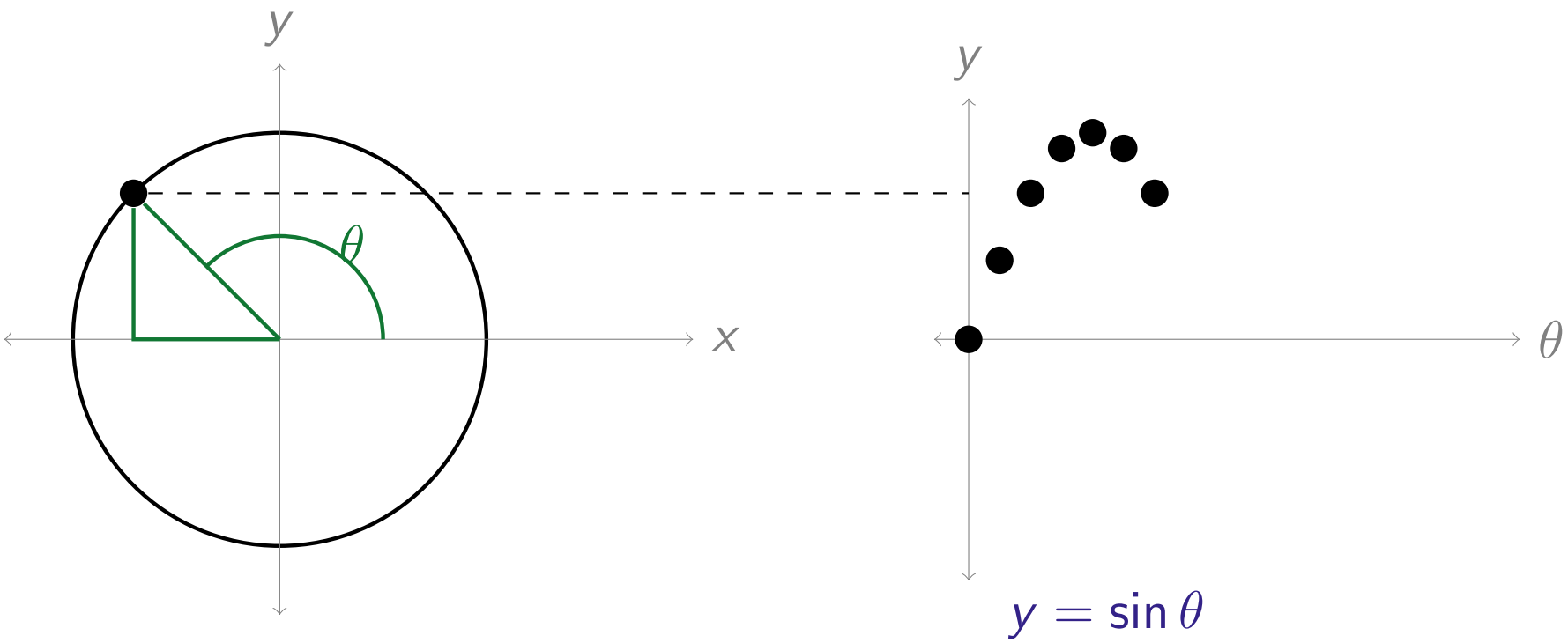
unit circle, $r = 1$

Unit circle, sines and cosines



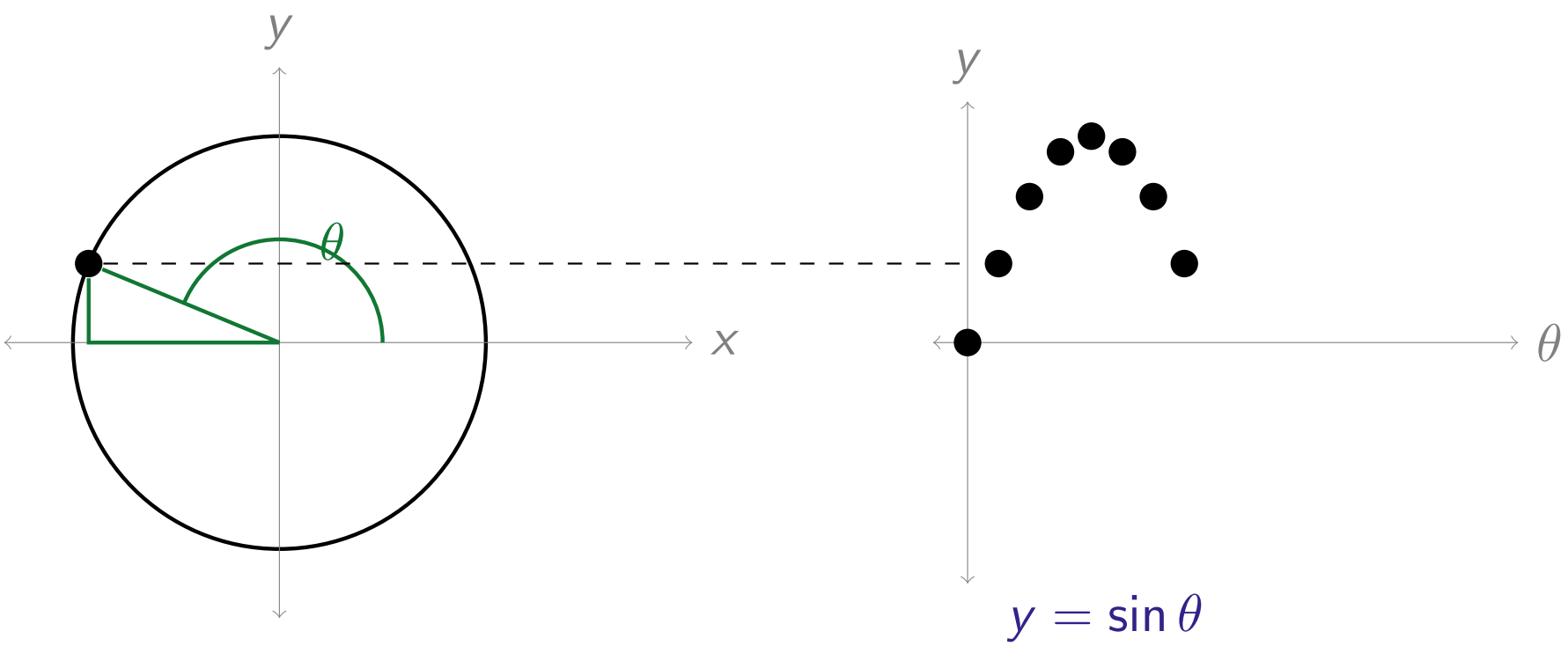
unit circle, $r = 1$

Unit circle, sines and cosines



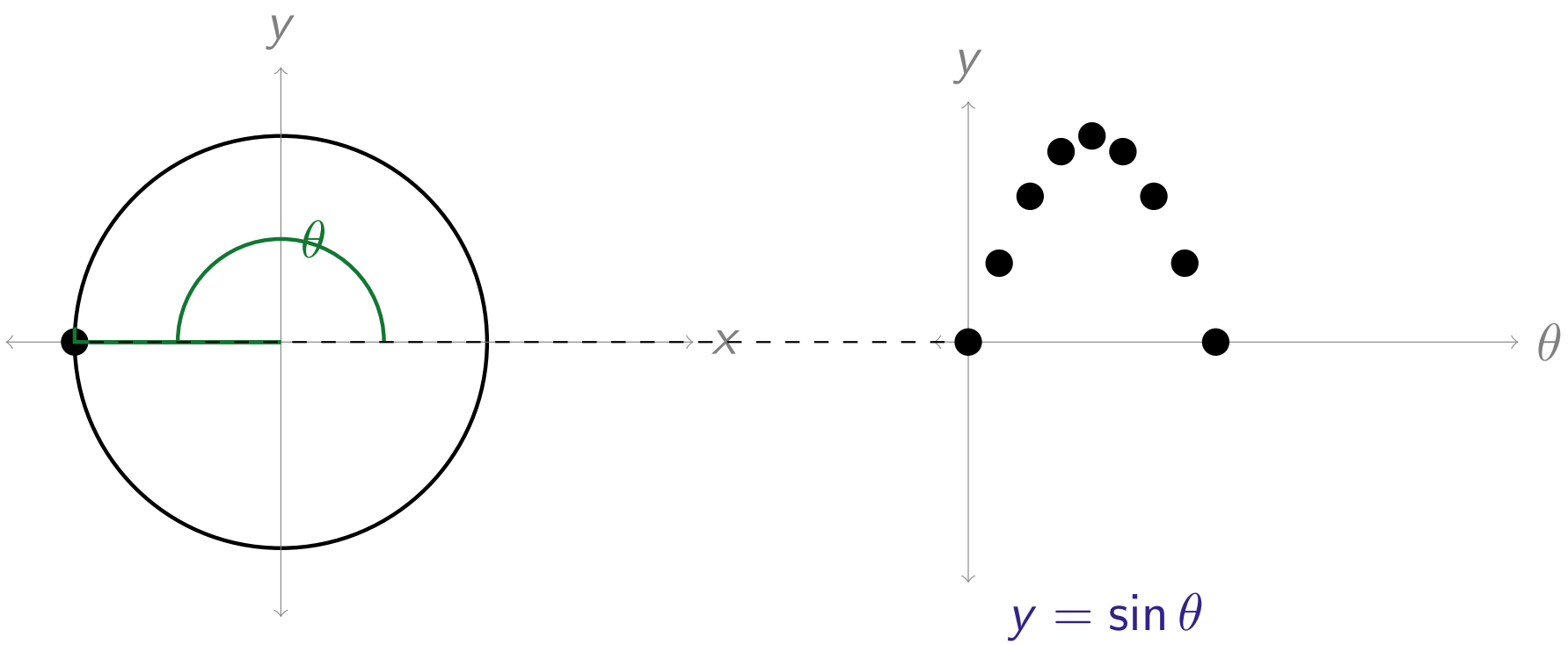
unit circle, $r = 1$

Unit circle, sines and cosines



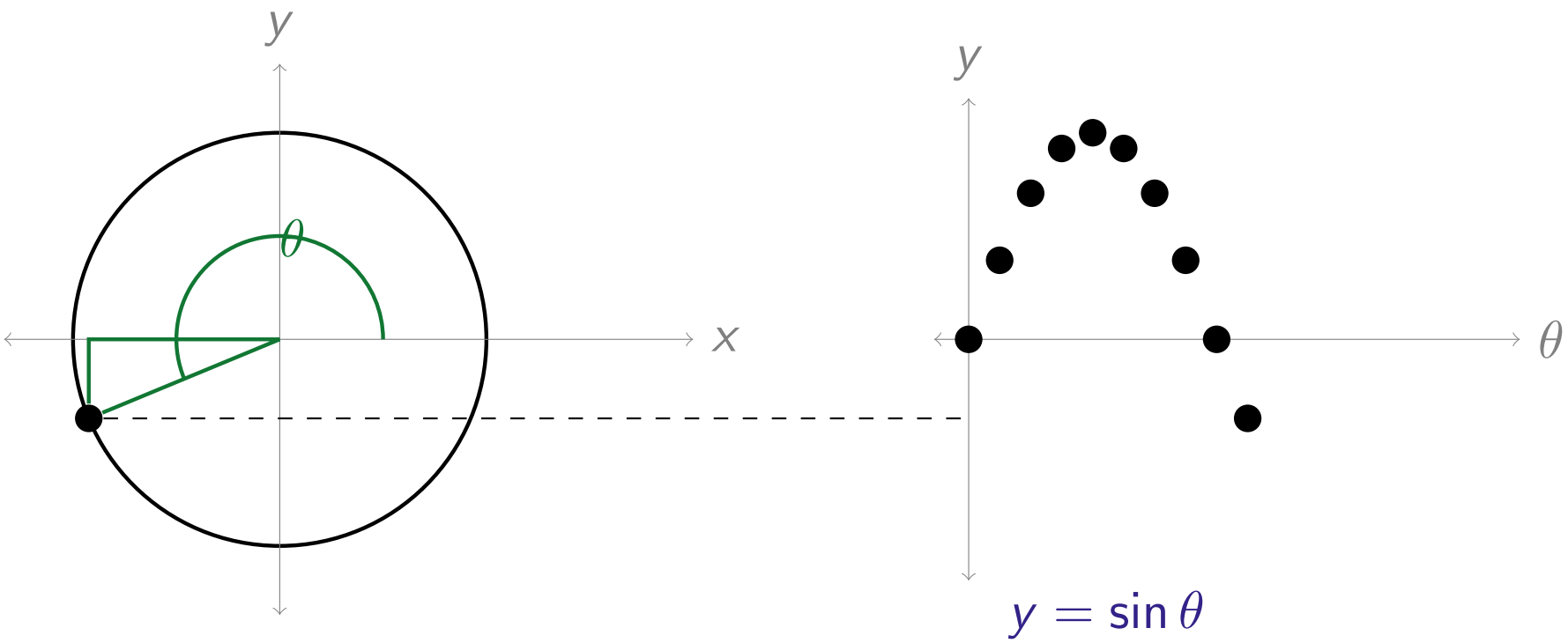
unit circle, $r = 1$

Unit circle, sines and cosines



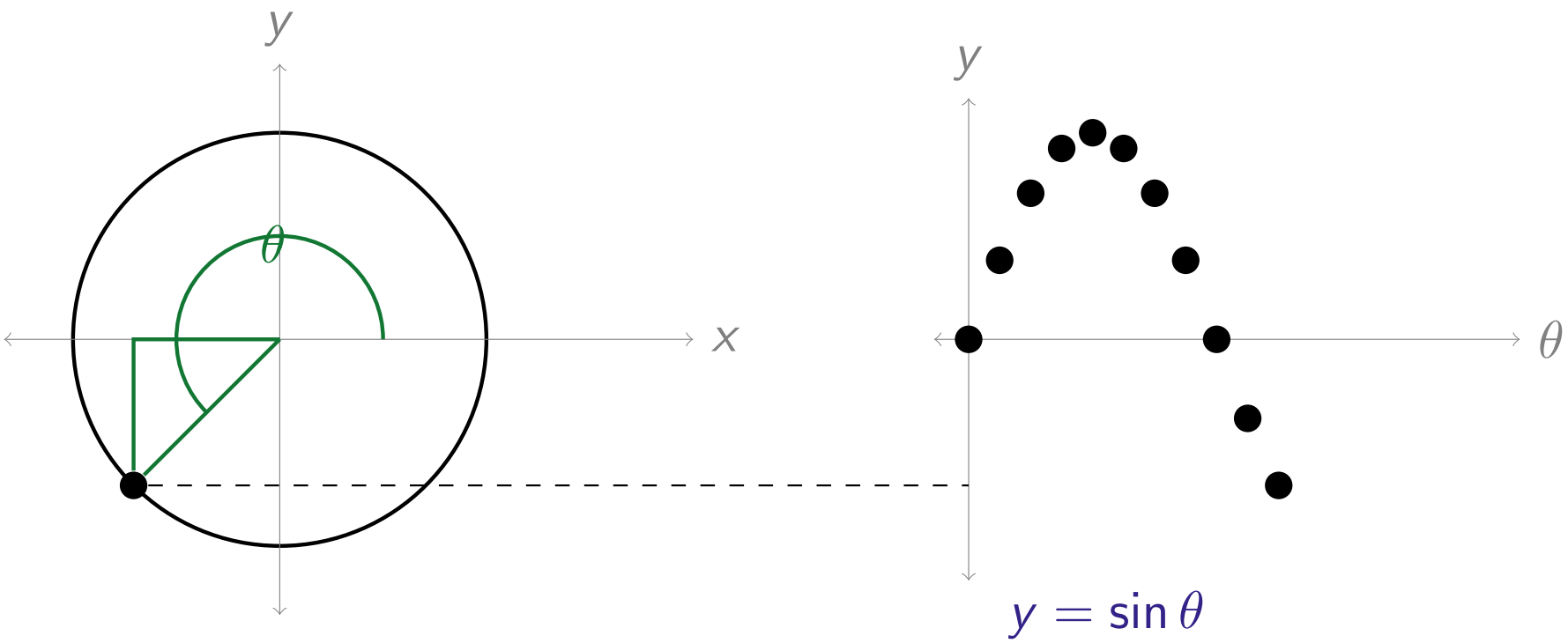
unit circle, $r = 1$

Unit circle, sines and cosines



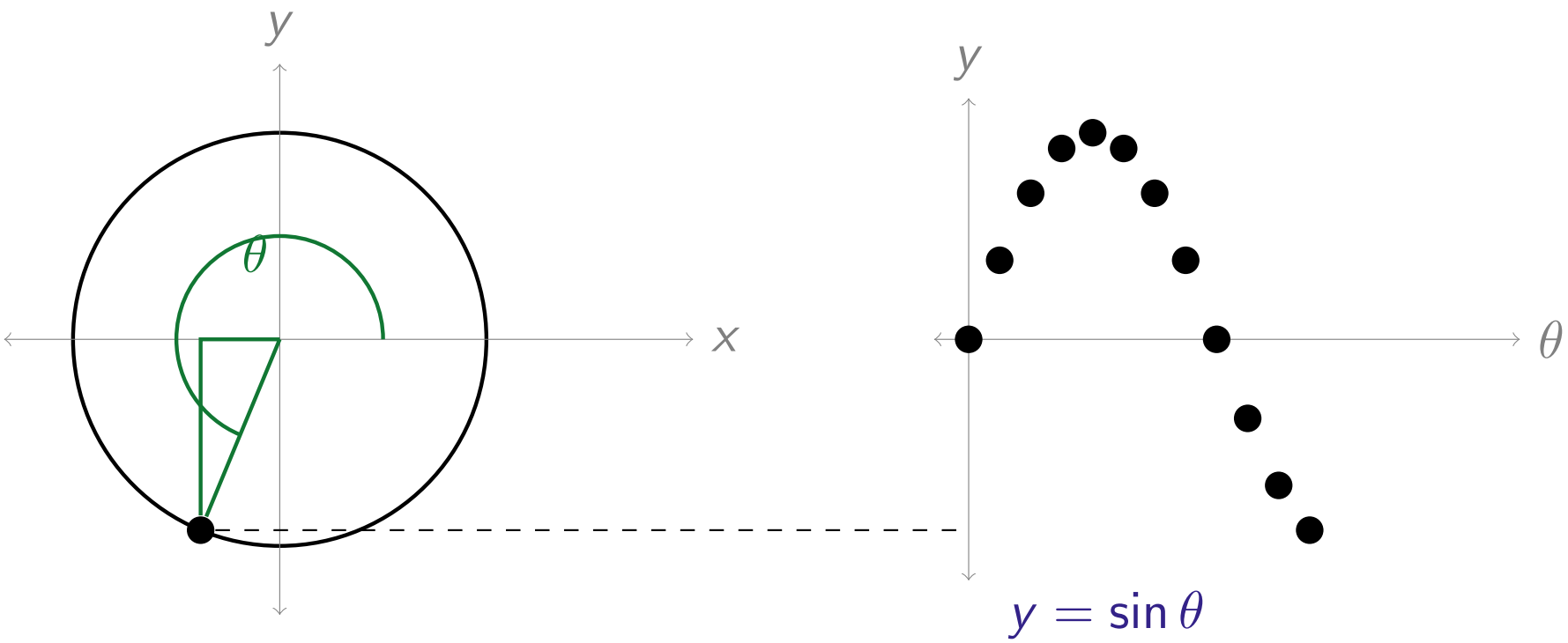
unit circle, $r = 1$

Unit circle, sines and cosines



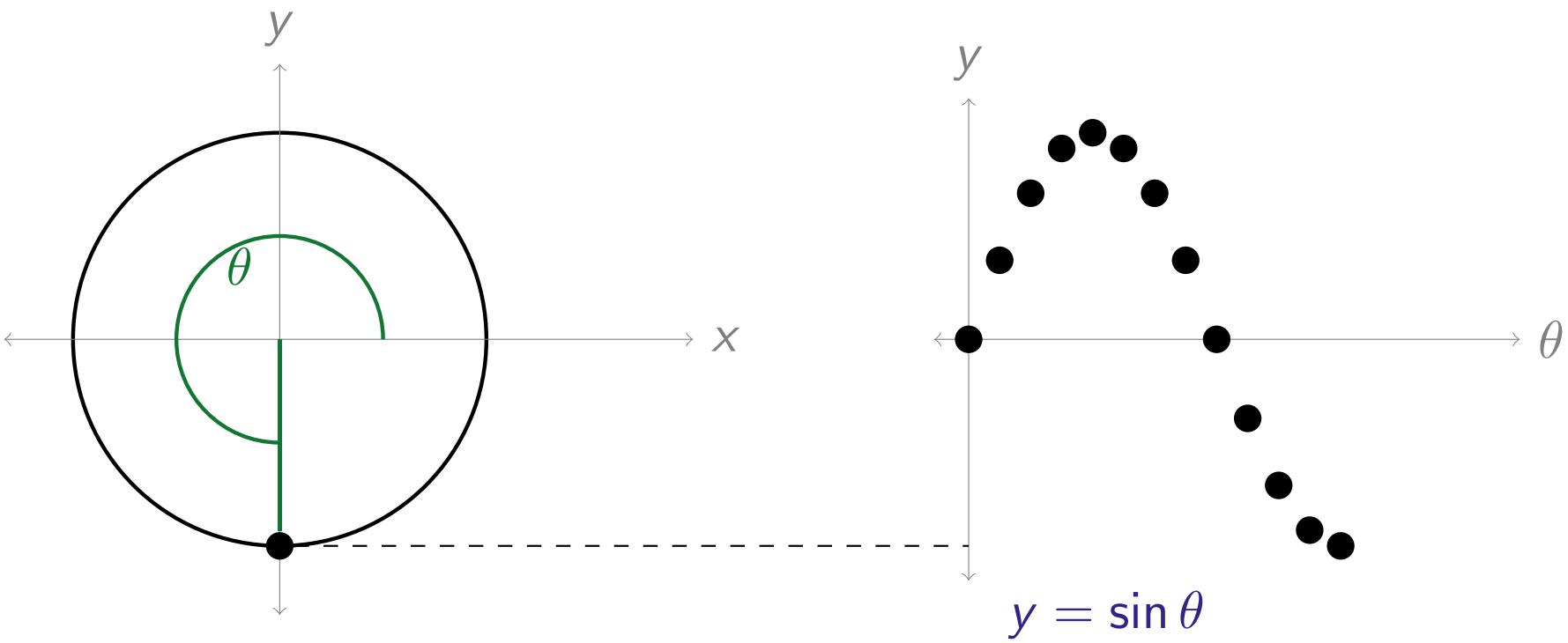
unit circle, $r = 1$

Unit circle, sines and cosines



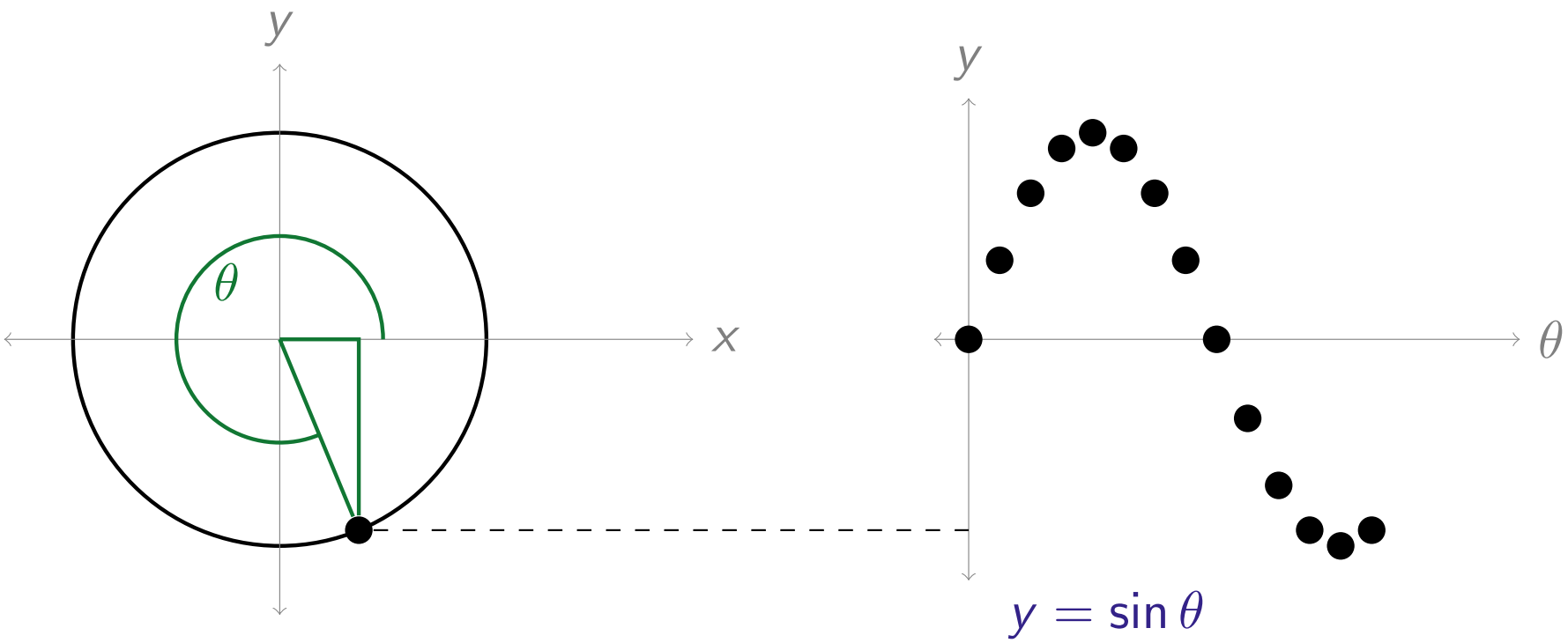
unit circle, $r = 1$

Unit circle, sines and cosines



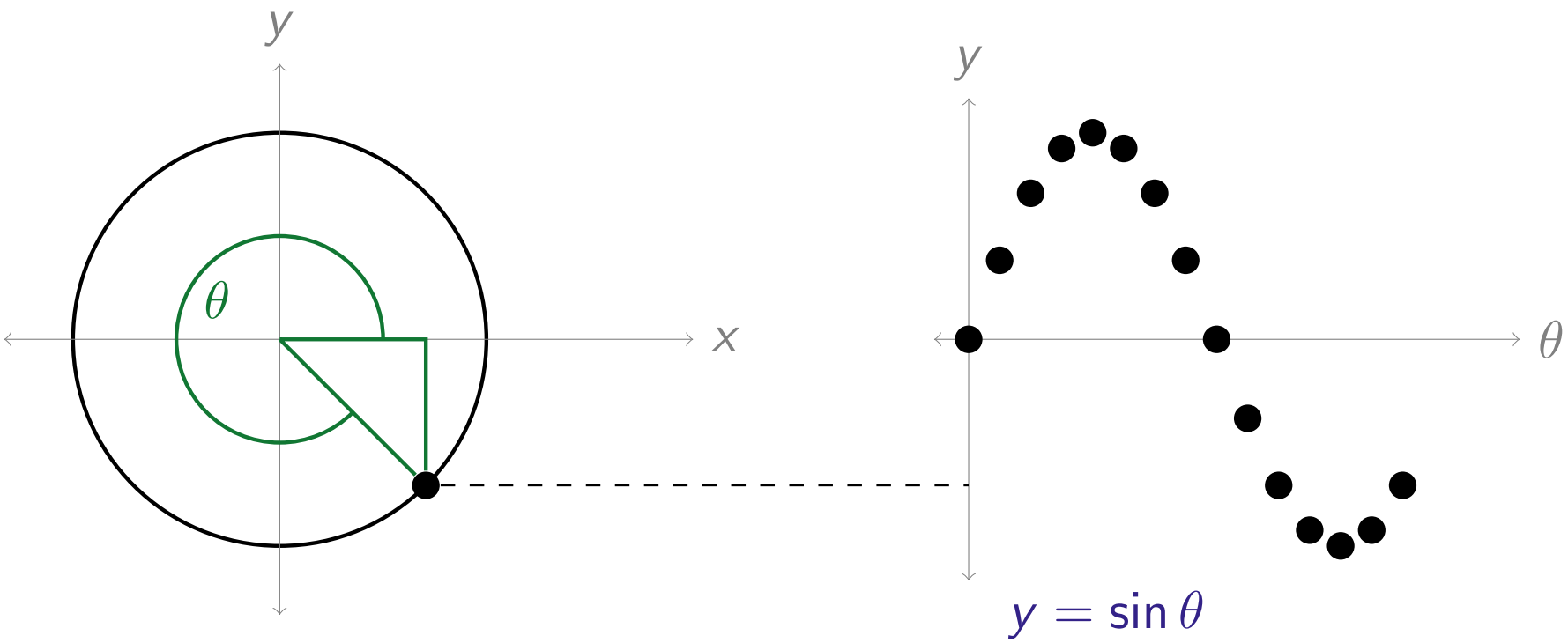
unit circle, $r = 1$

Unit circle, sines and cosines



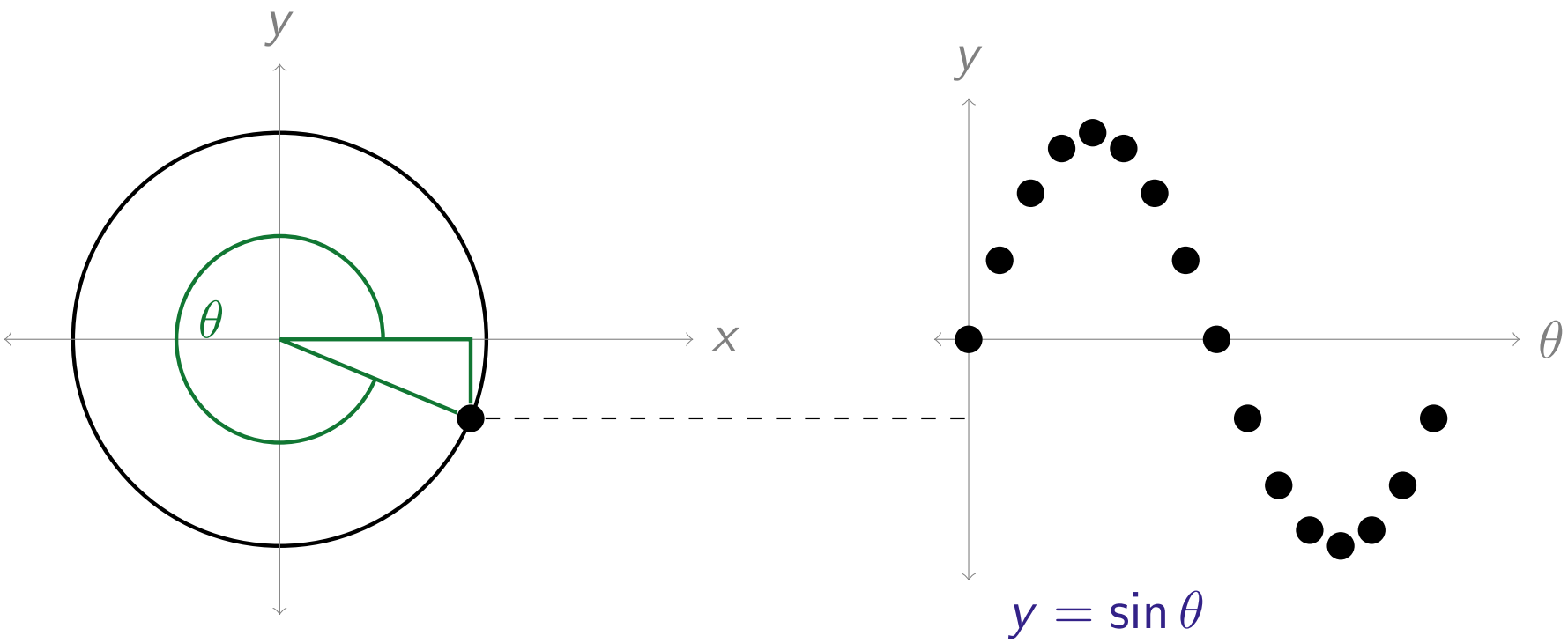
unit circle, $r = 1$

Unit circle, sines and cosines



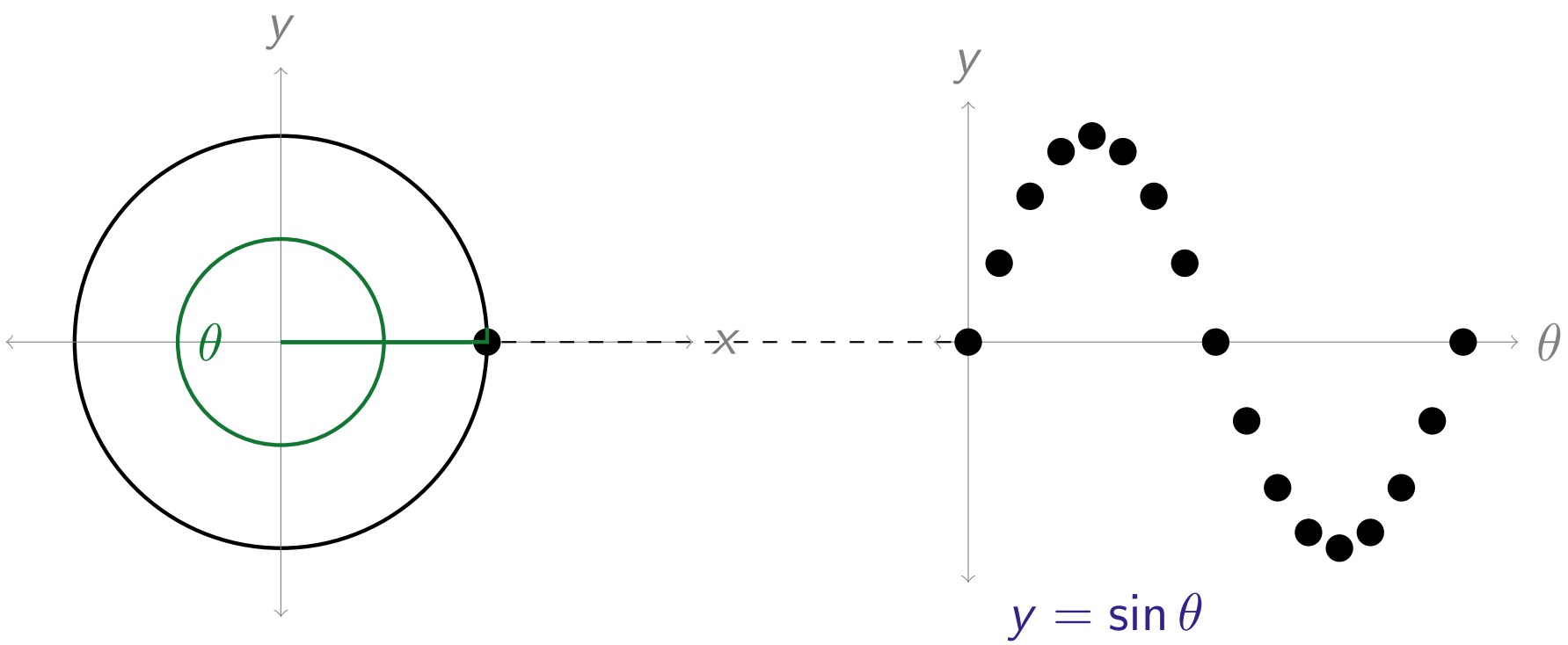
unit circle, $r = 1$

Unit circle, sines and cosines



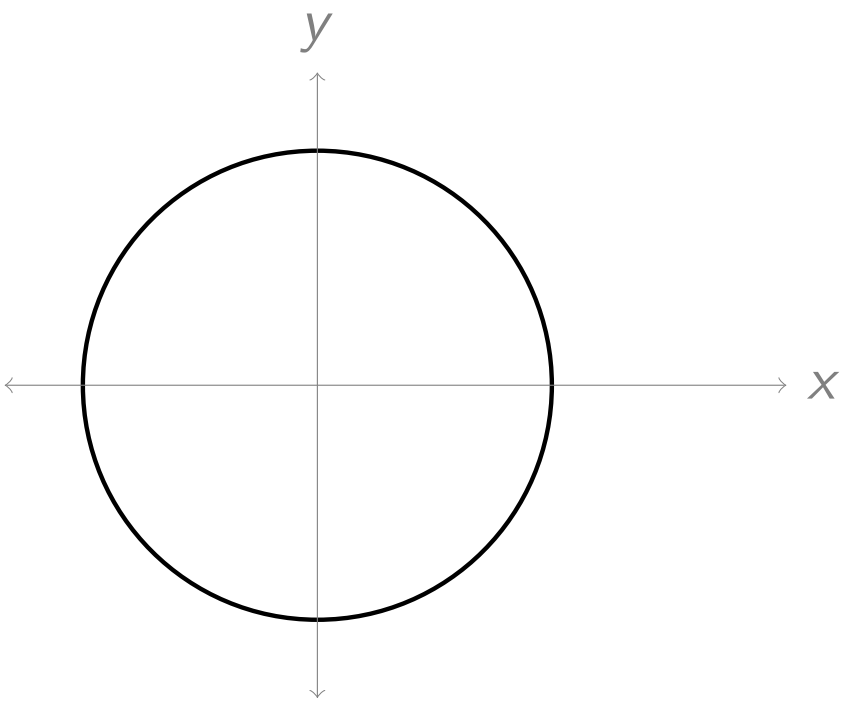
unit circle, $r = 1$

Unit circle, sines and cosines

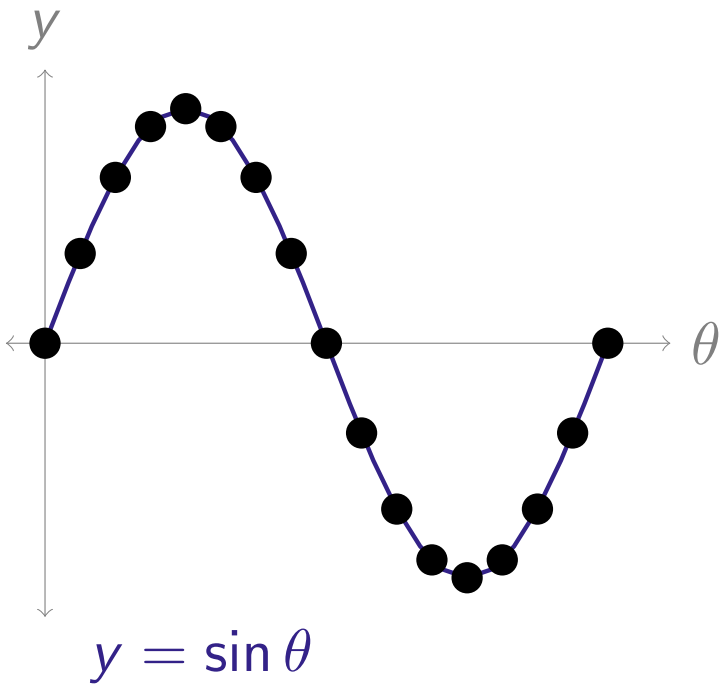


unit circle, $r = 1$

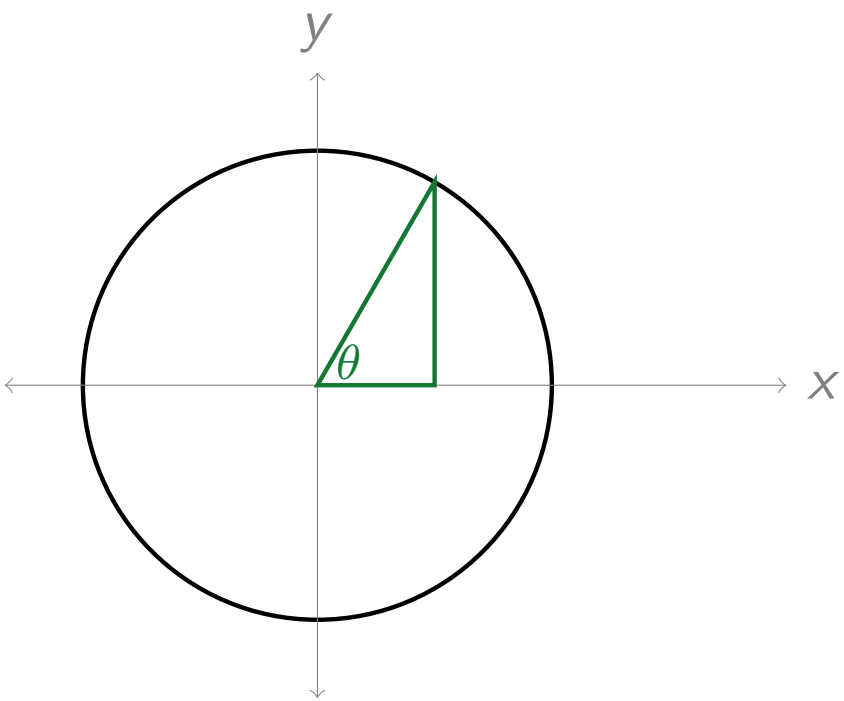
Unit circle, sines and cosines



unit circle, $r = 1$

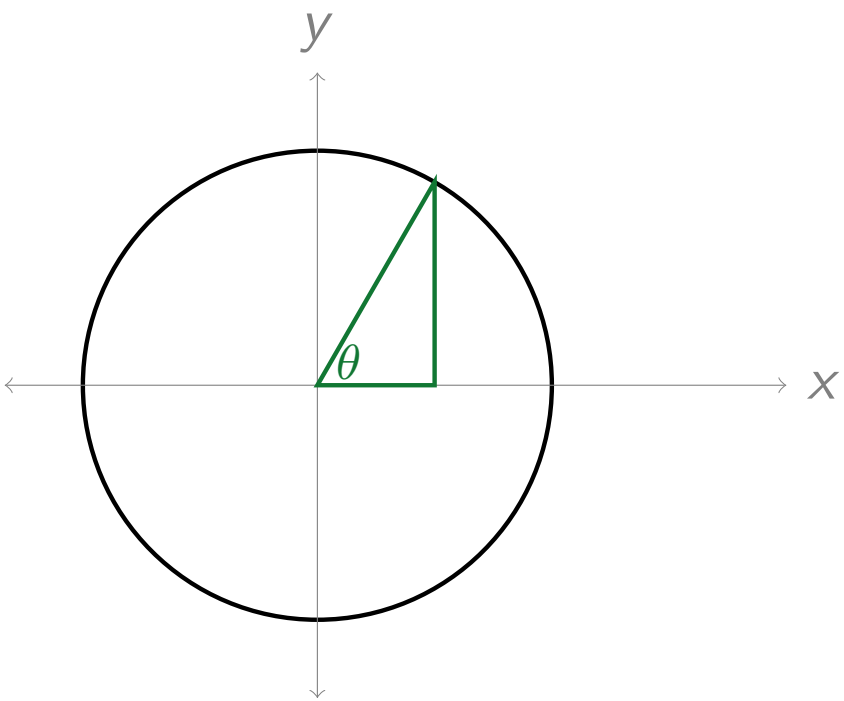


Unit circle, sines and cosines



unit circle, $r = 1$

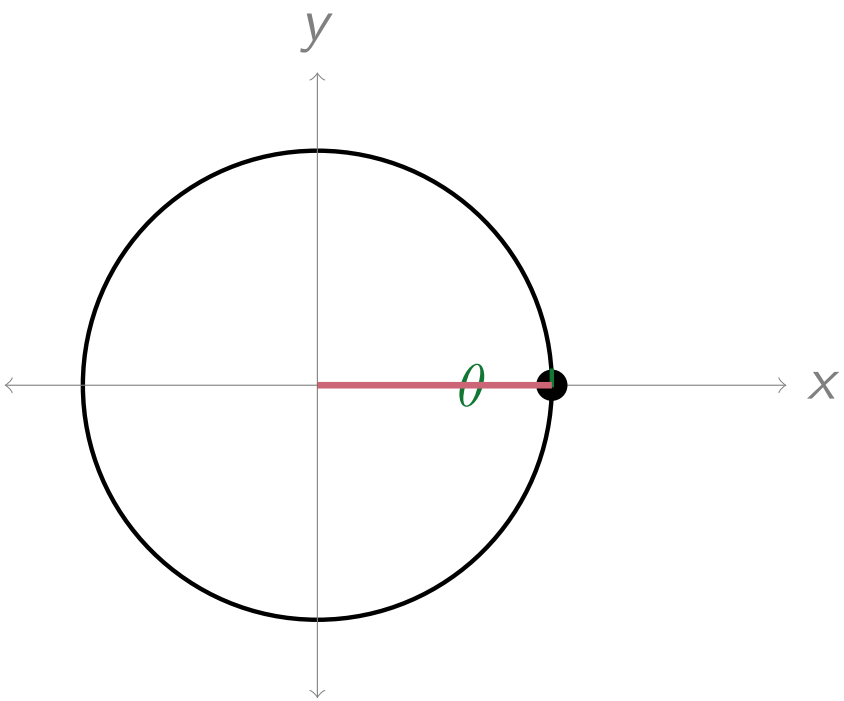
Unit circle, sines and cosines



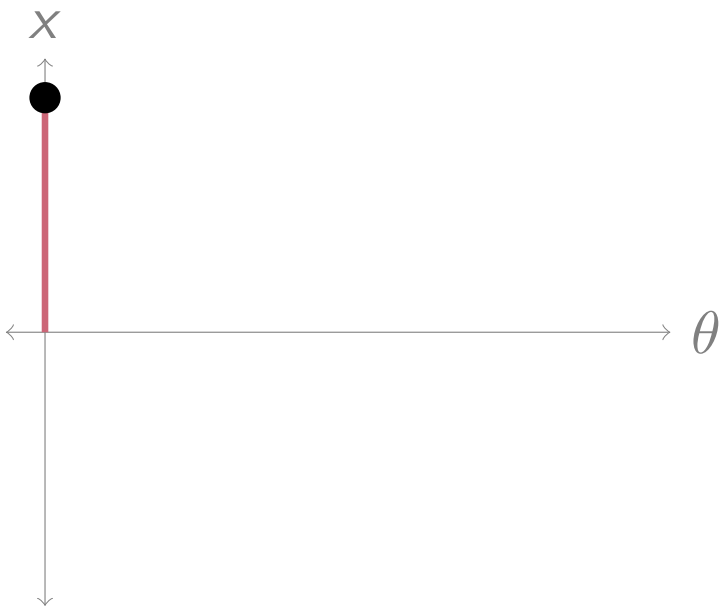
unit circle, $r = 1$

$x = \cos\theta$

Unit circle, sines and cosines

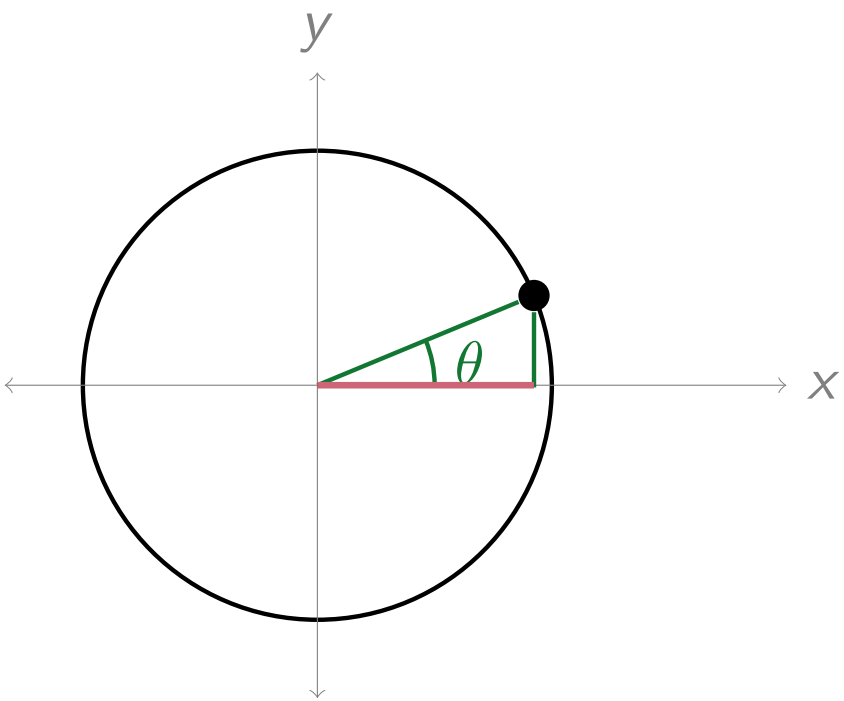


unit circle, $r = 1$

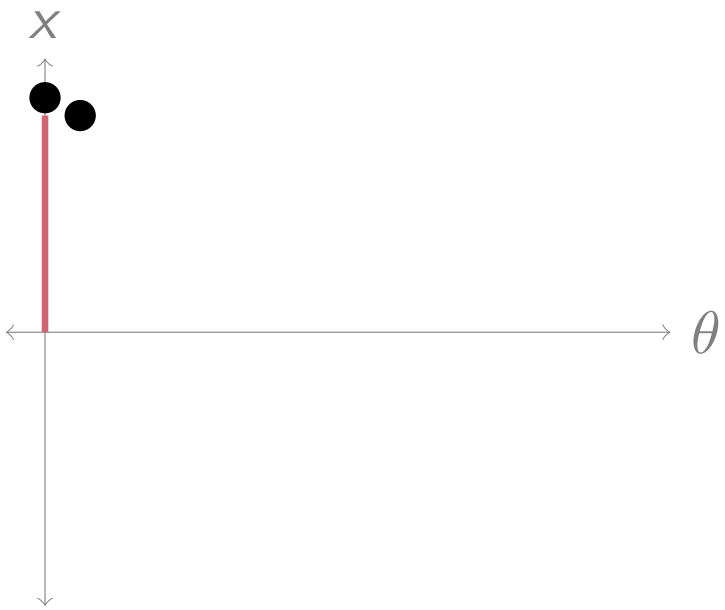


$x = \cos\theta$

Unit circle, sines and cosines

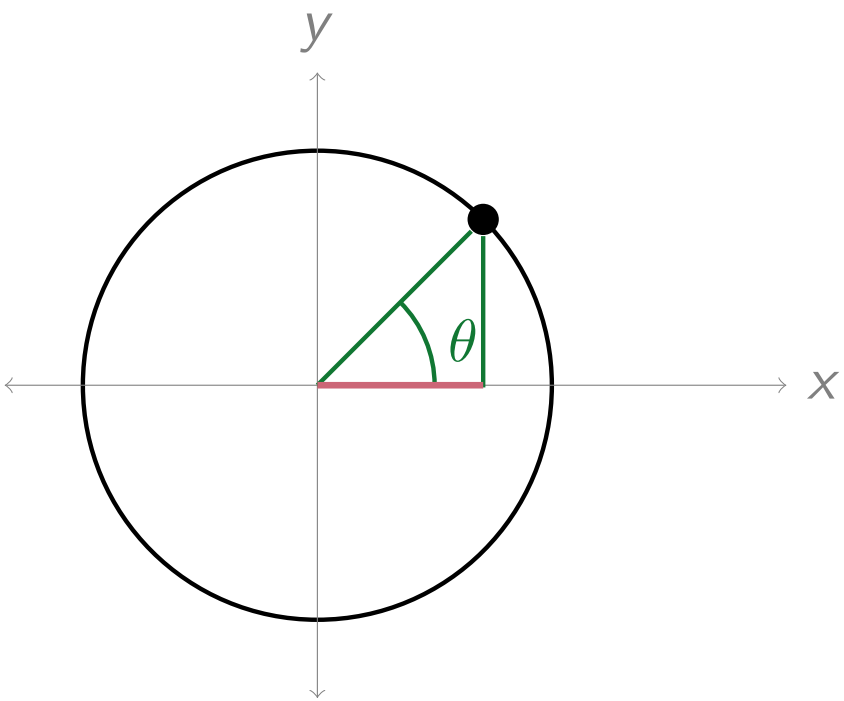


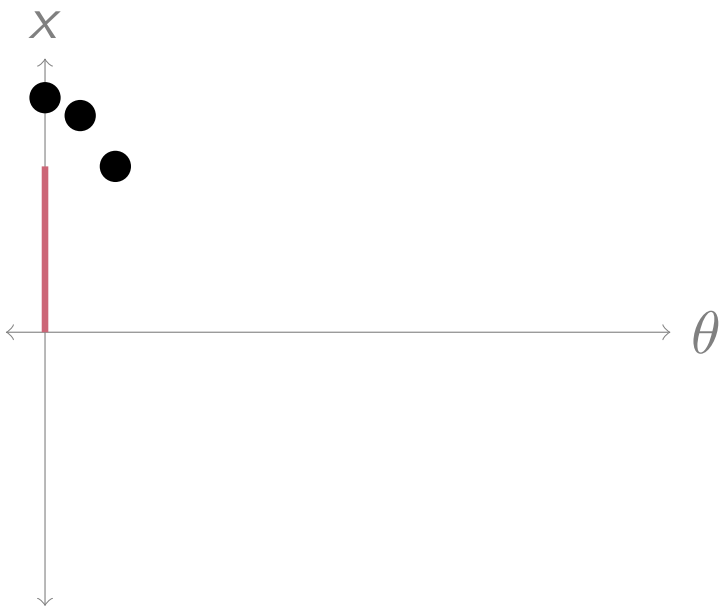
unit circle, $r = 1$



$x = \cos\theta$

Unit circle, sines and cosines

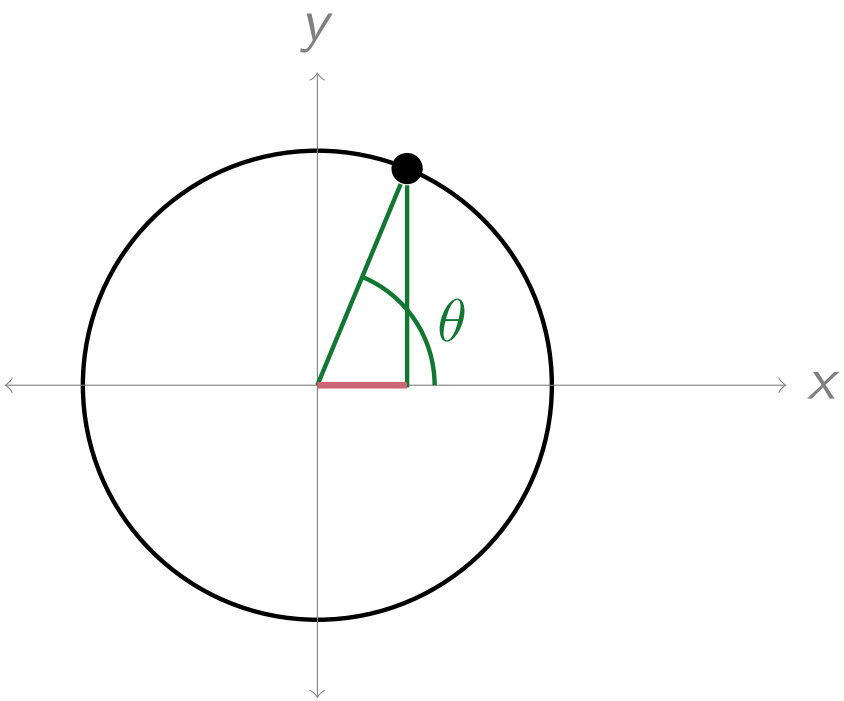




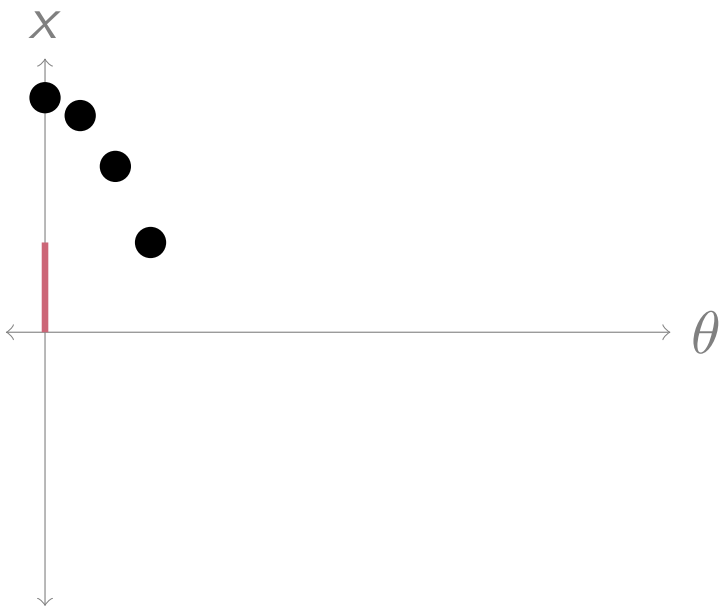
$x = \cos\theta$

unit circle, $r = 1$

Unit circle, sines and cosines

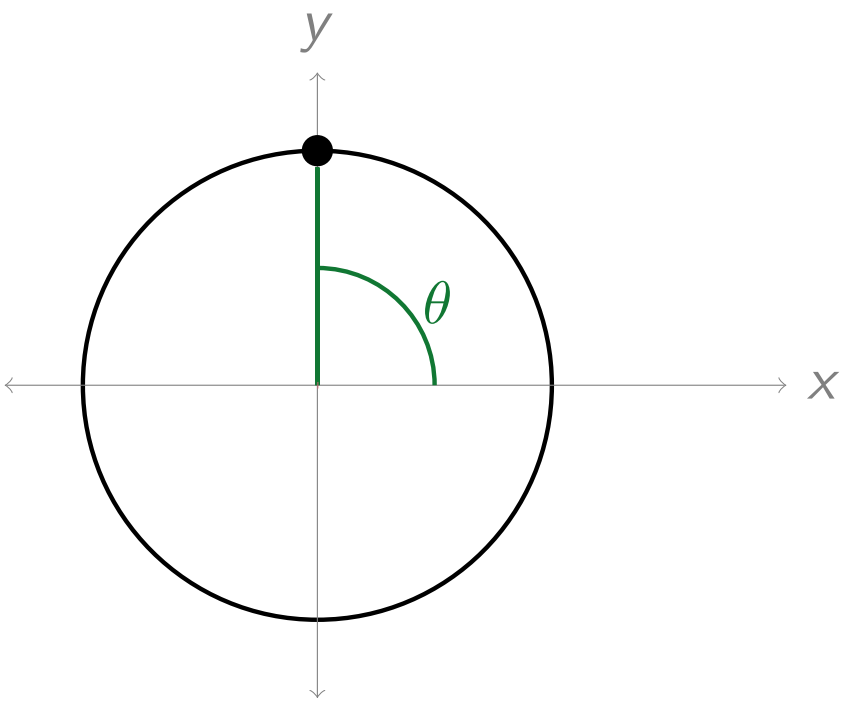


unit circle, $r = 1$

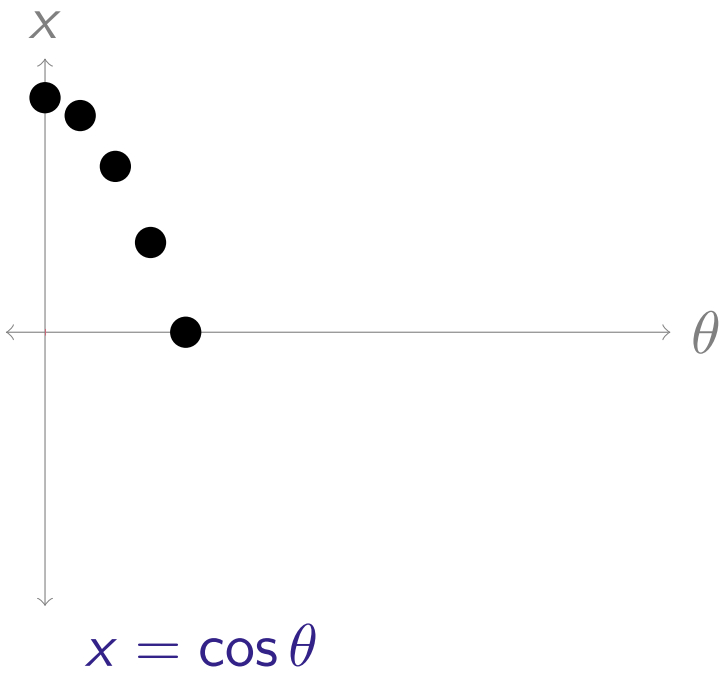


$x = \cos\theta$

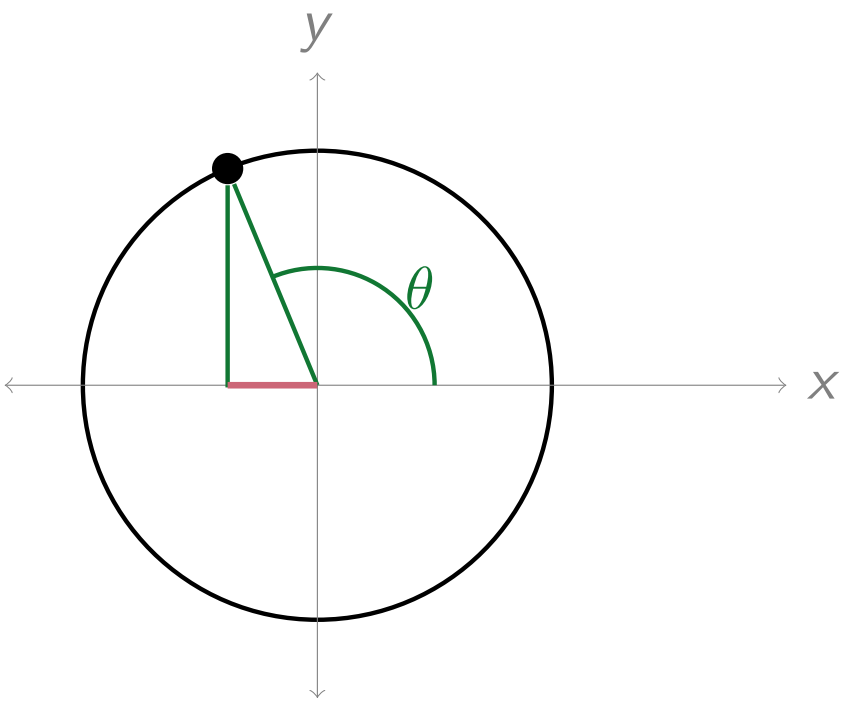
Unit circle, sines and cosines



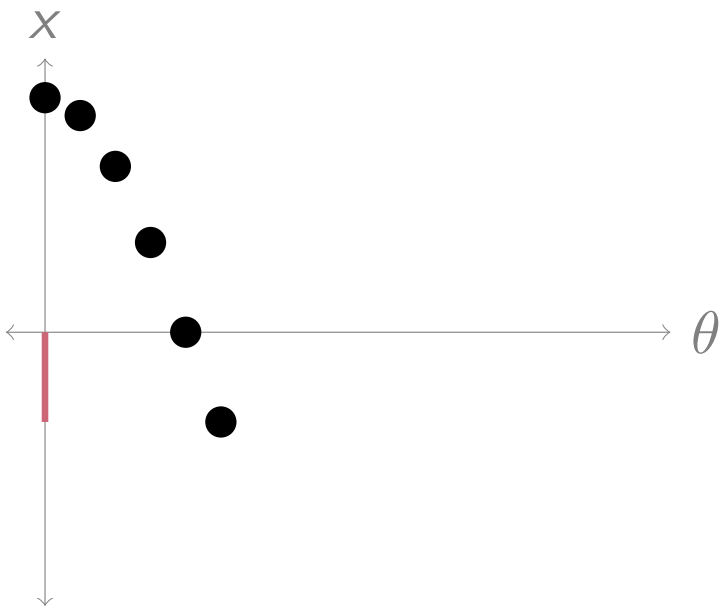
unit circle, $r = 1$



Unit circle, sines and cosines

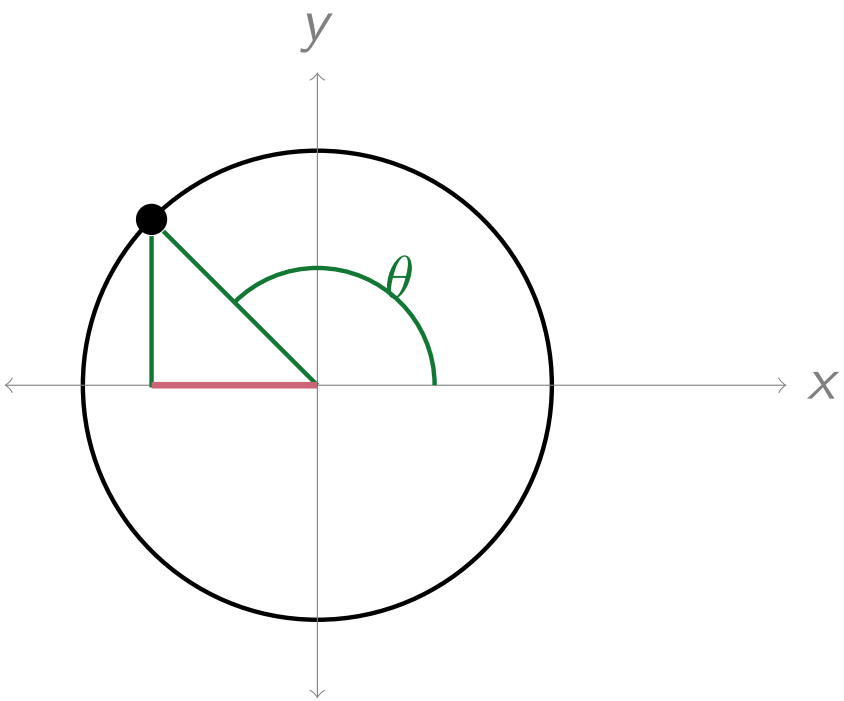


unit circle, $r = 1$

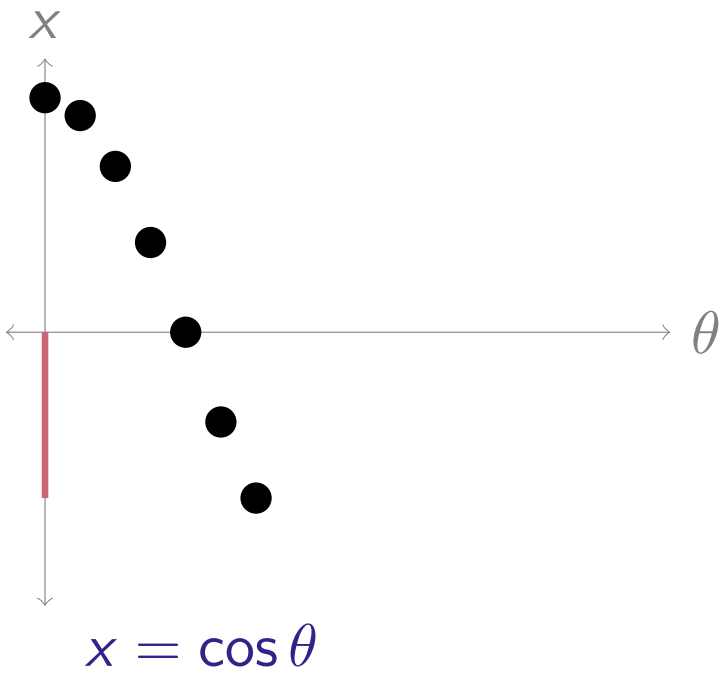


$x = \cos\theta$

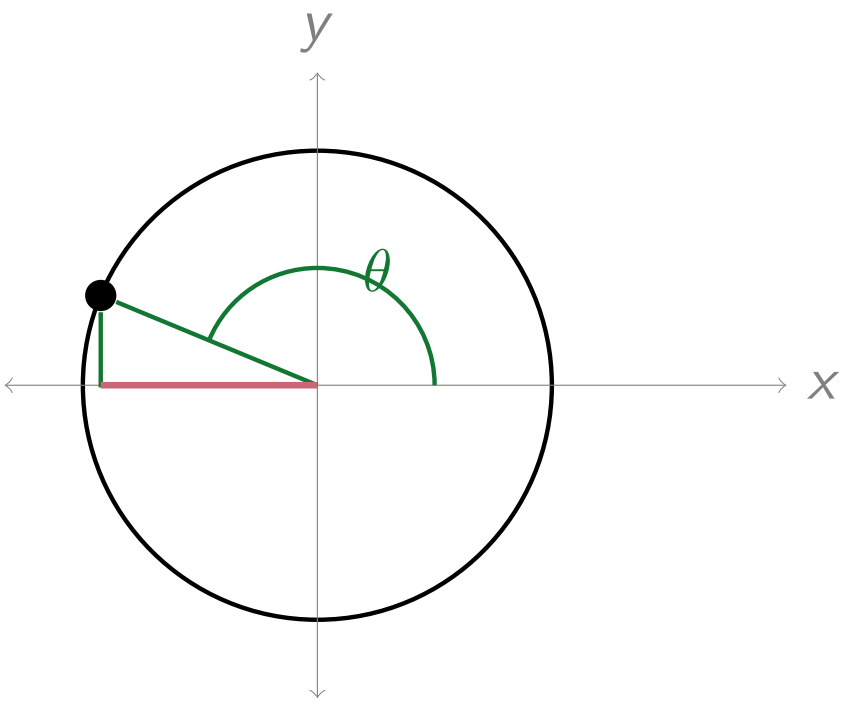
Unit circle, sines and cosines



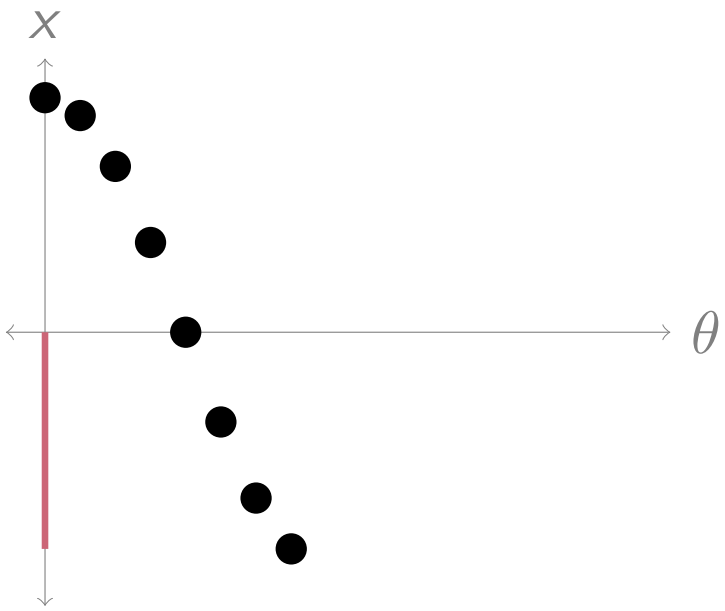
unit circle, $r = 1$



Unit circle, sines and cosines

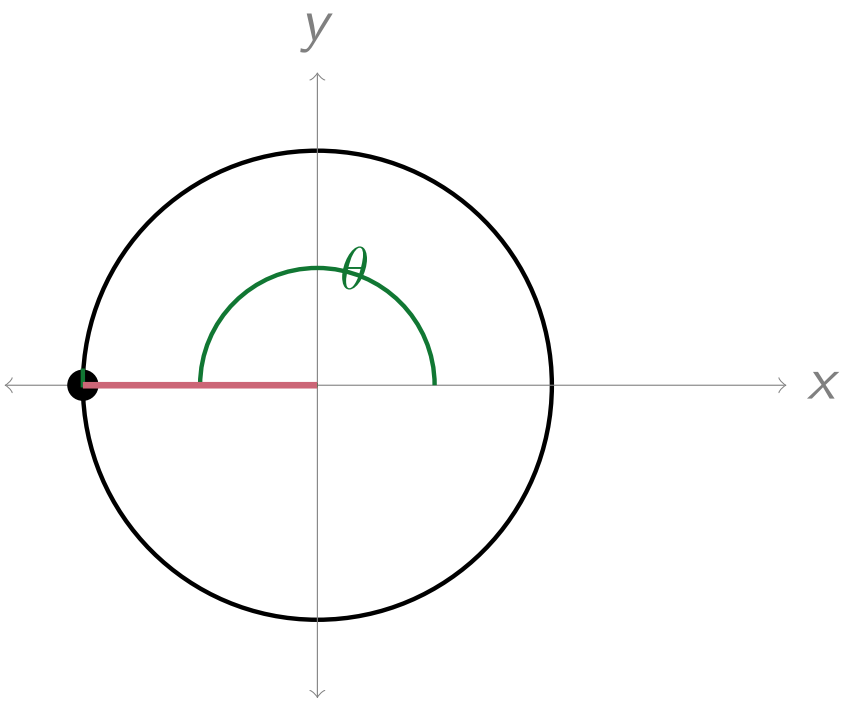


unit circle, $r = 1$

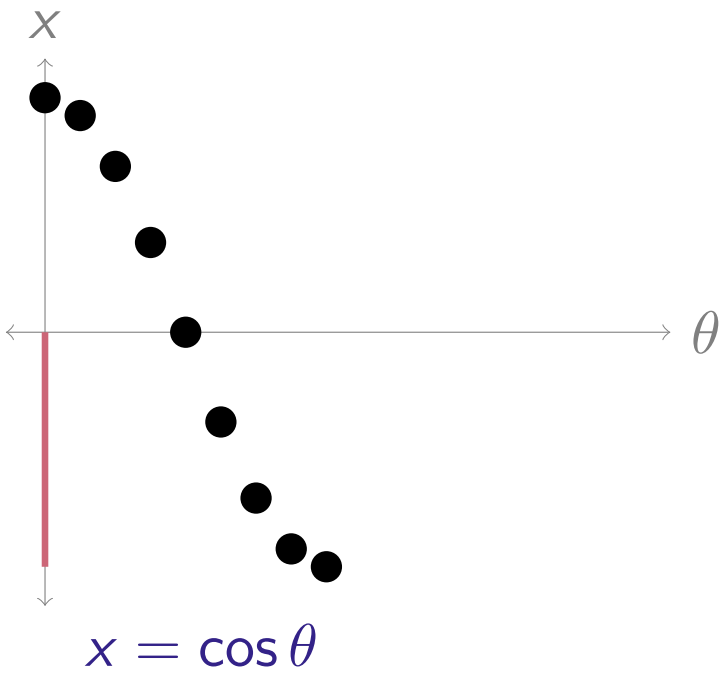


$x = \cos\theta$

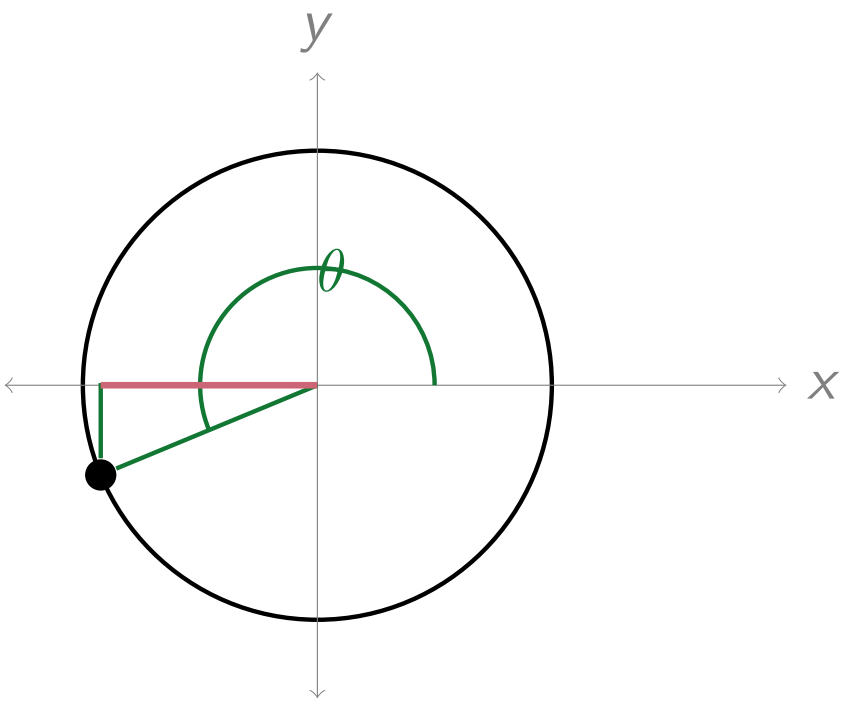
Unit circle, sines and cosines



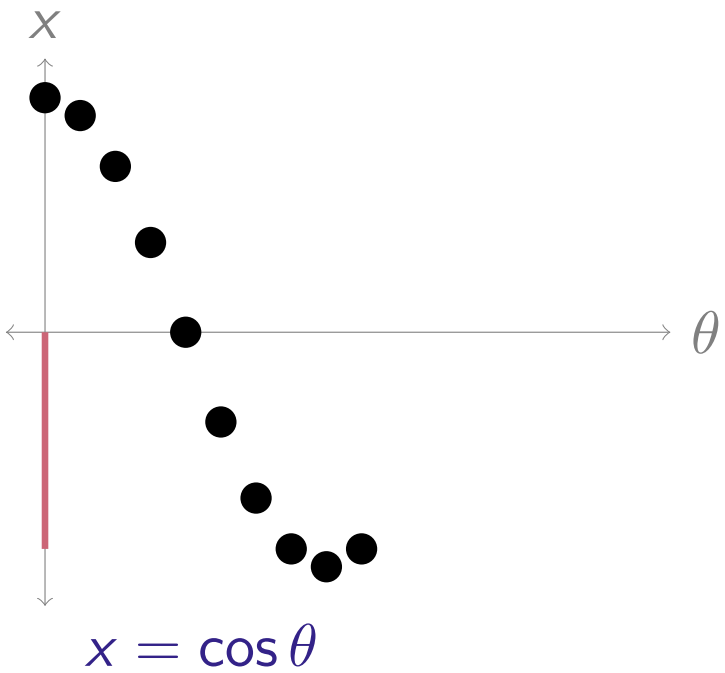
unit circle, $r = 1$



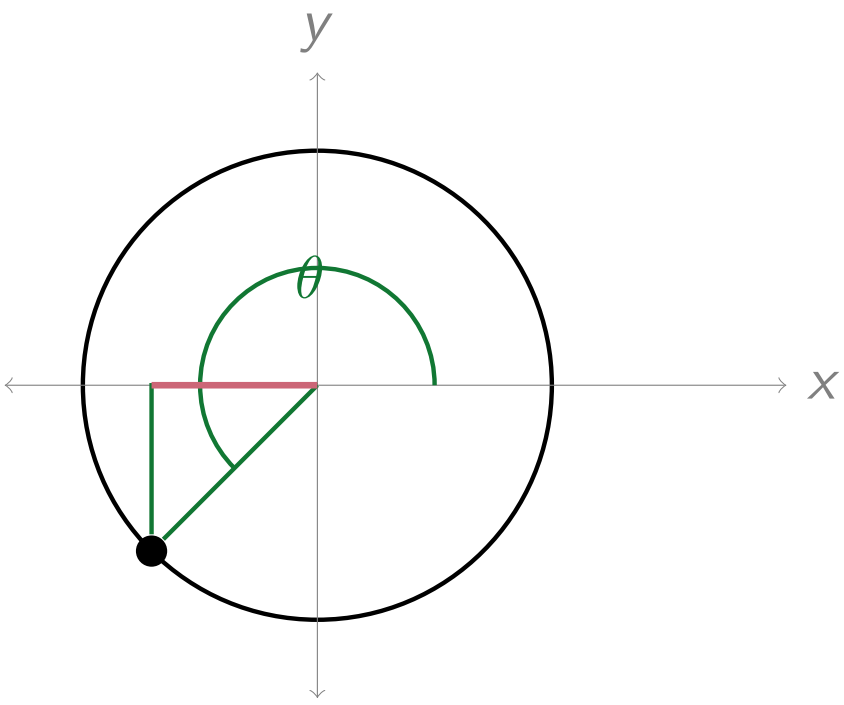
Unit circle, sines and cosines



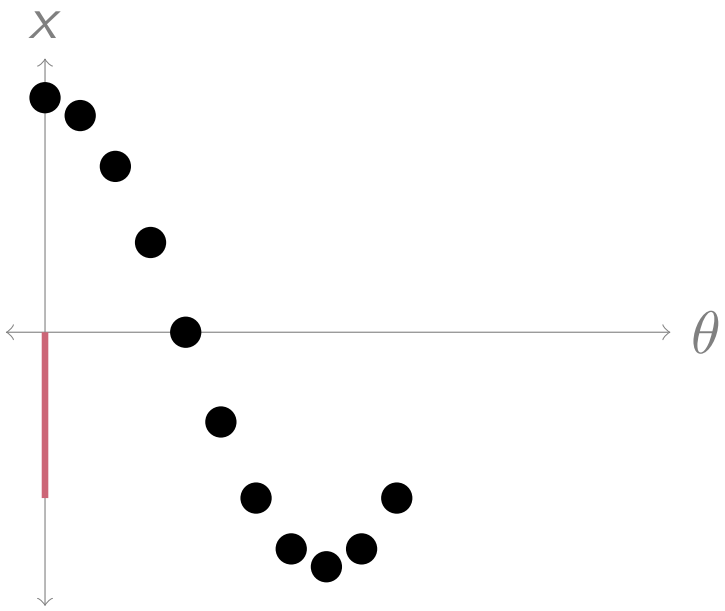
unit circle, $r = 1$



Unit circle, sines and cosines

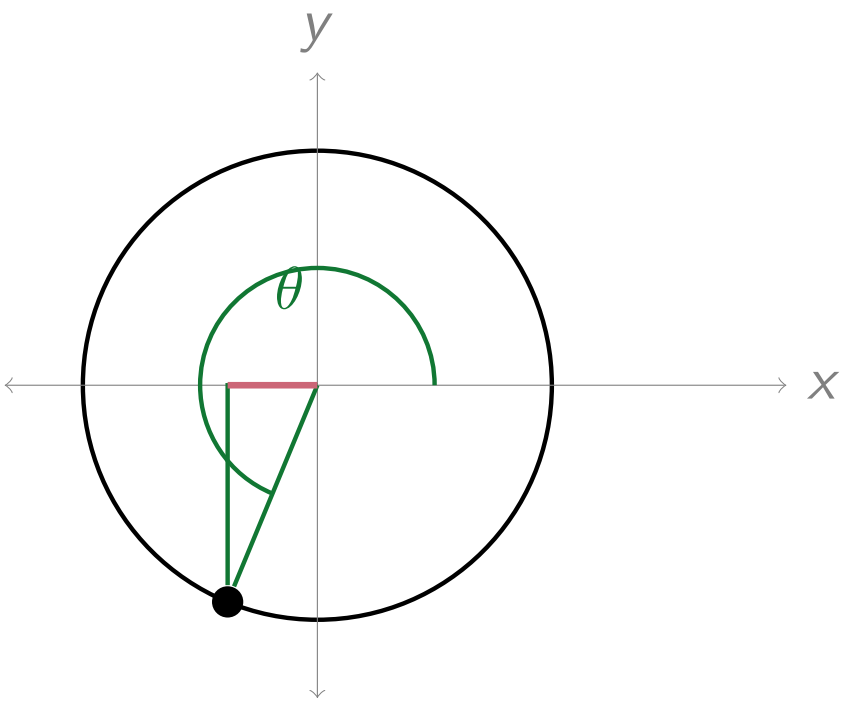


unit circle, $r = 1$

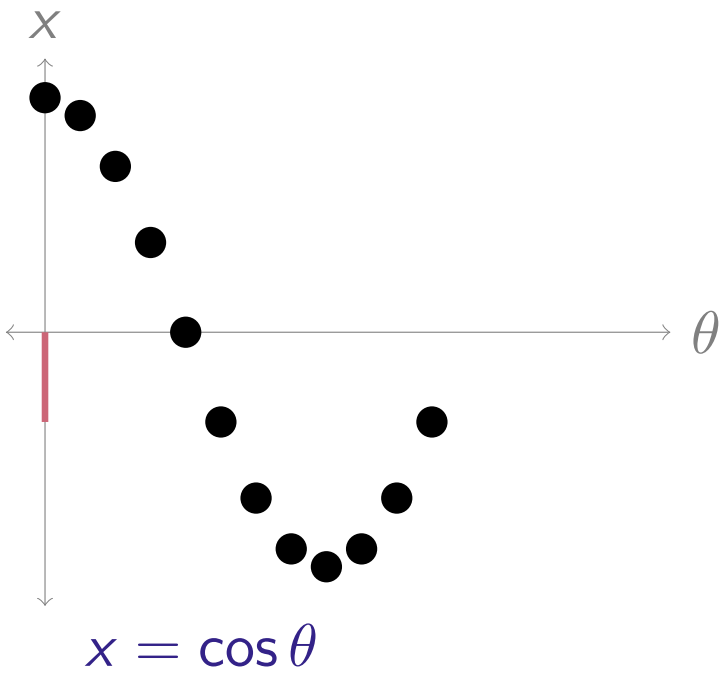


$x = \cos\theta$

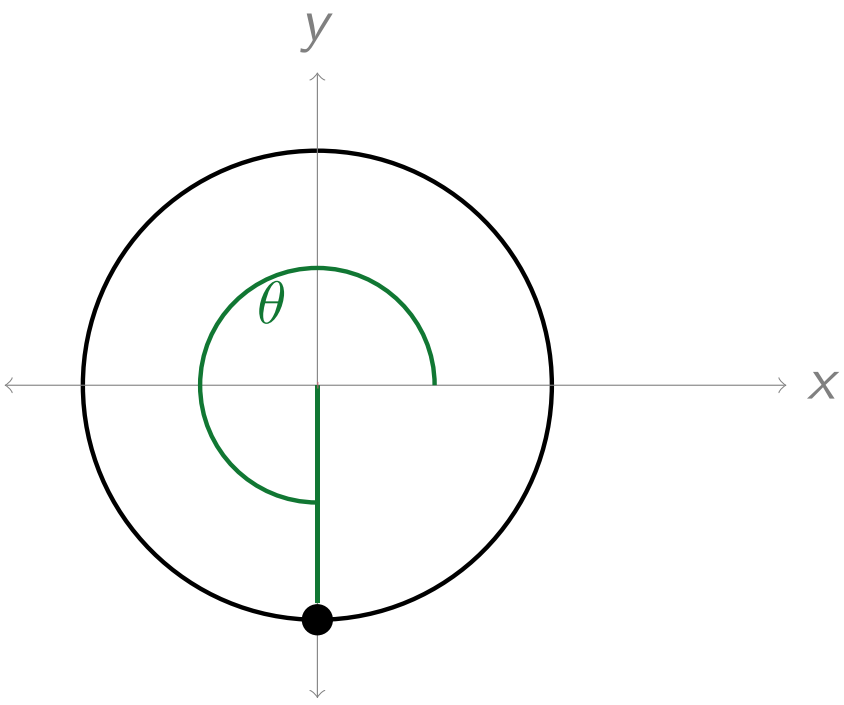
Unit circle, sines and cosines



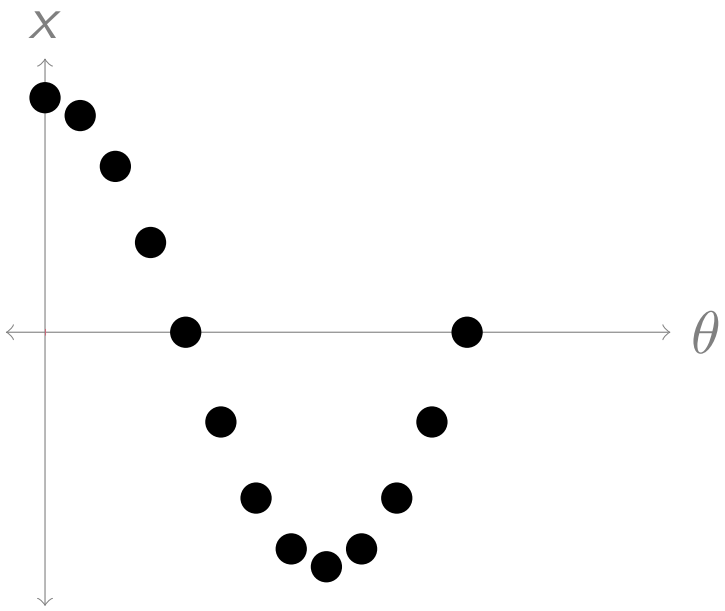
unit circle, $r = 1$



Unit circle, sines and cosines

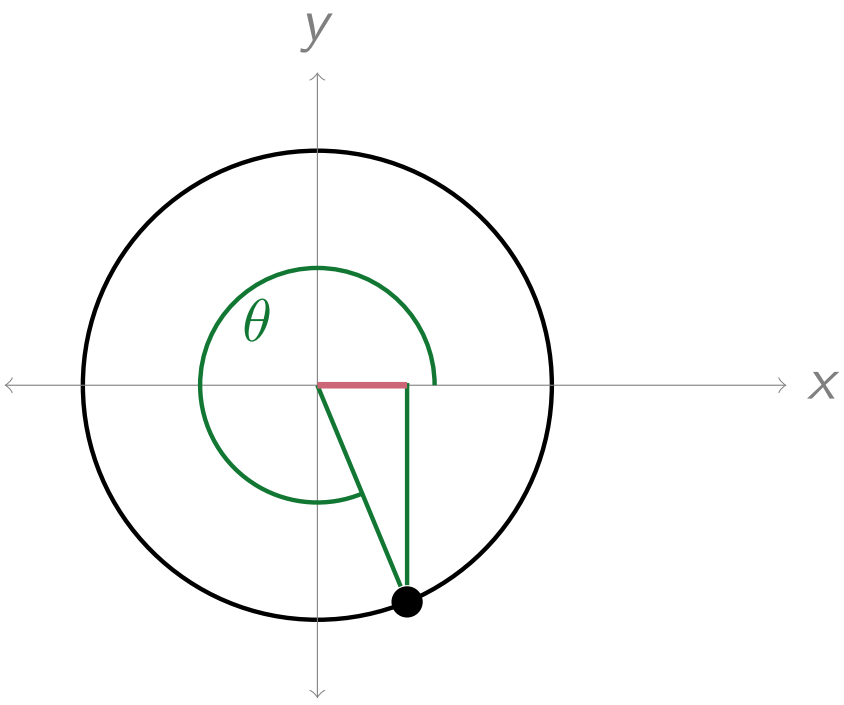


unit circle, $r = 1$

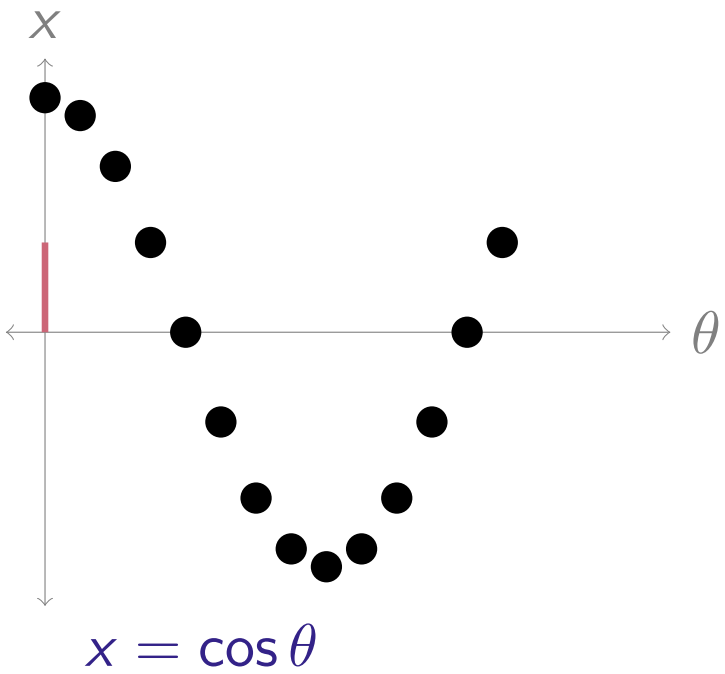


$x = \cos\theta$

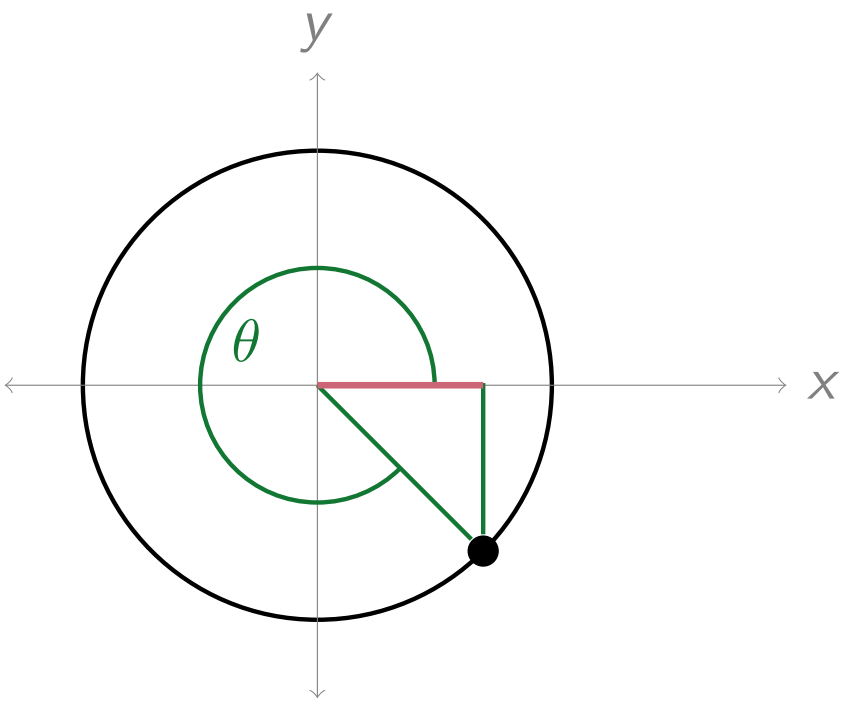
Unit circle, sines and cosines



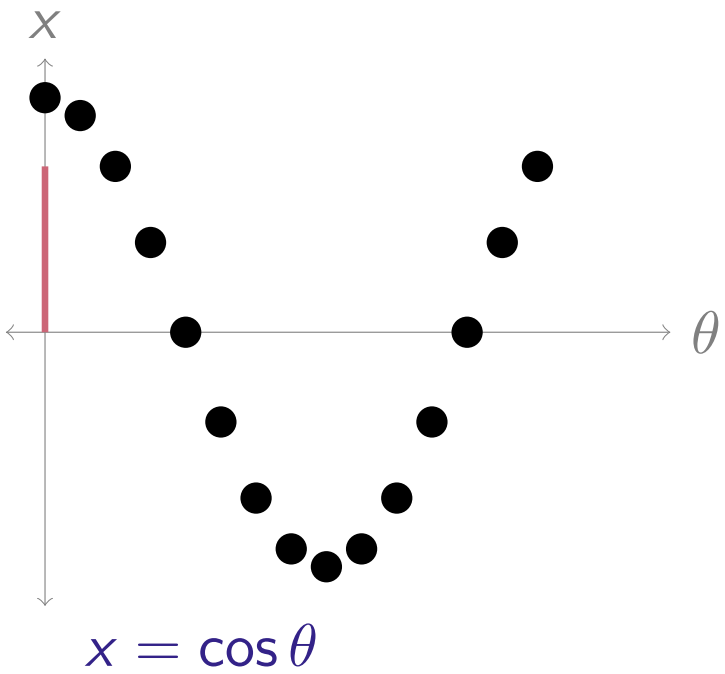
unit circle, $r = 1$



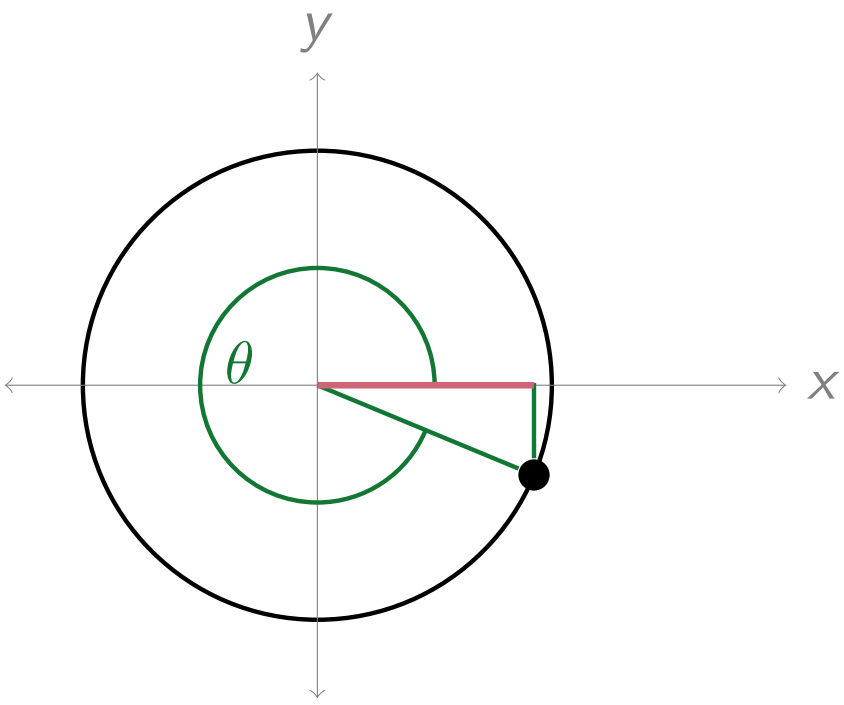
Unit circle, sines and cosines



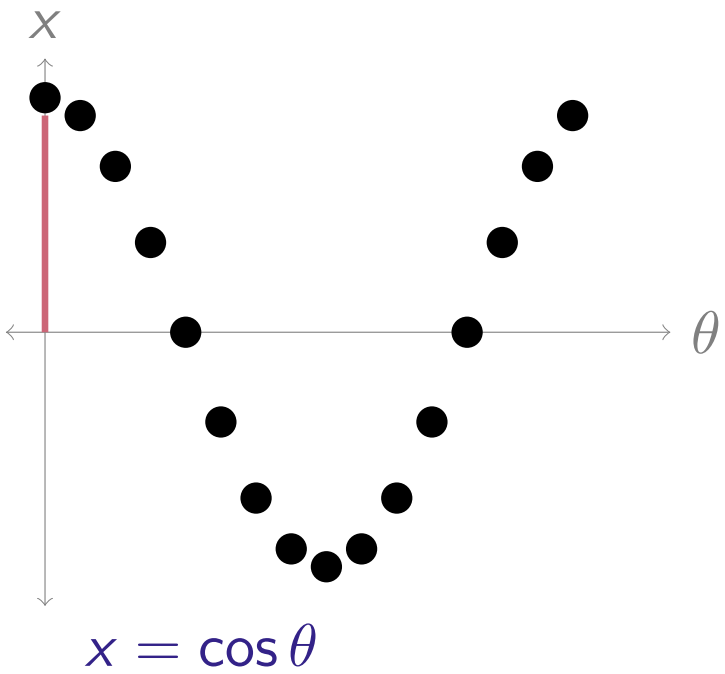
unit circle, $r = 1$



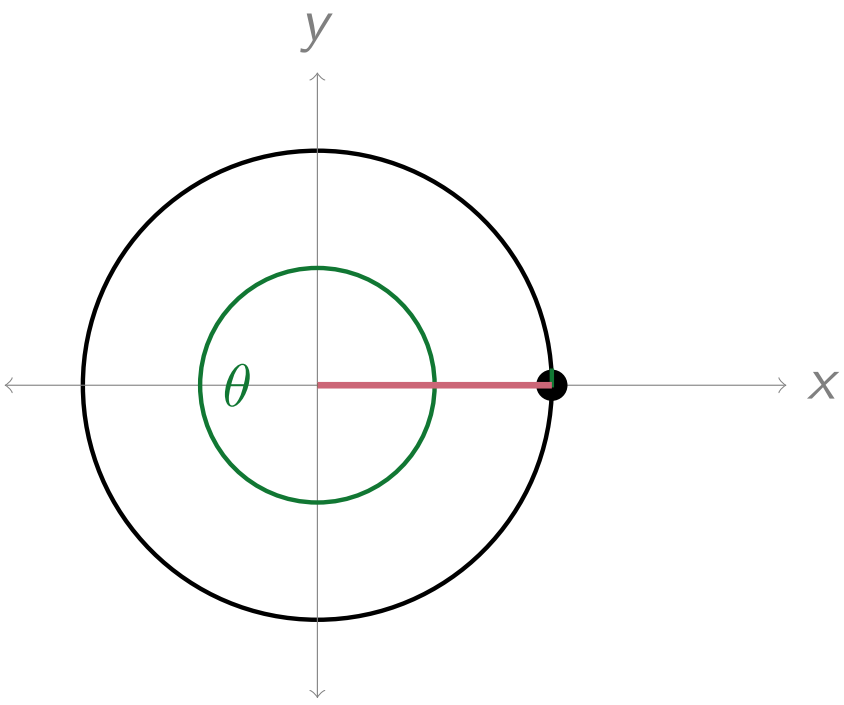
Unit circle, sines and cosines



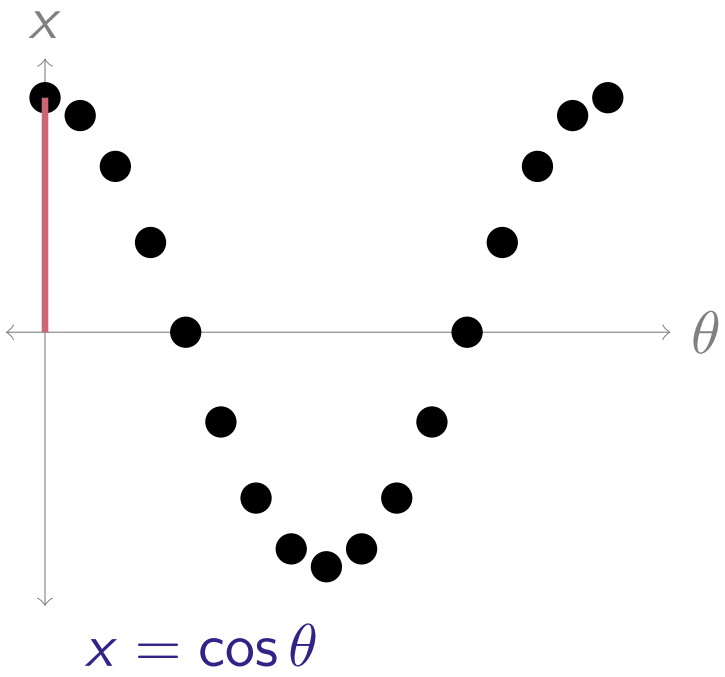
unit circle, $r = 1$



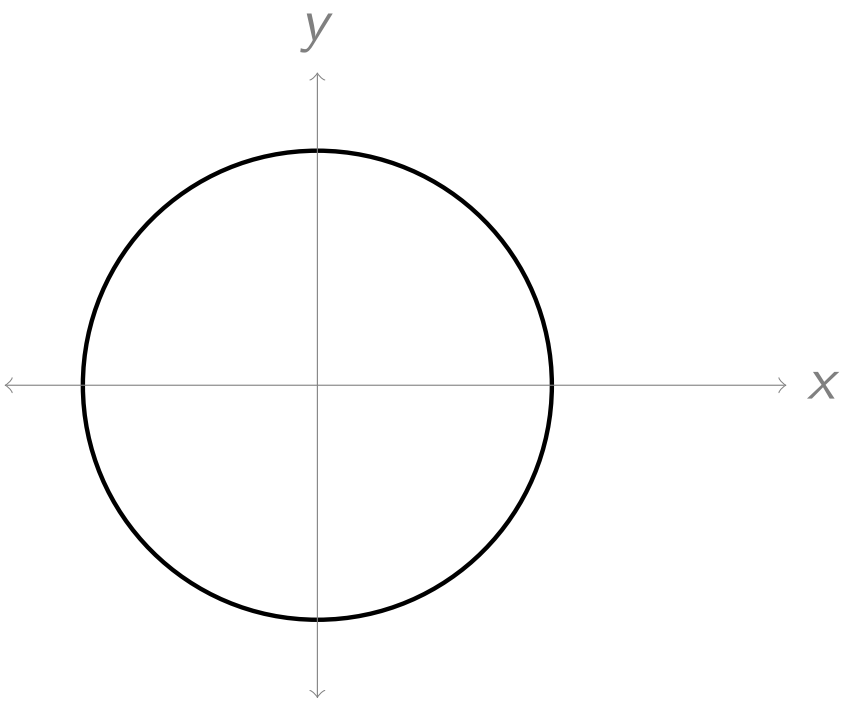
Unit circle, sines and cosines



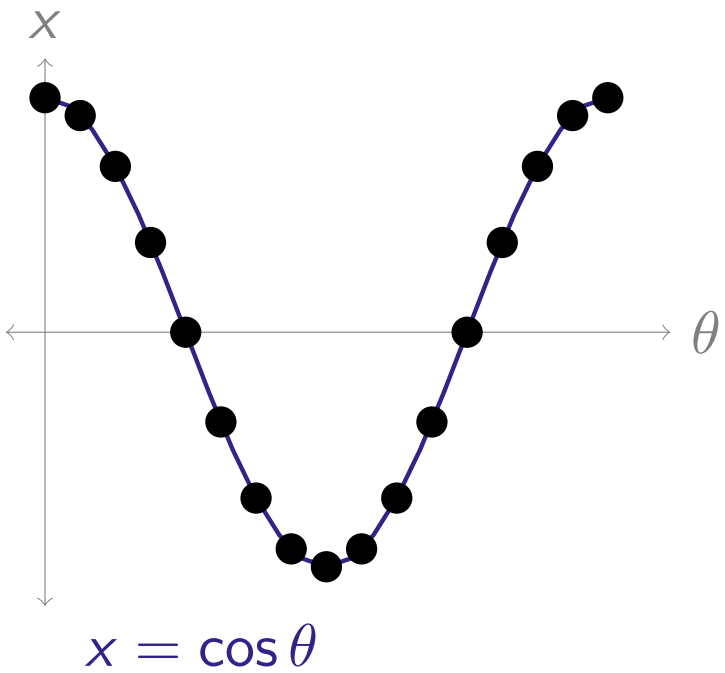
unit circle, $r = 1$



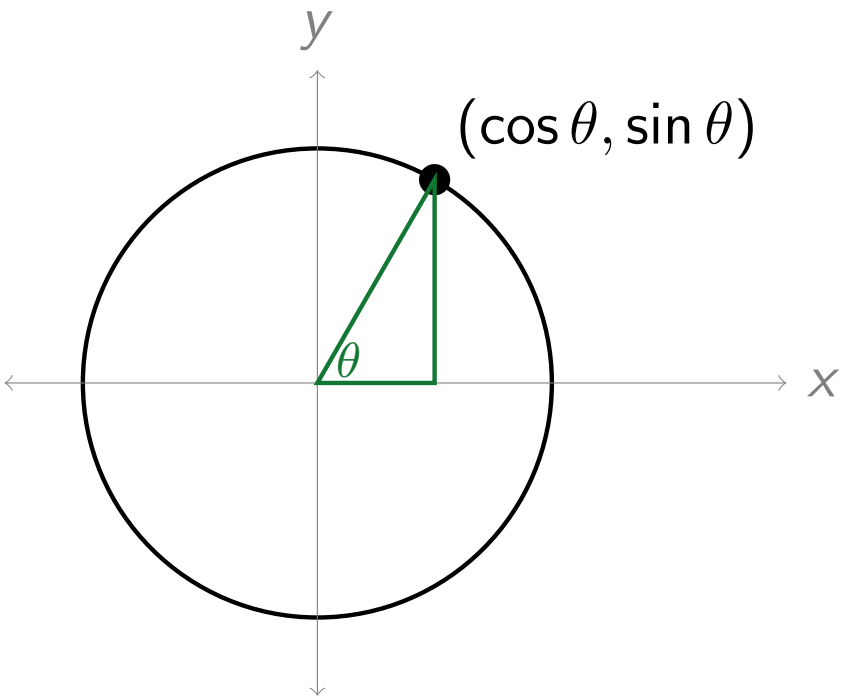
Unit circle, sines and cosines



unit circle, $r = 1$



Unit circle, sines and cosines

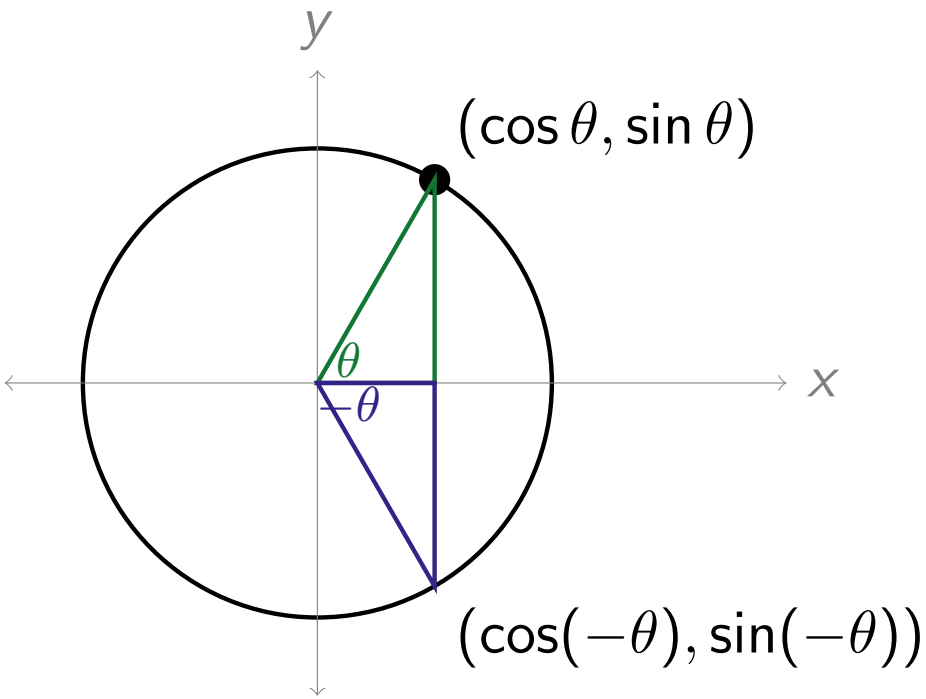


unit circle, $r = 1$

Even function: $f(-\theta) = f(\theta)$

Odd function: $f(-\theta) = -f(\theta)$

Unit circle, sines and cosines



unit circle, $r = 1$

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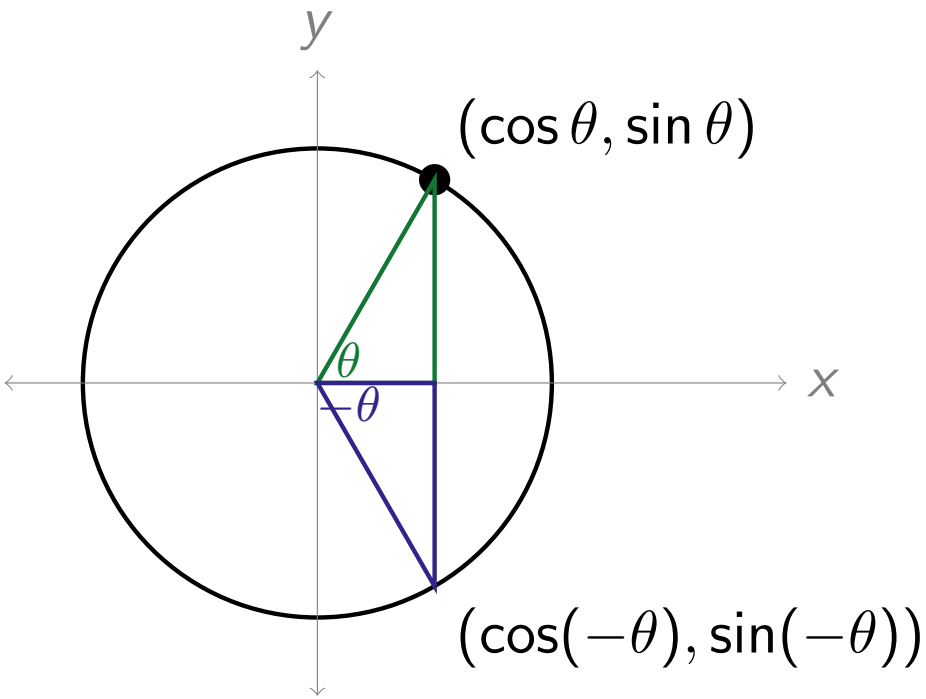
Unit circle, sines and cosines



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Cosine is an even function: $\cos(-\theta) = \cos(\theta)$.

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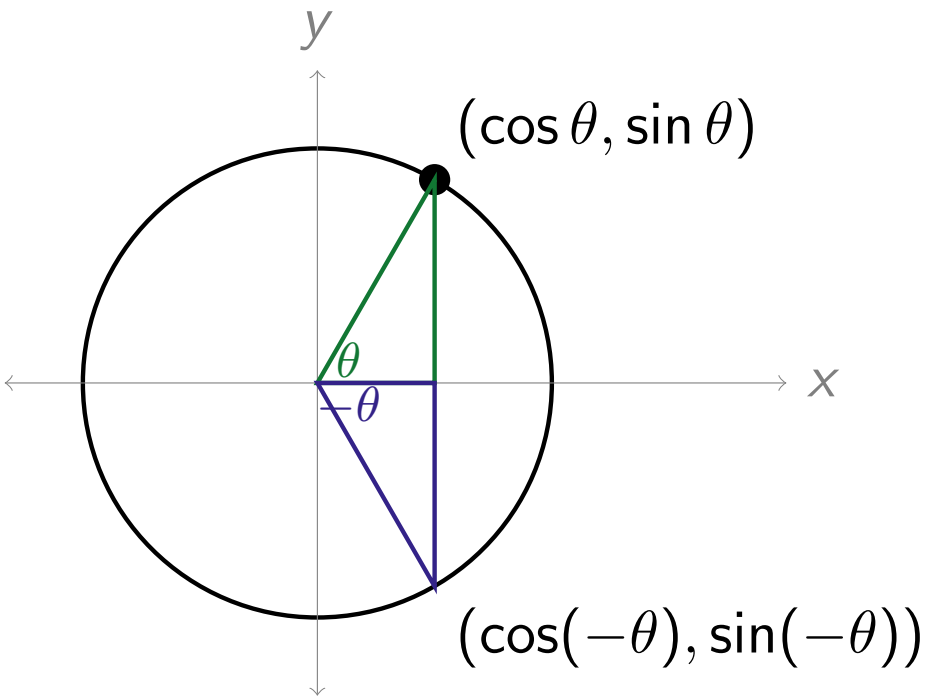

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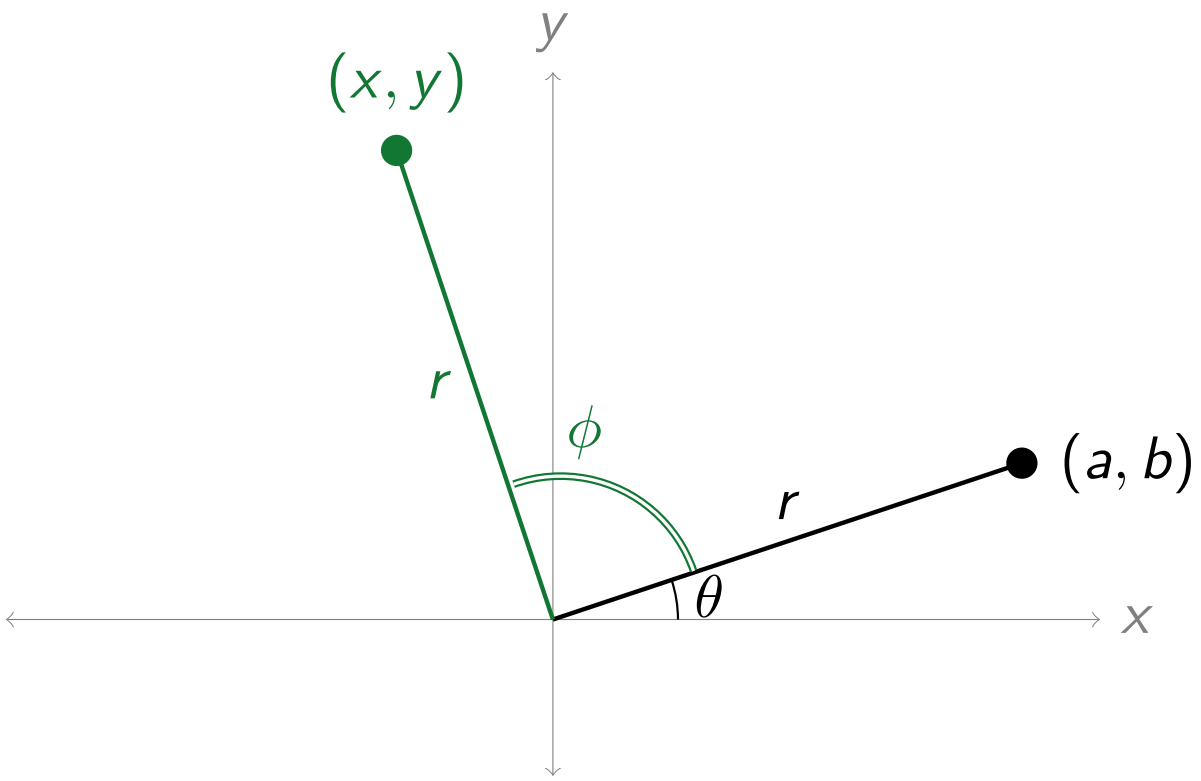
Sine is an odd function: $\sin(-\theta) = -\sin(\theta)$.

Practice

A point (a, b) in the plane is r centimetres from the origin, at an angle of θ. It is rotated ϕ radians. What is its new coordinate (x, y)?



Recall $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

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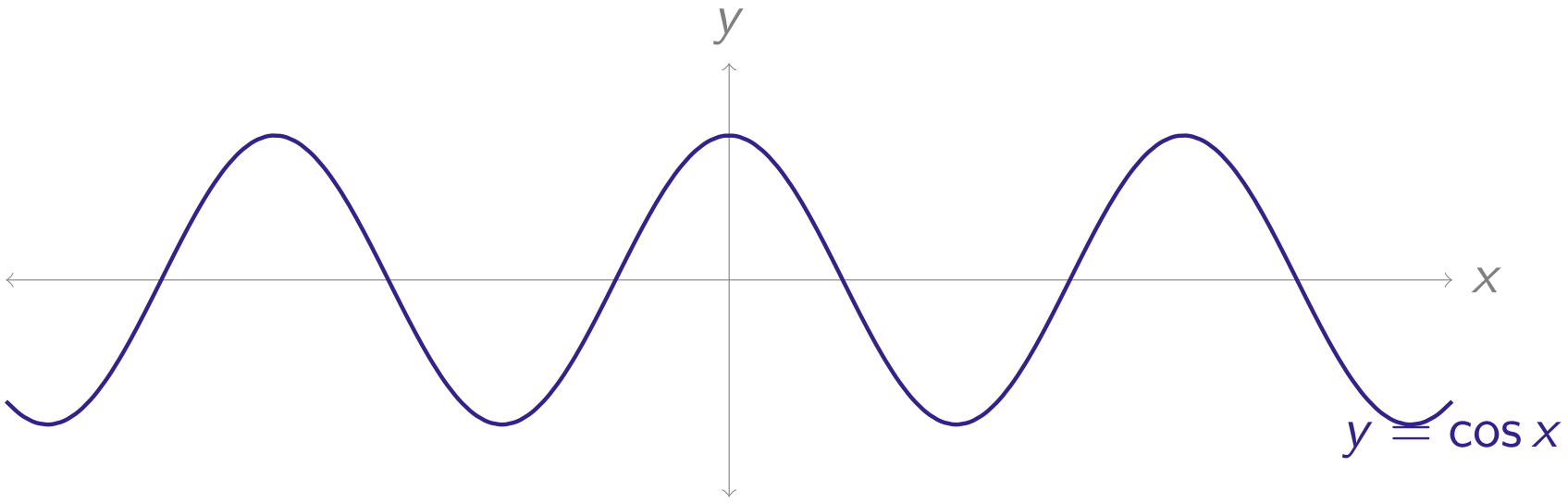
Recall $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\sin(A + B) = \sin A \cos B + \cos A \sin B$. link: explanation of 2d rotation

$$
\begin{aligned}
x &= r\cos\left(\theta + \phi\right) = r\left(\cos\theta\cos\phi - \sin\theta\sin\phi\right) \\
&= a\cos\phi - b\sin\phi \\
y &= r\left(\theta + \phi\right) = r\sin\left(\sin\theta\cos\phi + \cos\theta\sin\phi\right) \\
&= b\cos\theta + a\sin\theta
\end{aligned}
$$

Periodic functions

Sine and cosine are **periodic** functions, with **period** 2π.

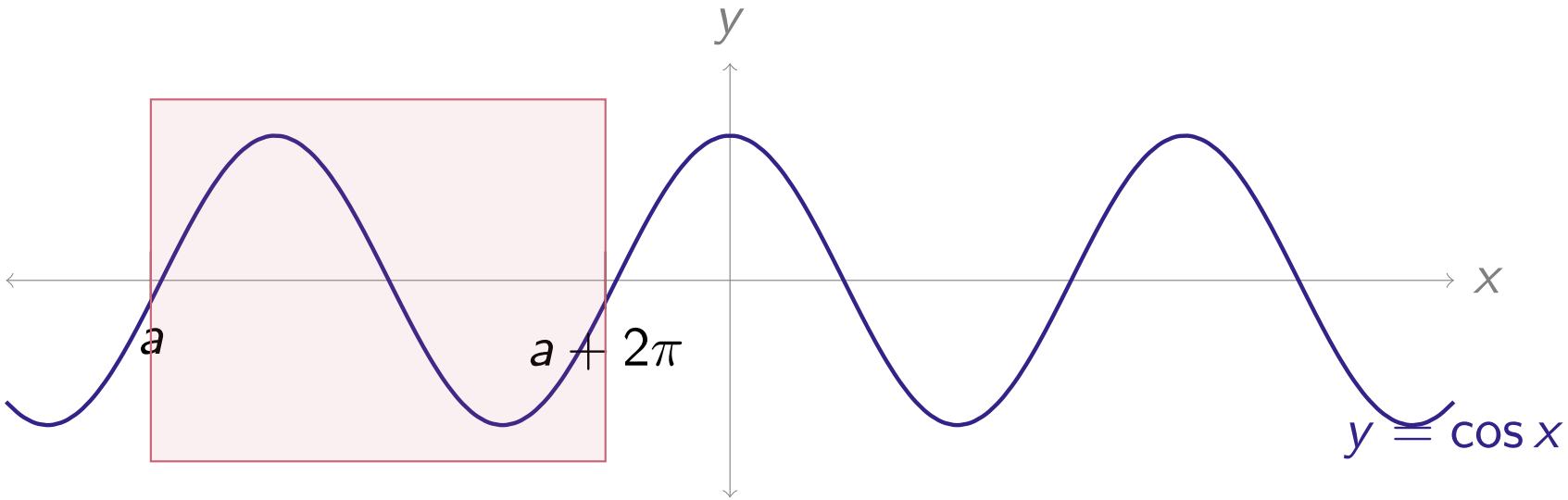


Periodic functions

Sine and cosine are **periodic** functions, with **period** 2π.
That is,

$$f(t) = f(t + 2\pi)$$

for every t.



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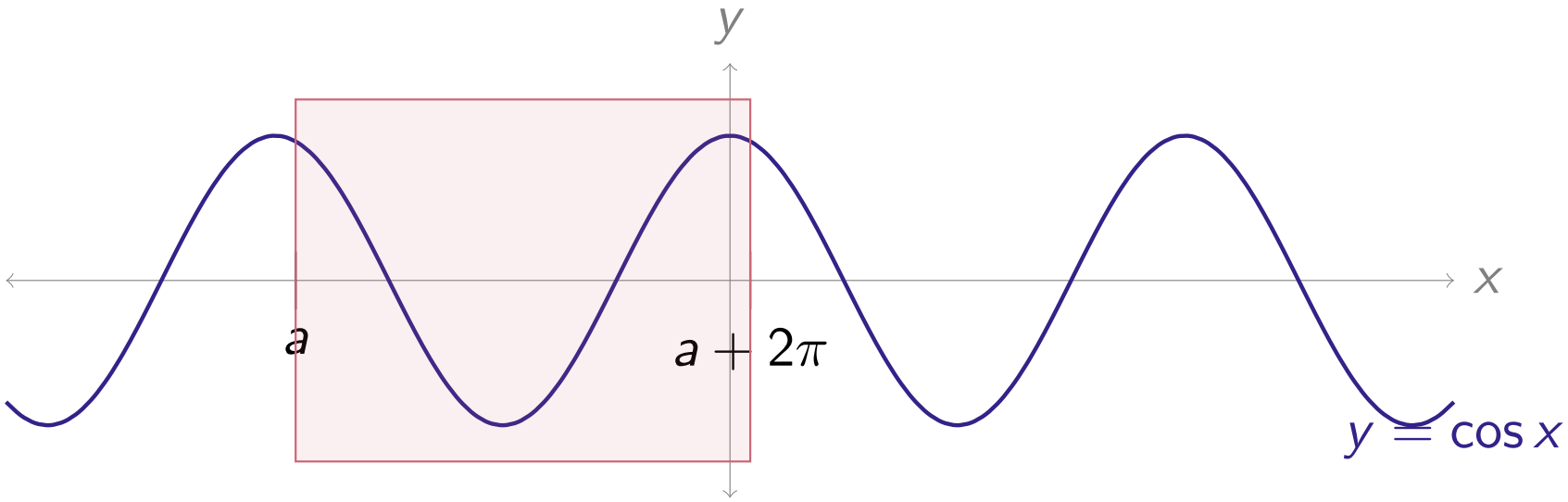

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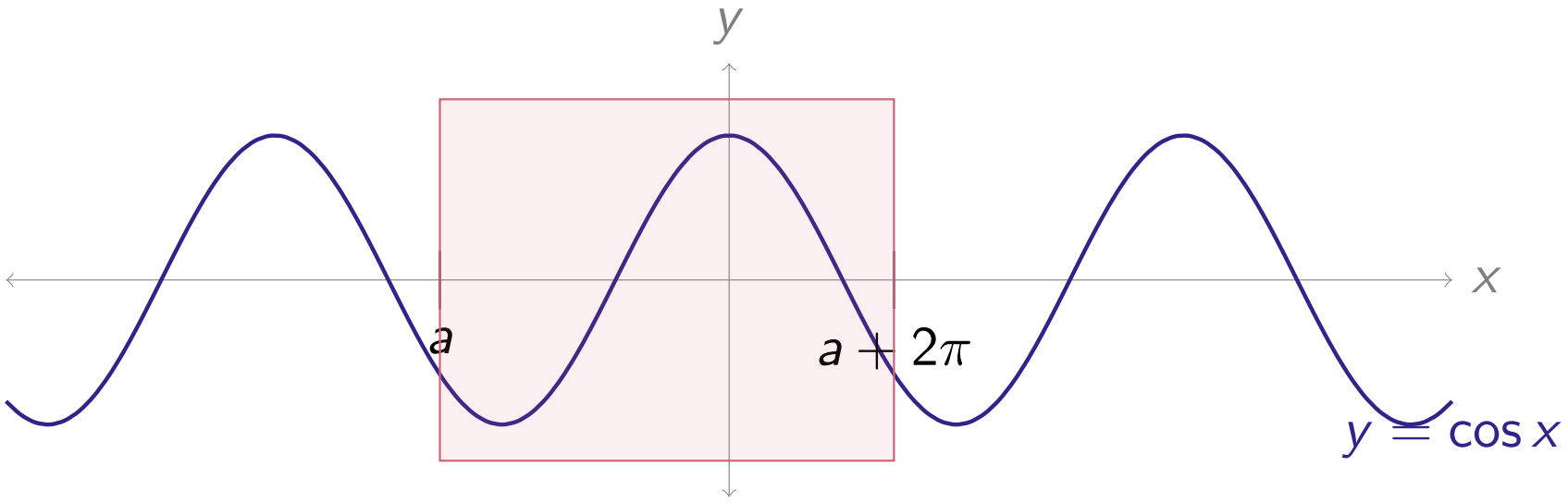

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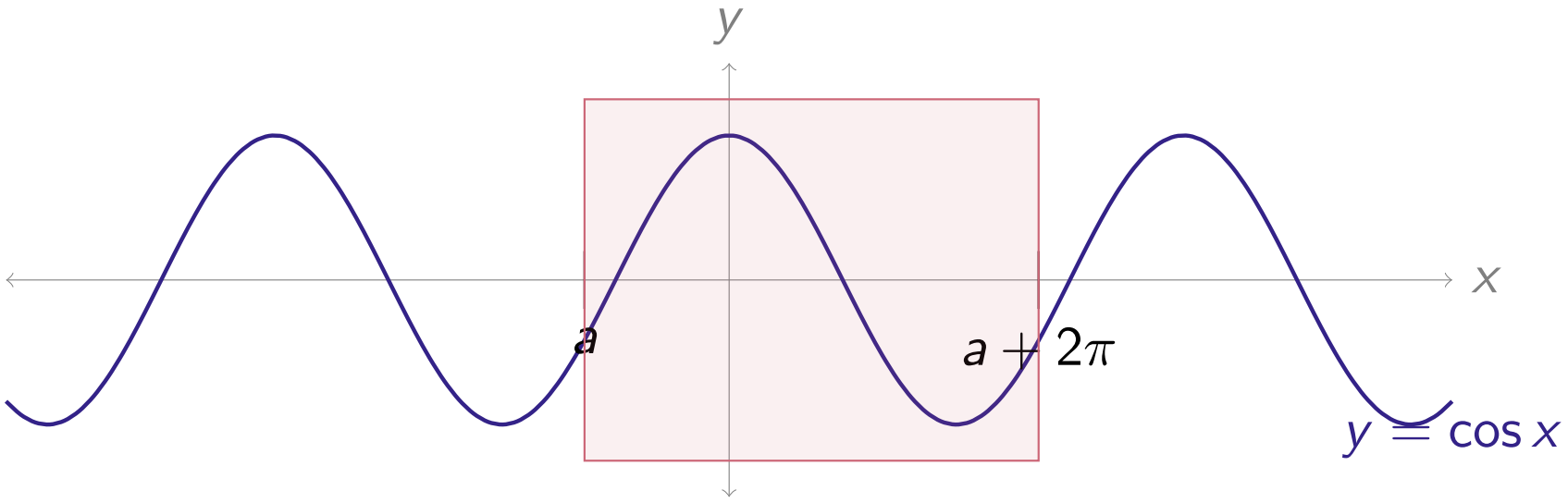

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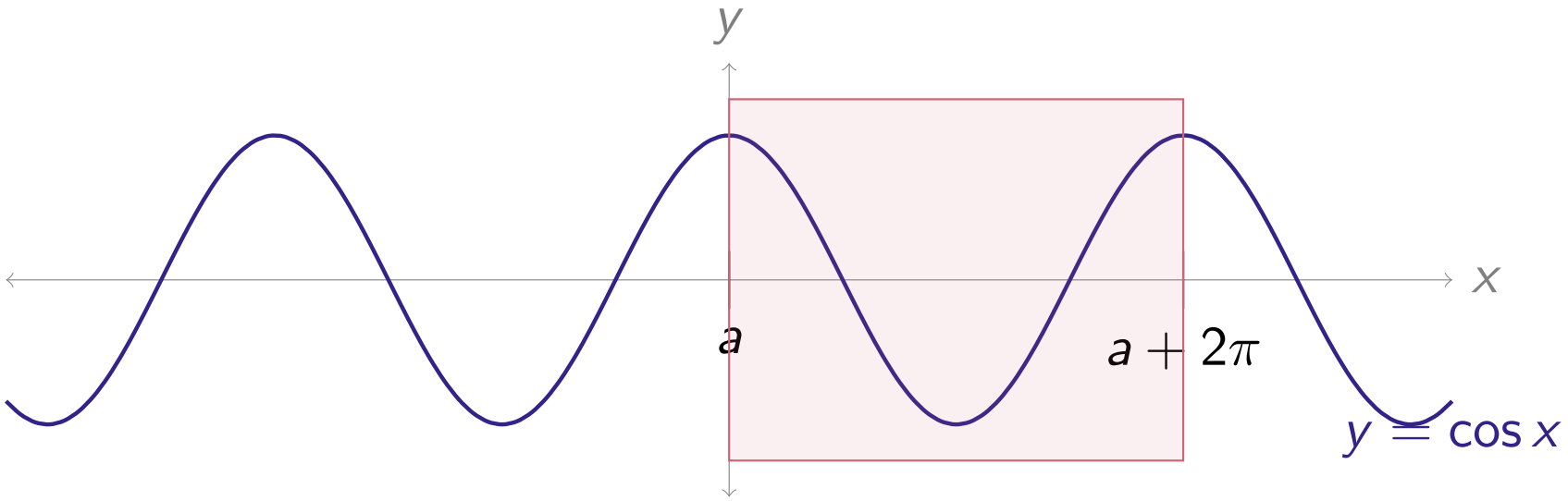
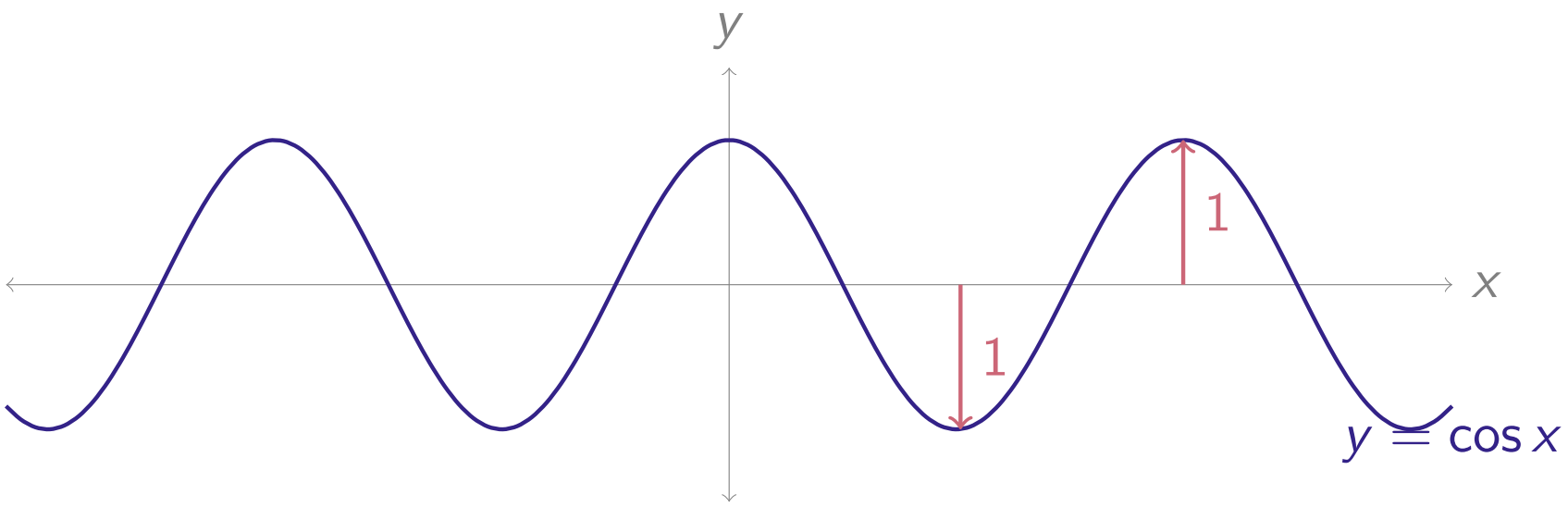
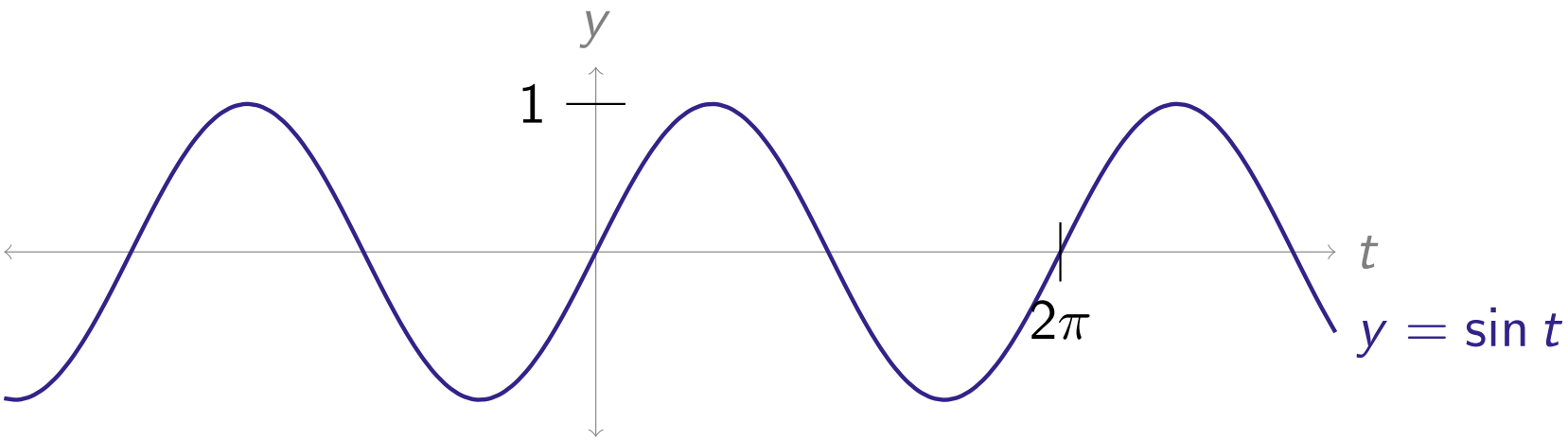


Periodic functions

Sine and cosine are have **amplitude** of 1, because the range of values they take on is from $(0 - 1)$ to $(0 + 1)$.



Transforming Periodic Functions



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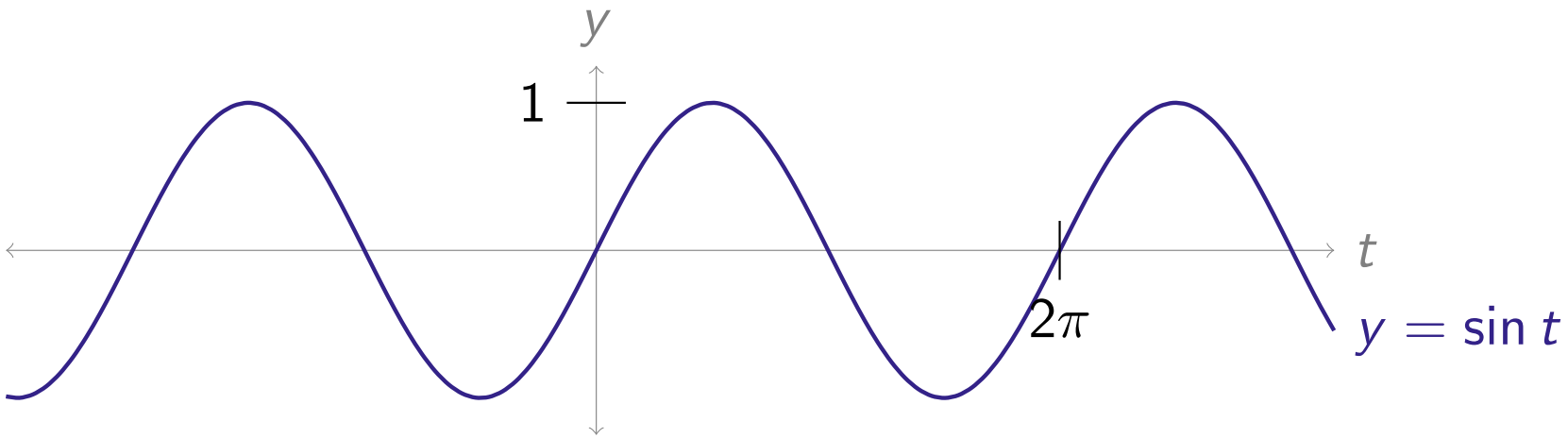

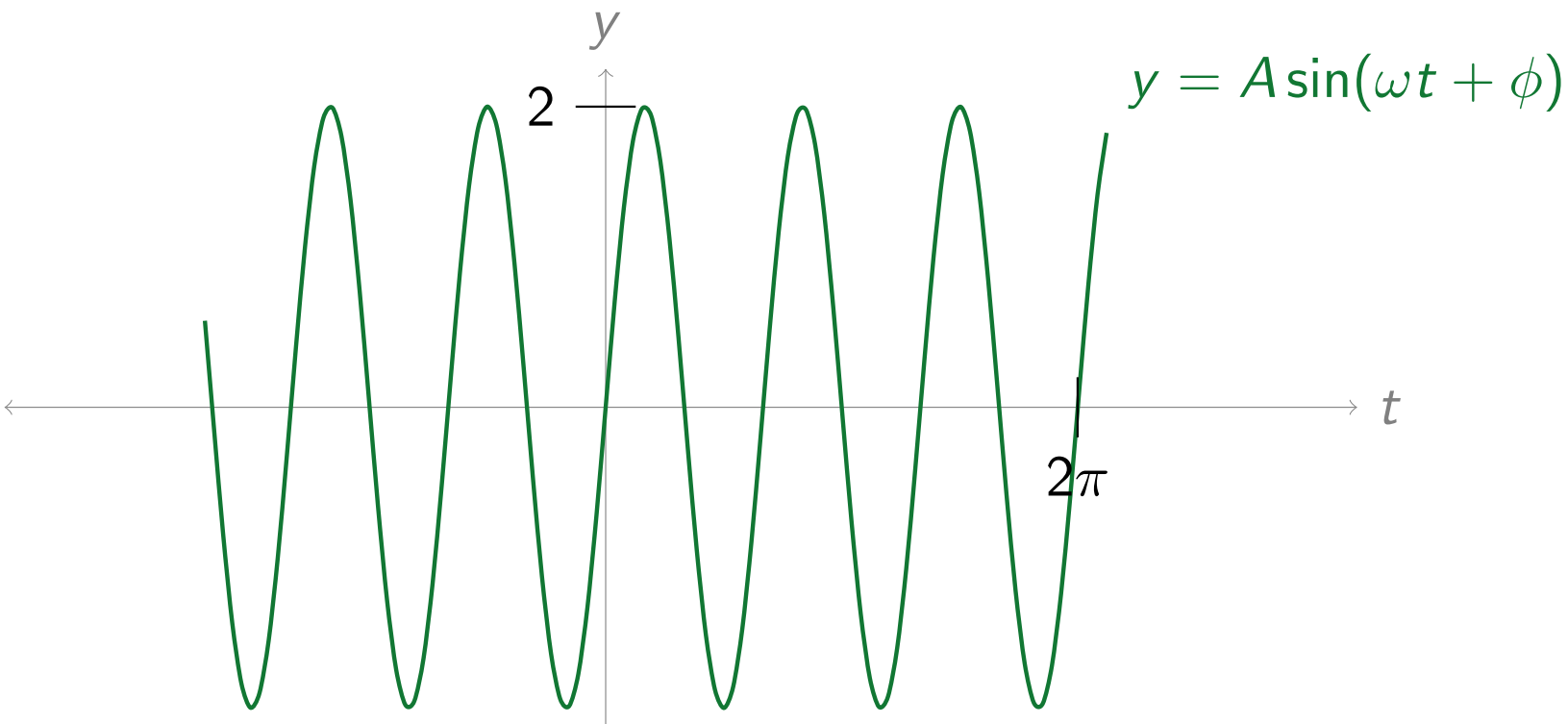

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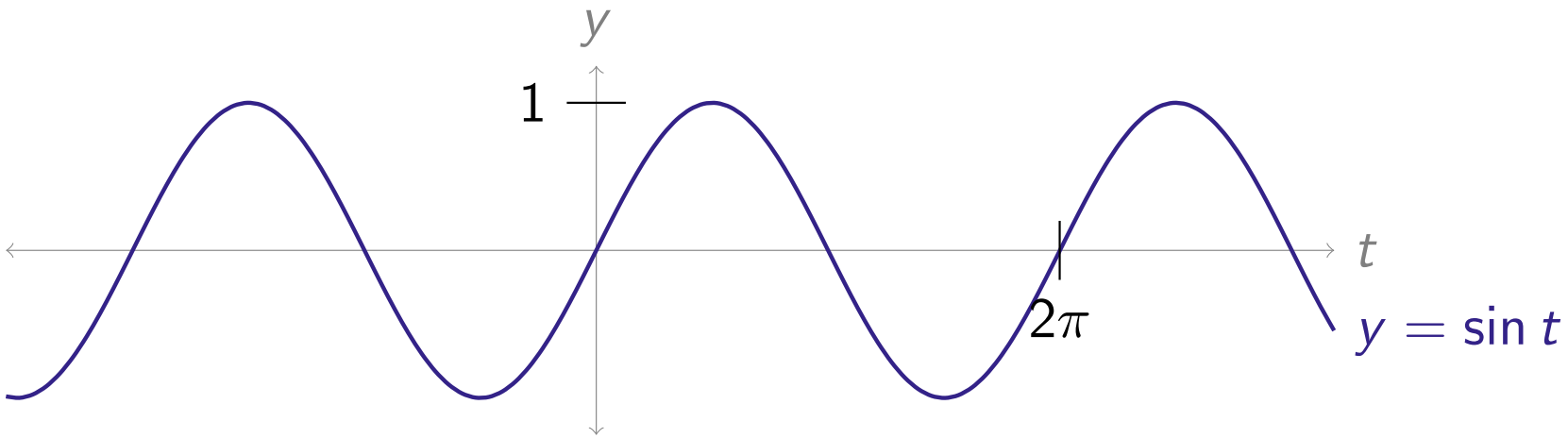
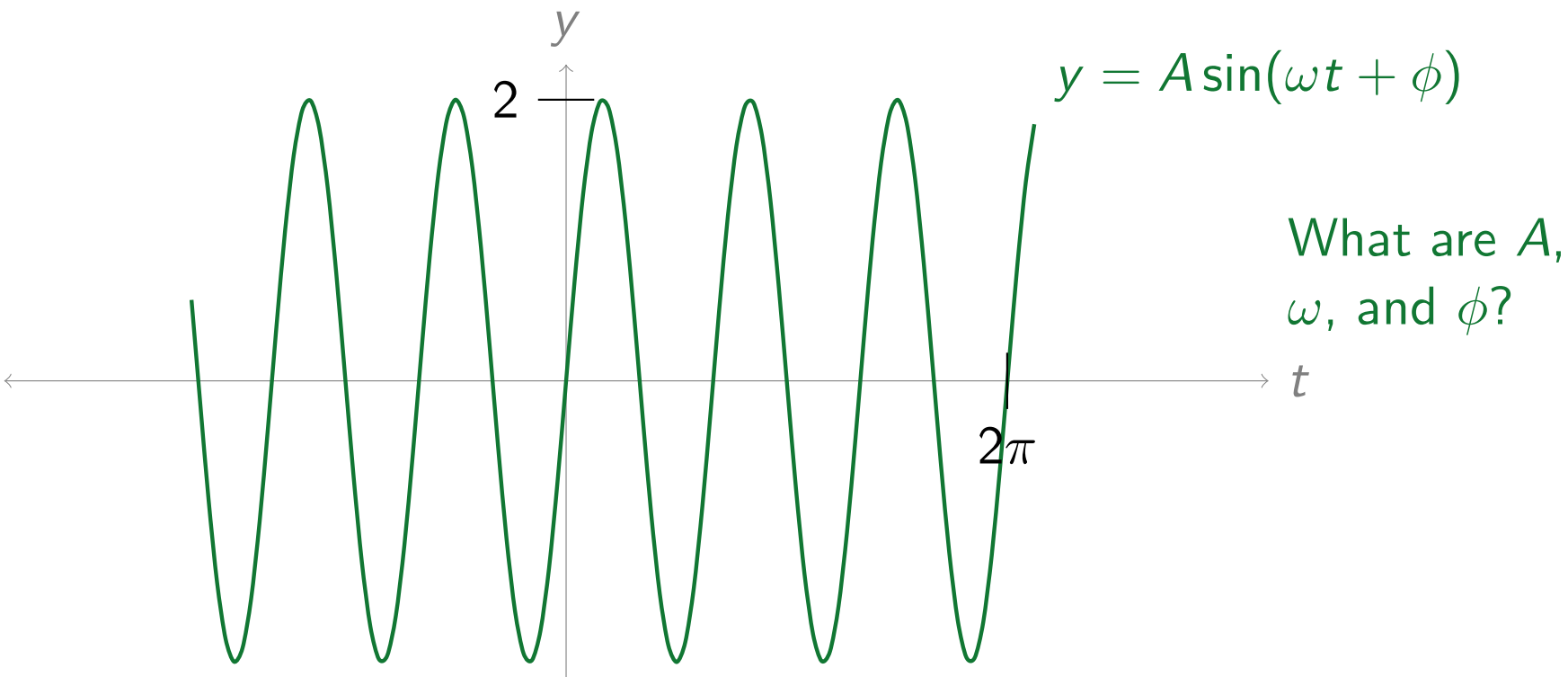
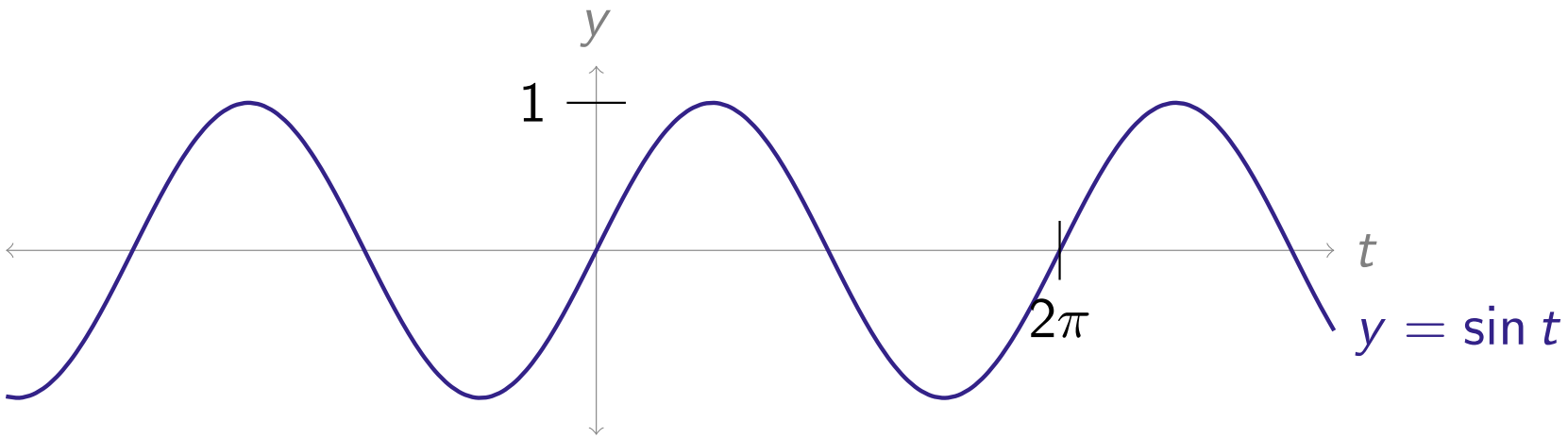
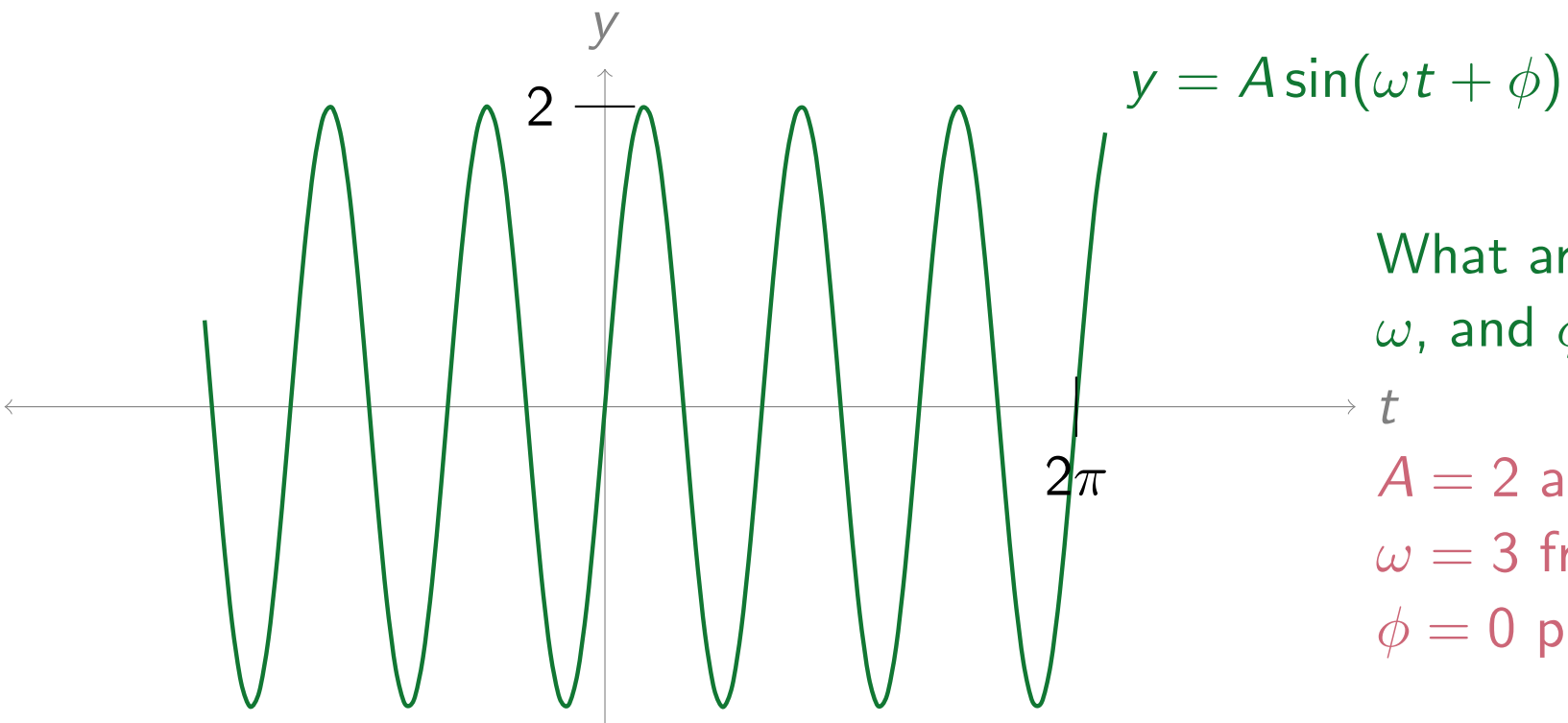




What are A, ω, and ϕ?

Transforming Periodic Functions





What are A, ω, and ϕ?

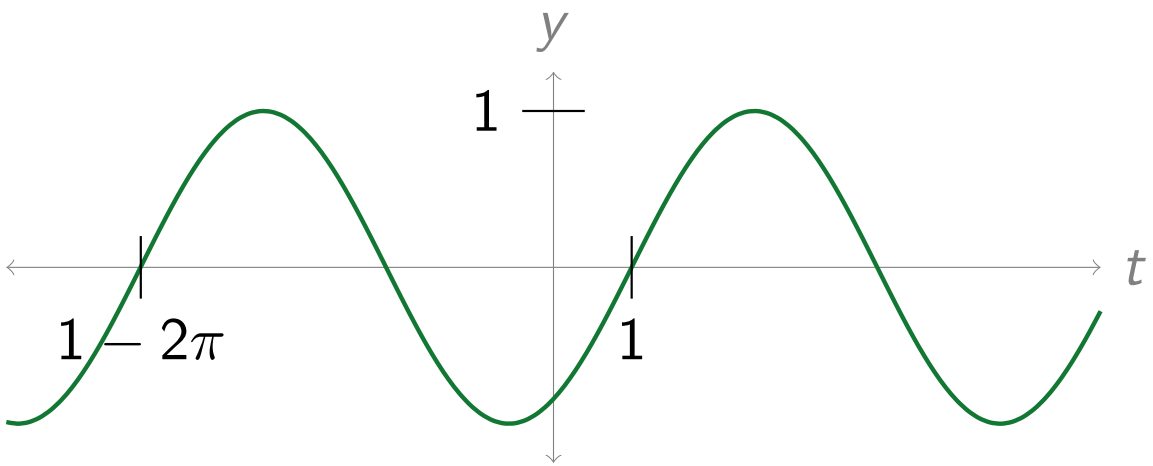
$A = 2$ amplitude

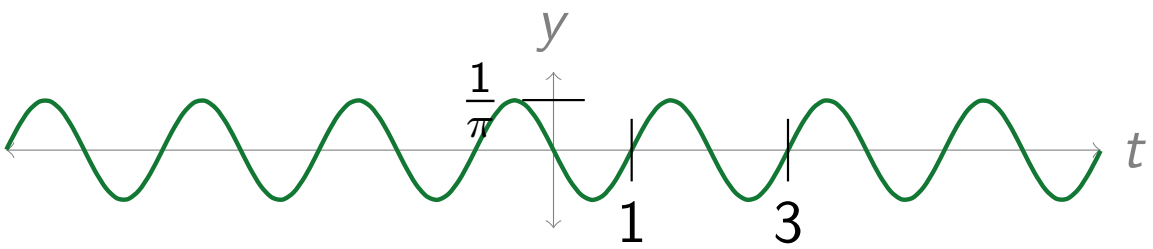
$\omega = 3$ frequency

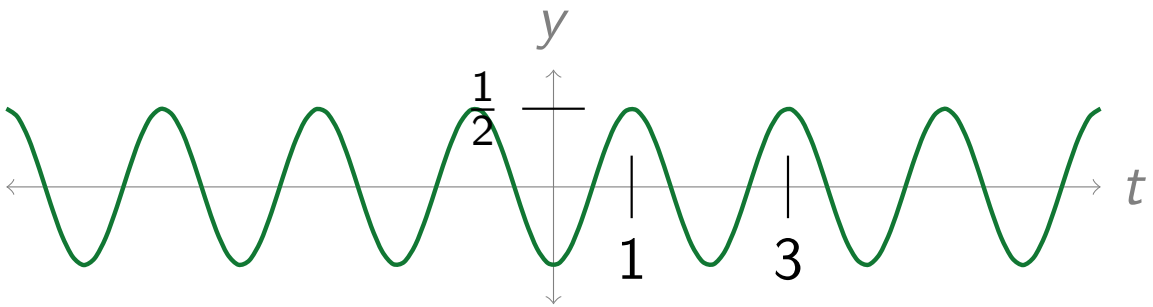
$\phi = 0$ phase shift

Transforming Periodic Functions

Example 1: Write each function below in the form $y = A\sin(\omega t + \phi)$.







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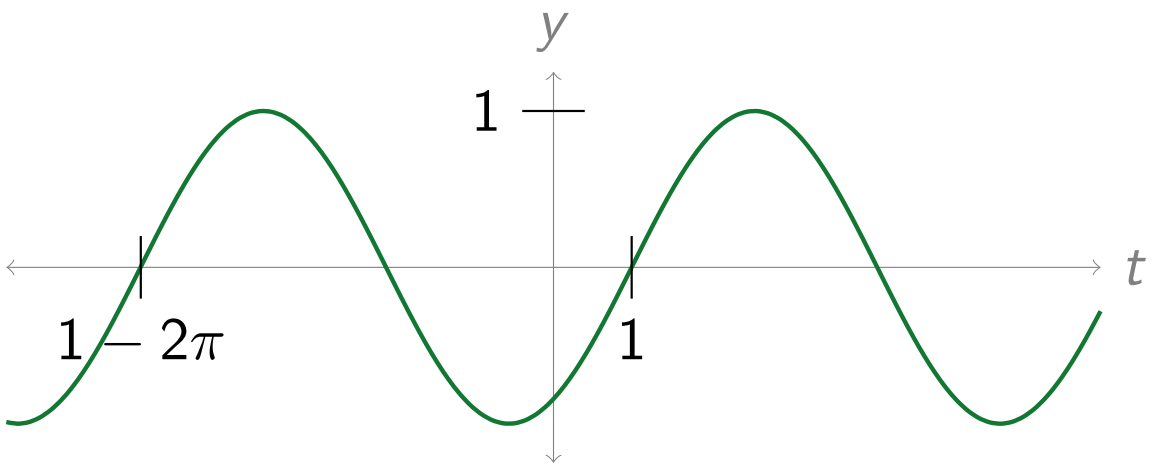
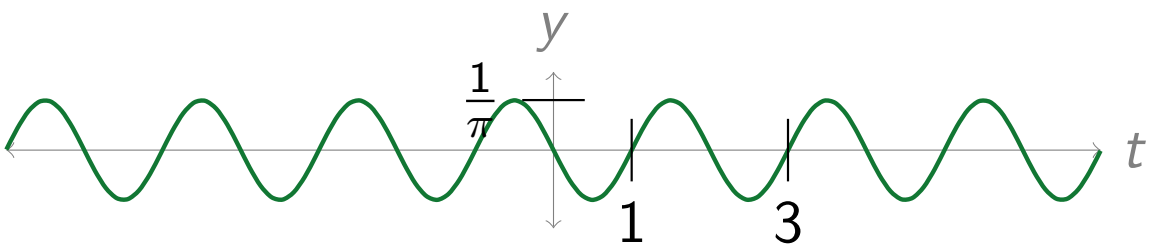
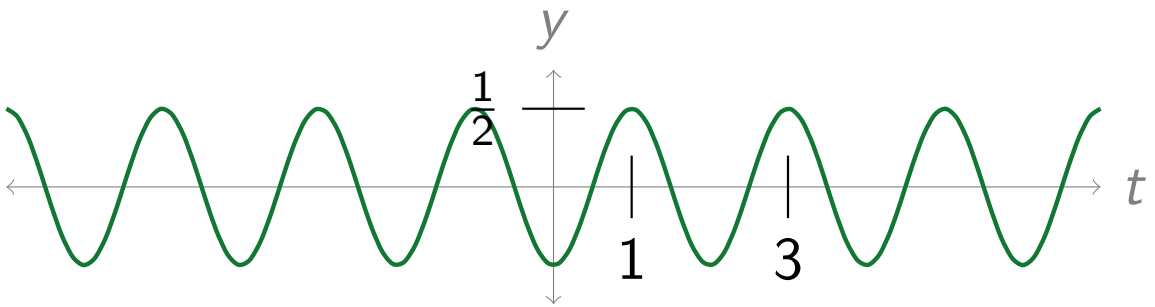


$y = \sin(t - 1)$





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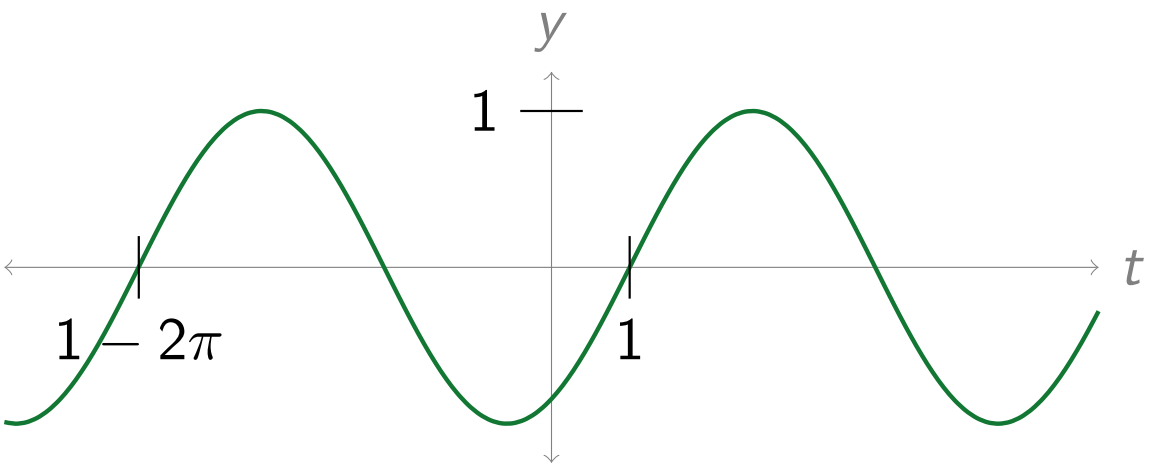

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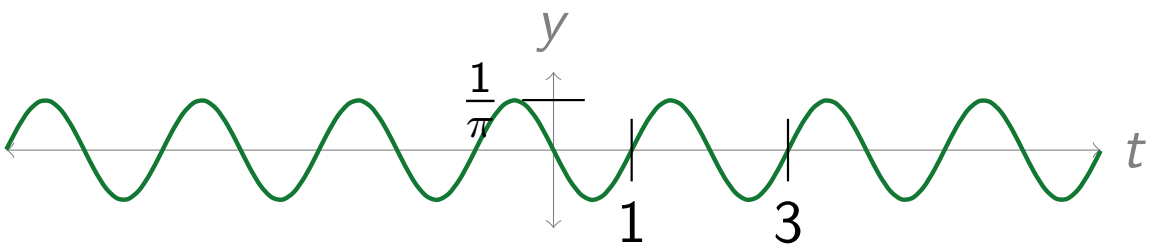

$y = -\frac{1}{\pi}\sin(\pi t)$ or
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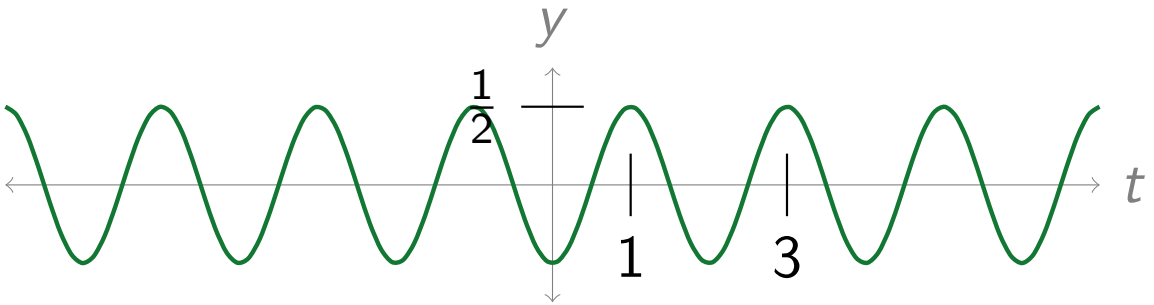

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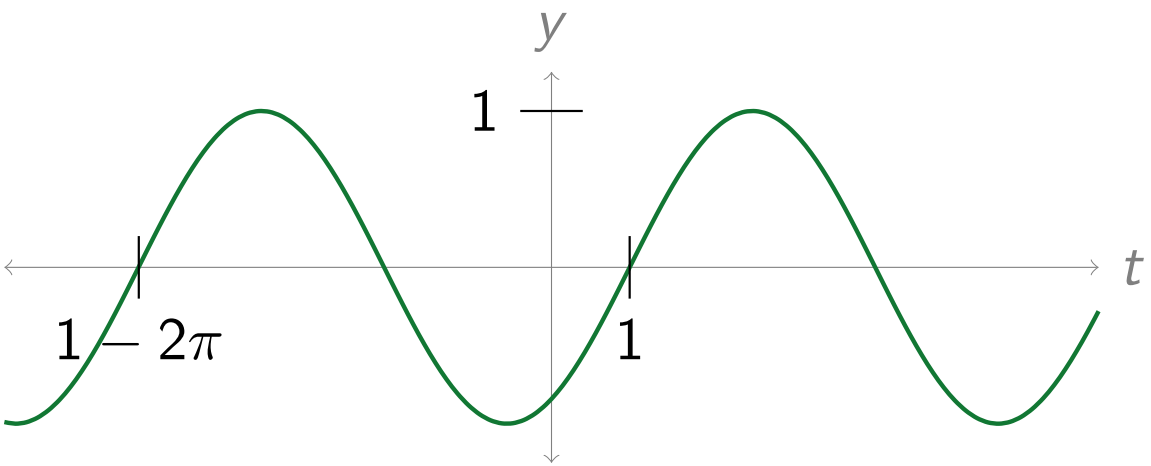

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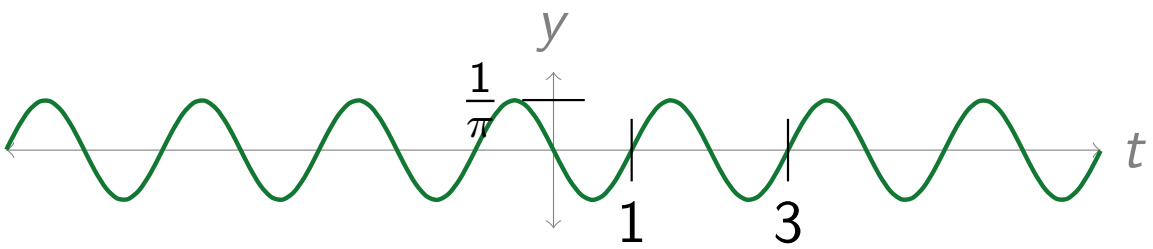

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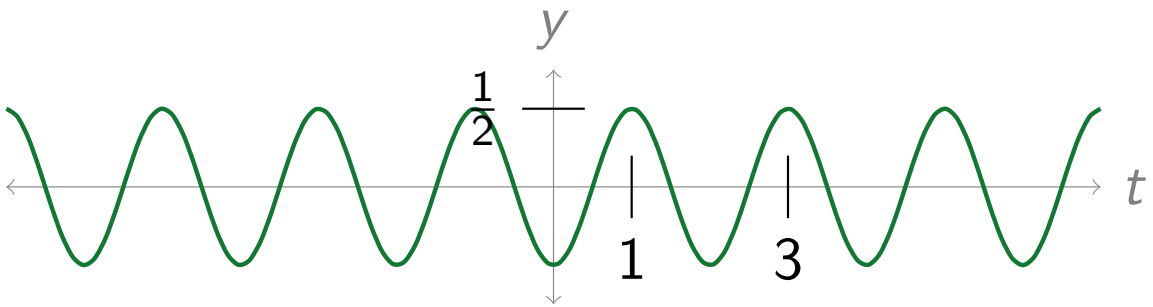

$y = \frac{1}{2}\sin\left(\pi t - \frac{\pi}{2}\right)$ or
$y = -\frac{1}{2}\sin\left(\pi t + \frac{\pi}{2}\right)$

Example 2: Sketch

(a) $y = \frac{1}{2}\sin(x)$

(b) $y = \frac{1}{2}\sin(2\pi x)$

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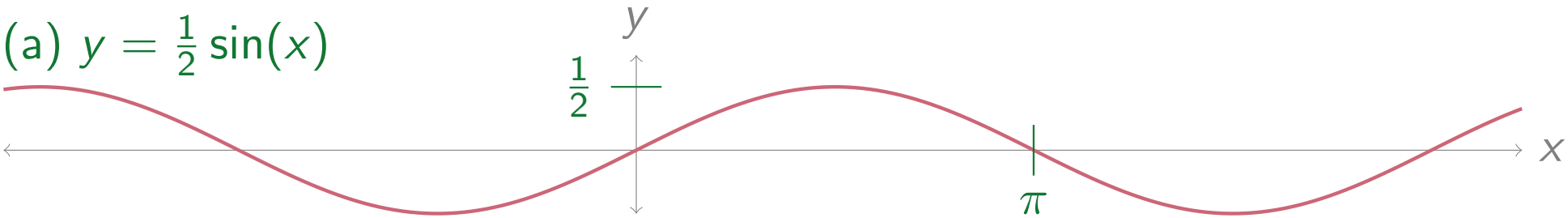

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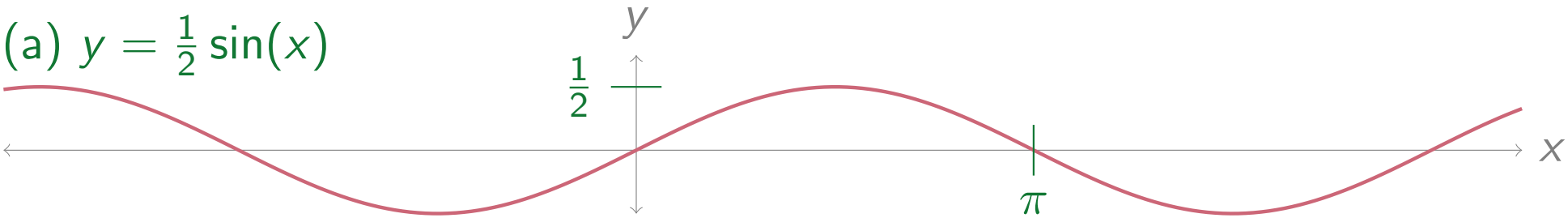

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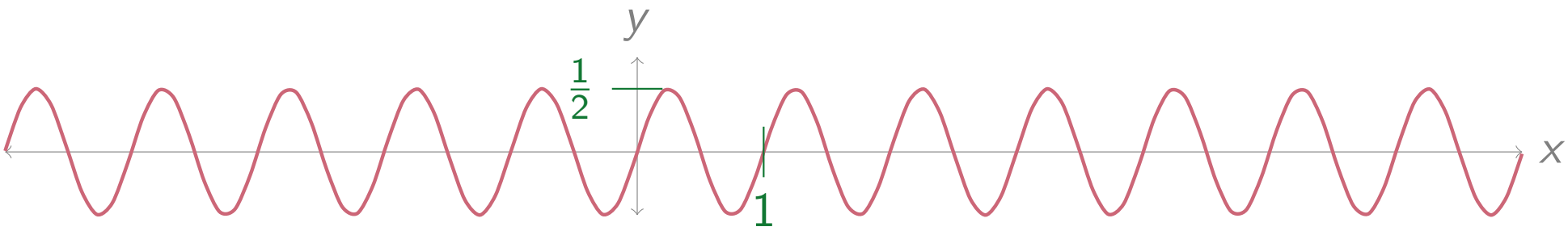

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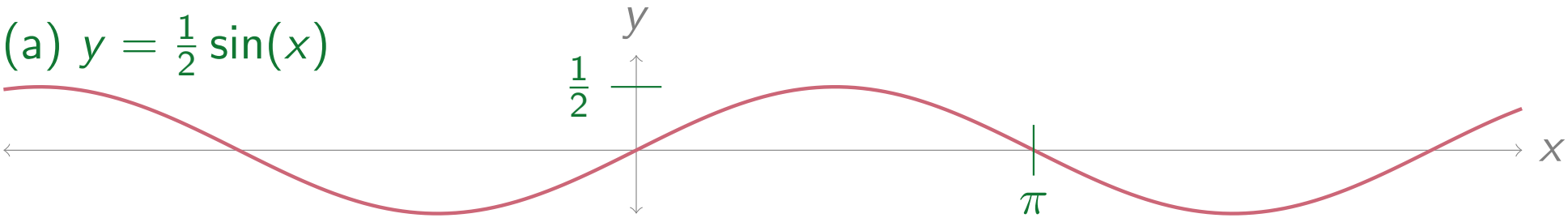

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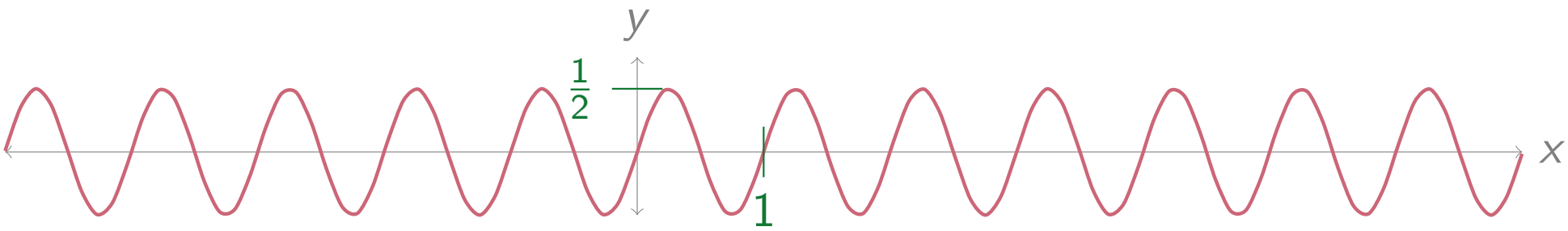

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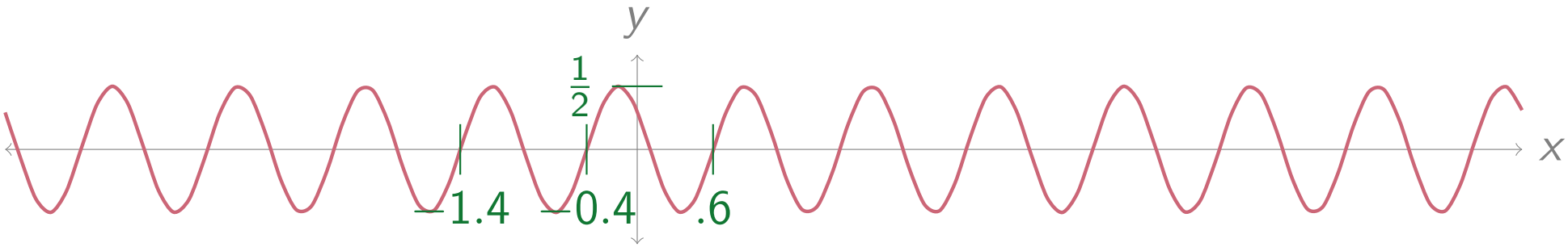
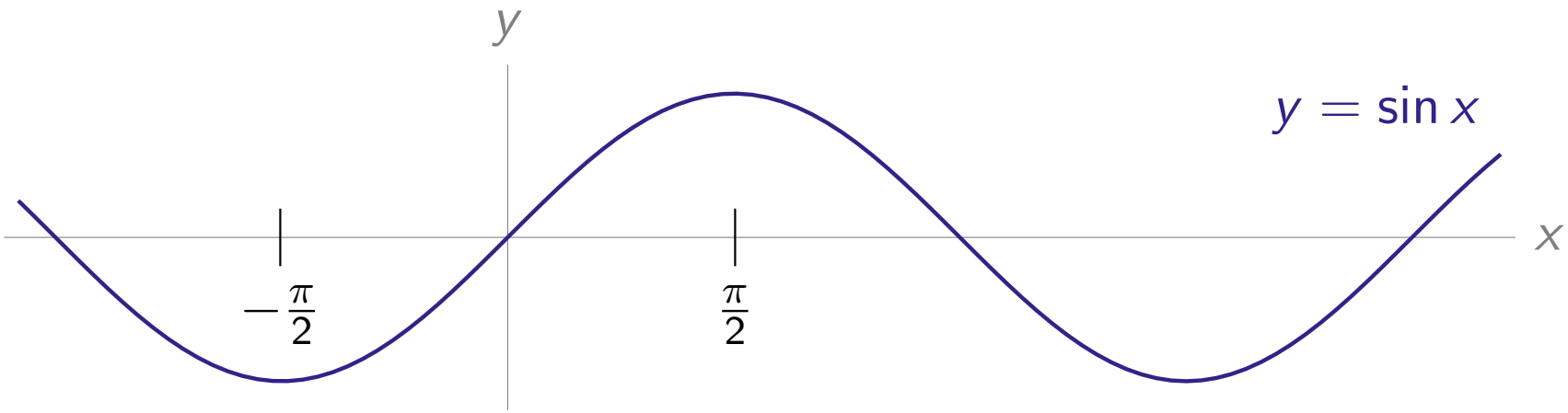


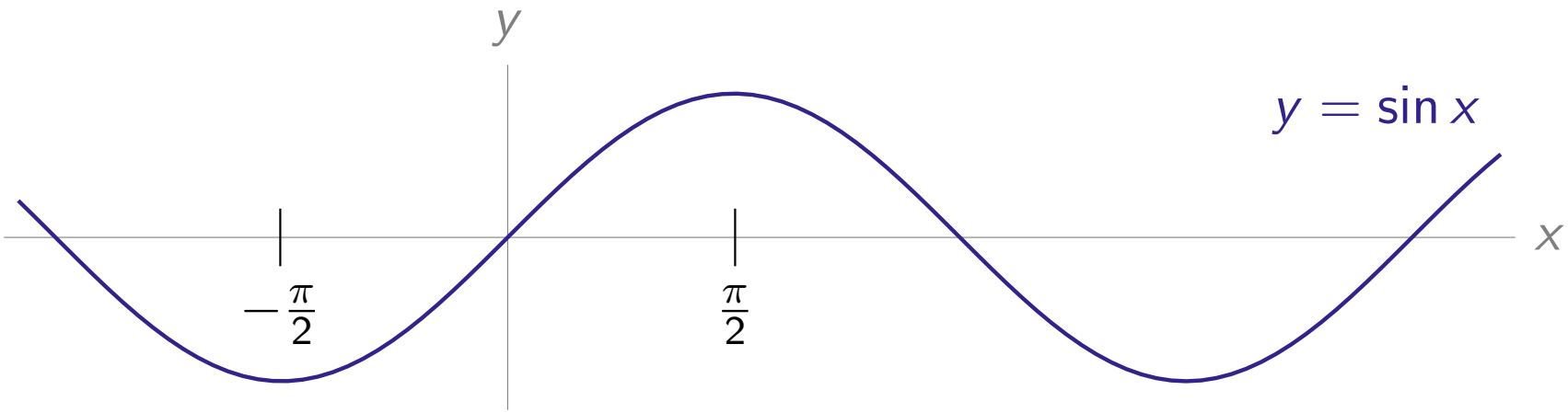
arcsine



Who wins at the inverse game?

- I'm thinking of an x, and $\sin x = 0$. What is my x?
- I'm thinking of an x, and $\sin x = \frac{1}{\sqrt{2}}$. What is my x?

arcsine



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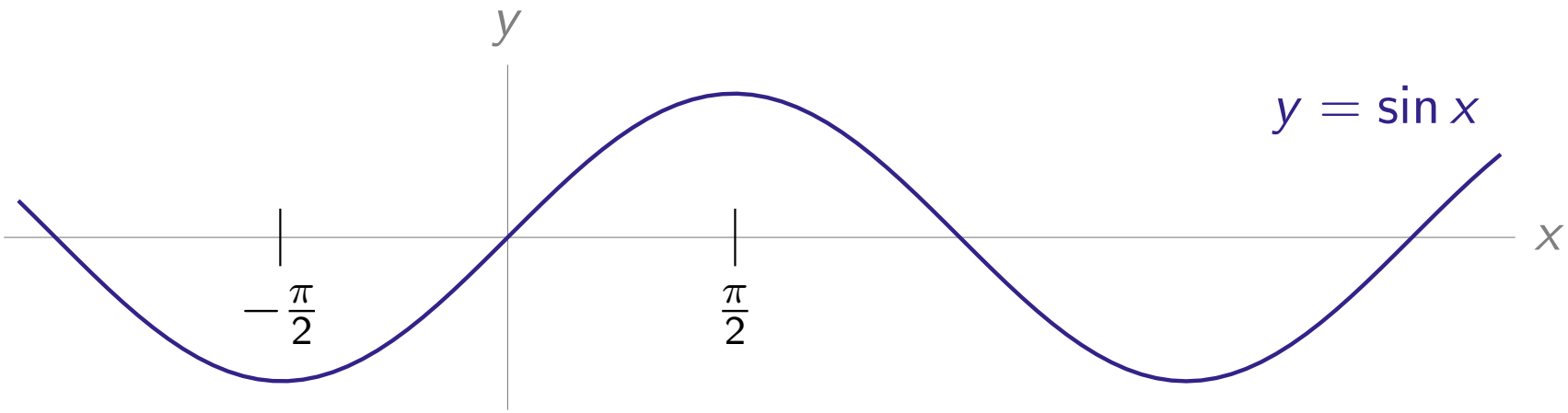

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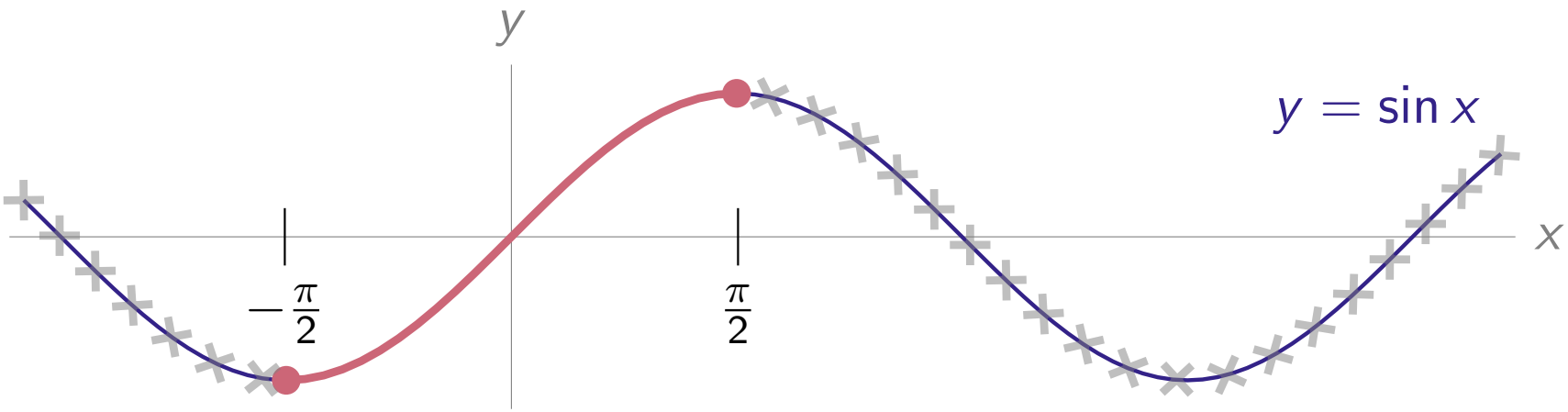
arcsine



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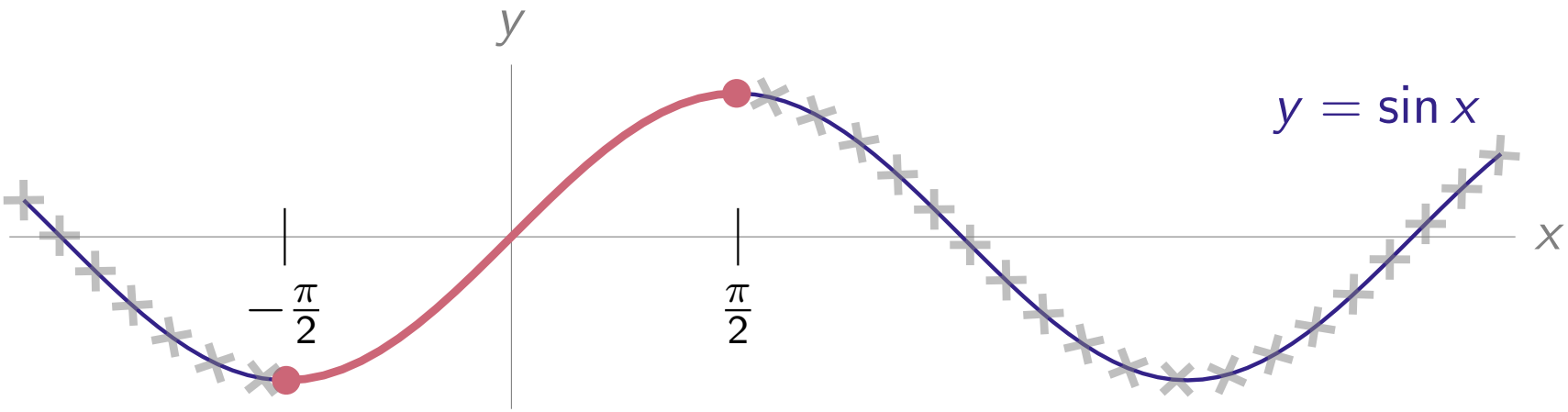

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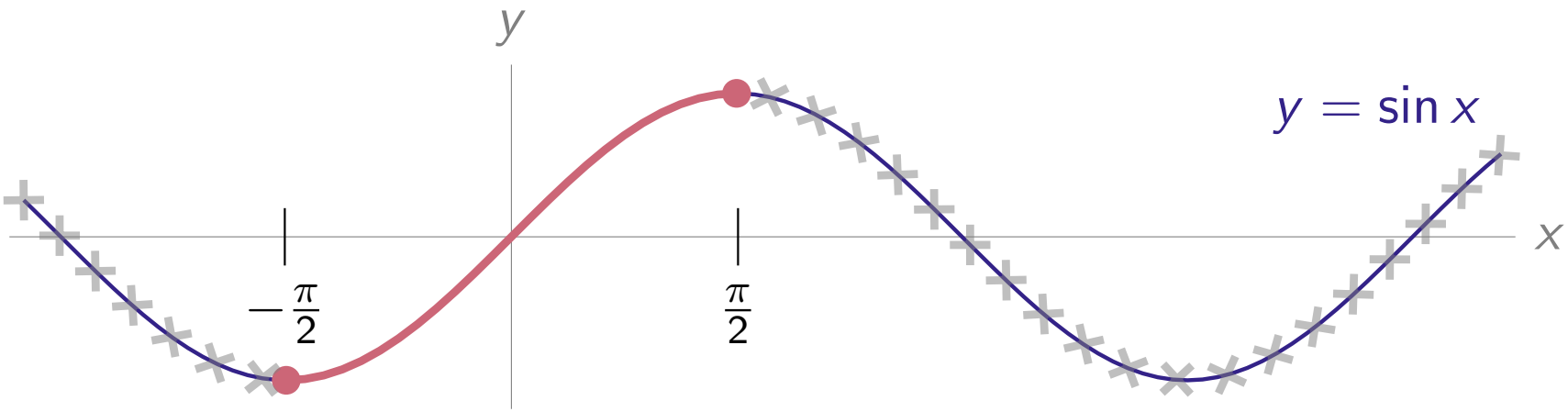

The function $f(x) = \sin(x)$ is invertible over the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and this is the domain we use to define $\arcsin(x)$.

$\arcsin(x)$ gives the number y such that:
(1) $\sin(y) = x$ and
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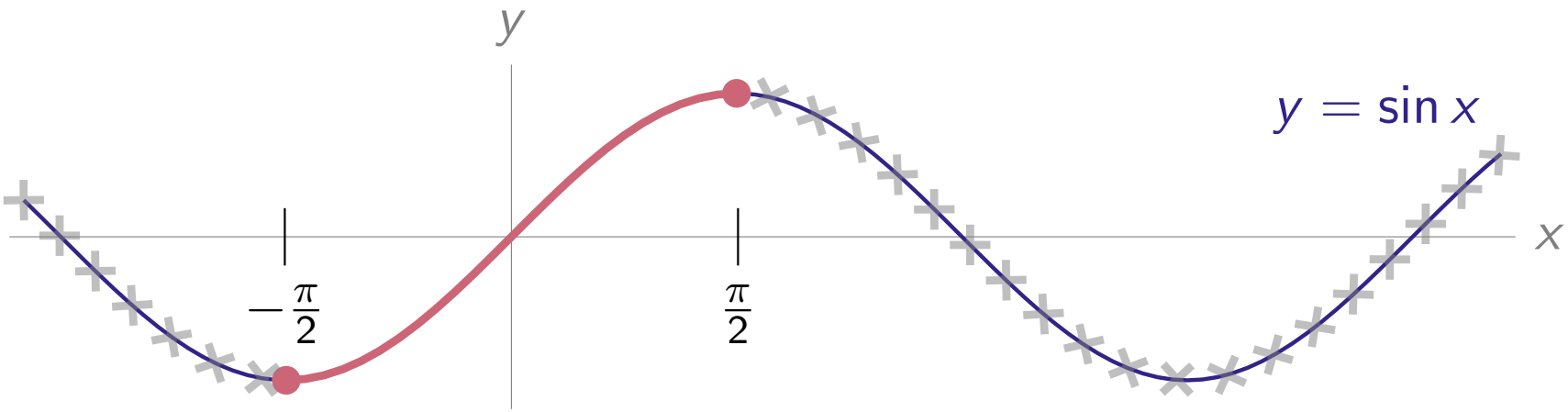
arcsine



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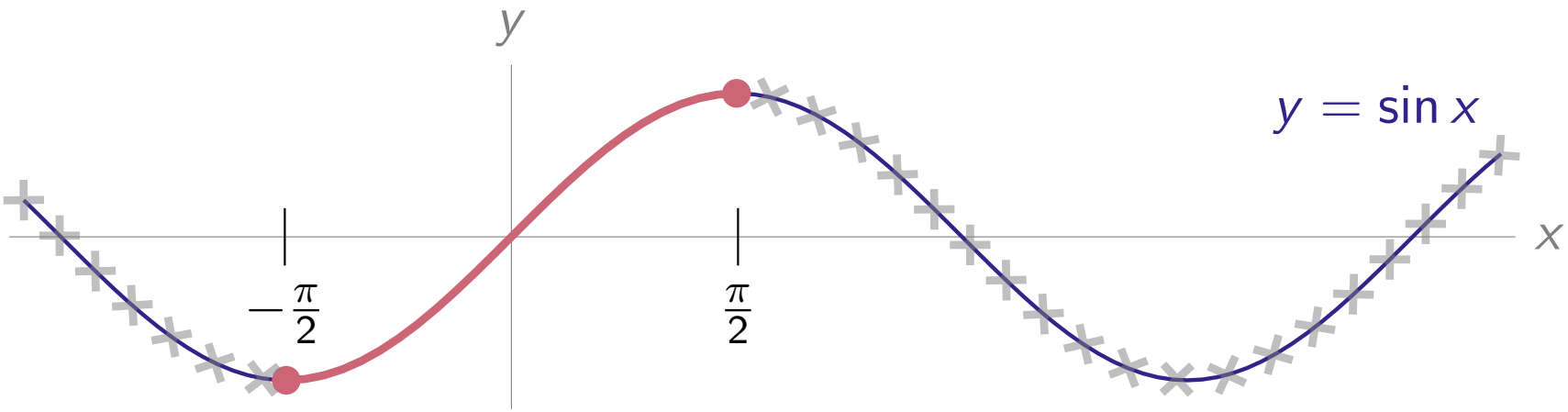
arcsine



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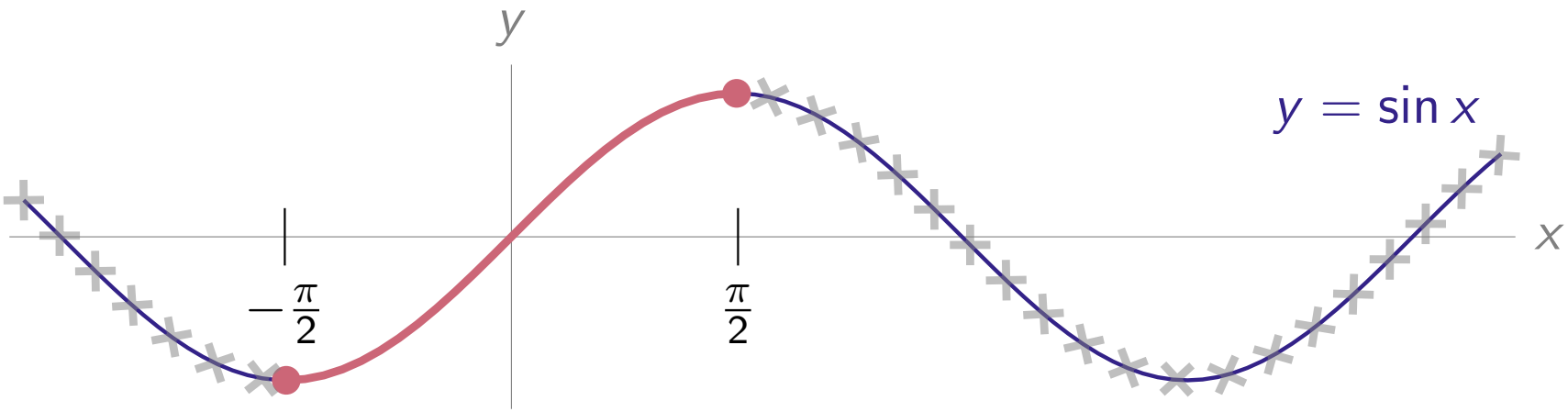
arcsine



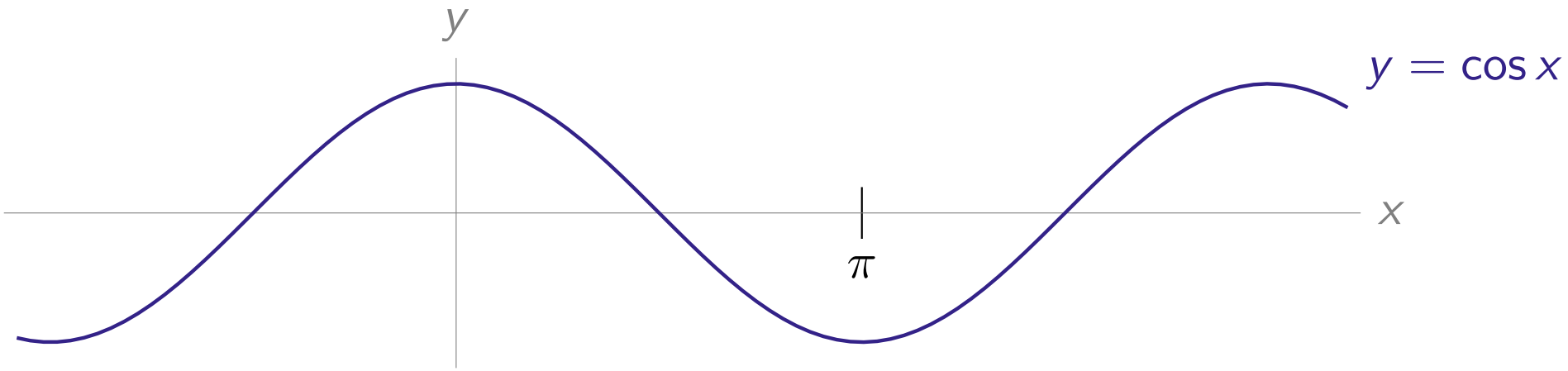
The function $f(x) = \sin(x)$ is invertible over the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and this is the domain we use to define $\arcsin(x)$.

$\arcsin(x)$ gives the number y such that:
(1) $\sin(y) = x$ and
(2) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

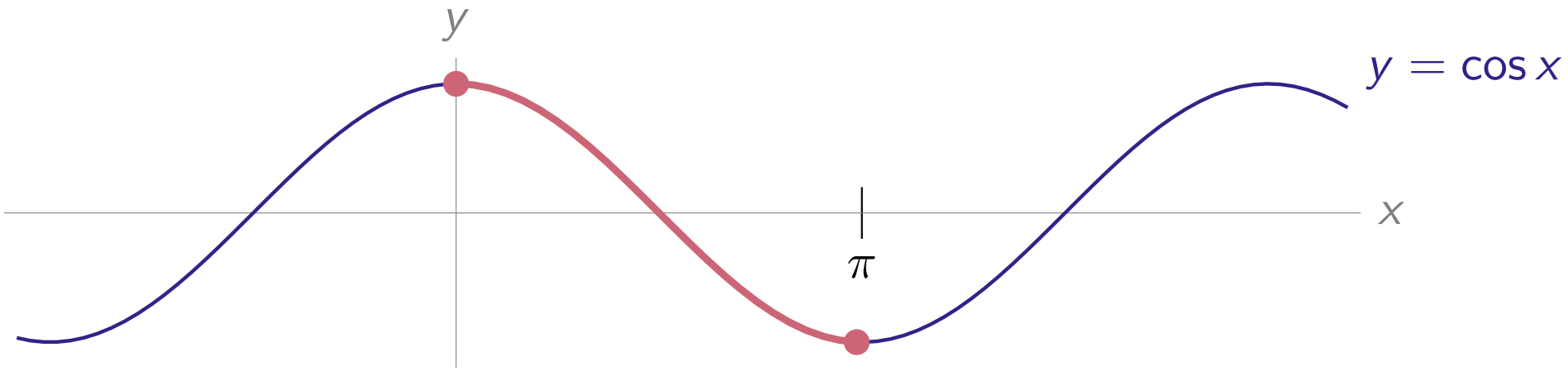
What is $\arcsin(\sin 0)$? 0

What is $\arcsin\left(\sin\left(\frac{3\pi}{2}\right)\right)$? $-\frac{\pi}{2}$

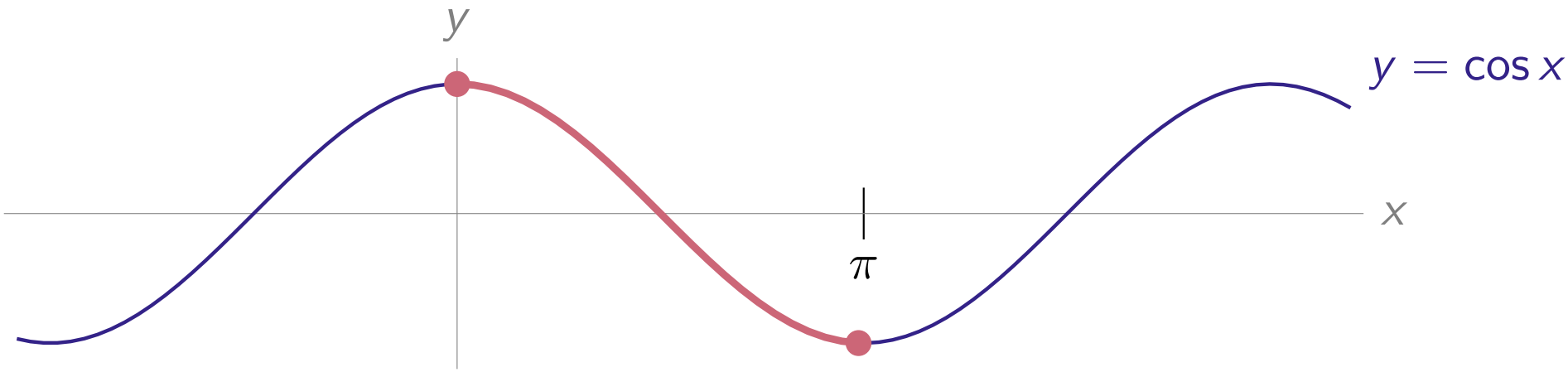
arccosine



arccosine



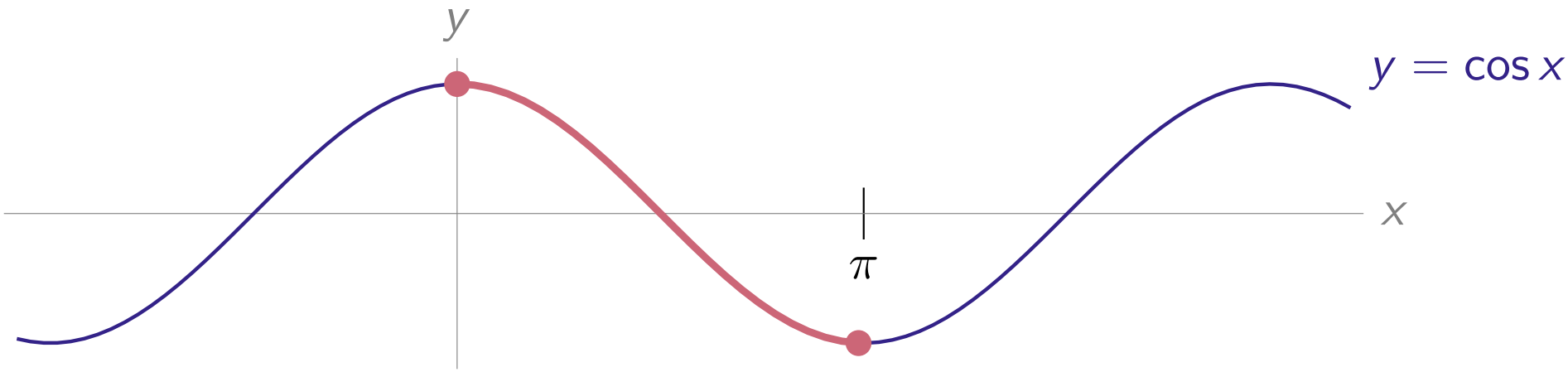
arccosine



The function $f(x) = \cos(x)$ is invertible over the domain $[0, \pi]$, and this is the domain we use to define $\arccos(x)$.

$\arccos(x)$ gives the number y such that:
(1) $\cos(y) = x$ and
(2) $0 \leq y \leq \pi$

arccosine



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$\arccos(x)$ gives the number y such that:
(1) $\cos(y) = x$ and
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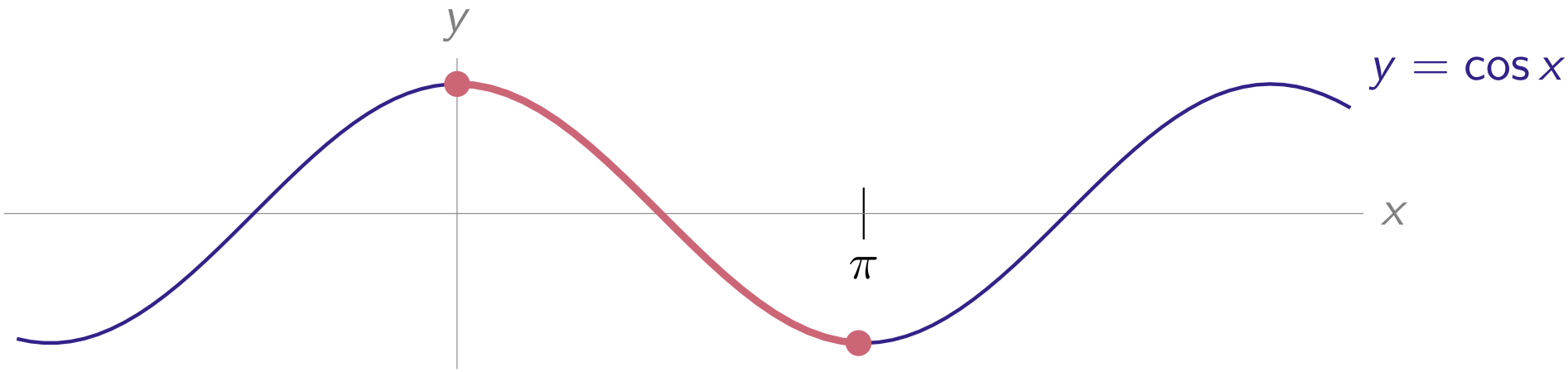
$\arccos\left(\cos \frac{5\pi}{4}\right) =$

$\tan\left(\arccos x\right) =$ (hint: draw a triangle)

arccosine



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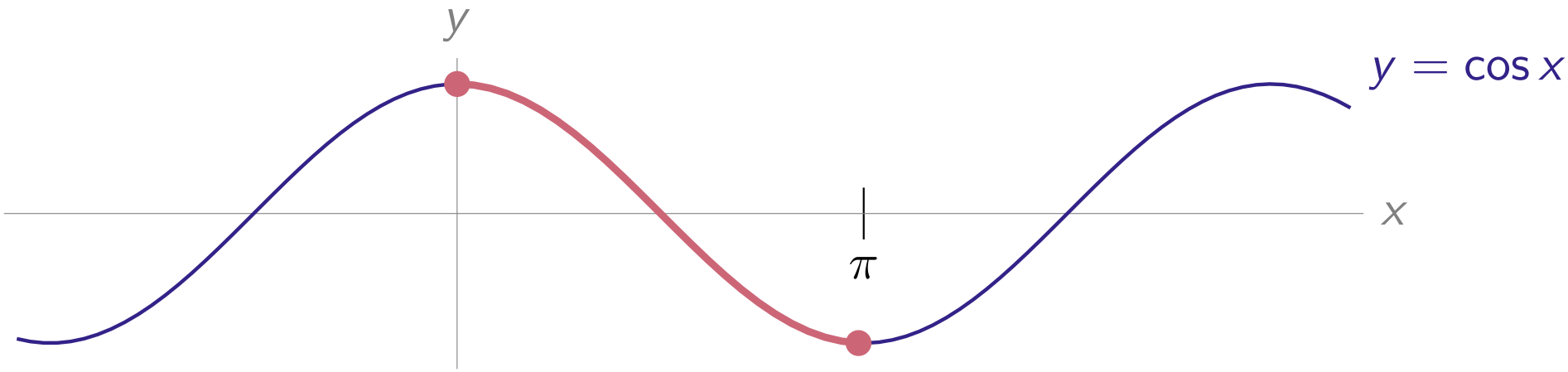

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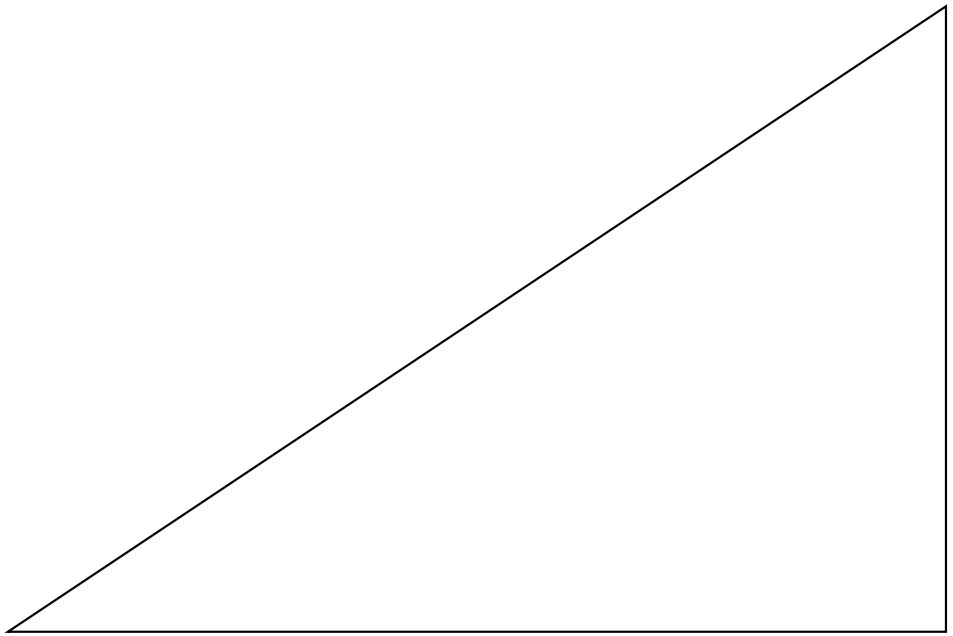
$\tan(\arccos x) = \dfrac{\sqrt{1-x^2}}{x}$ (hint: draw a triangle)

$\tan(\arccos x)$



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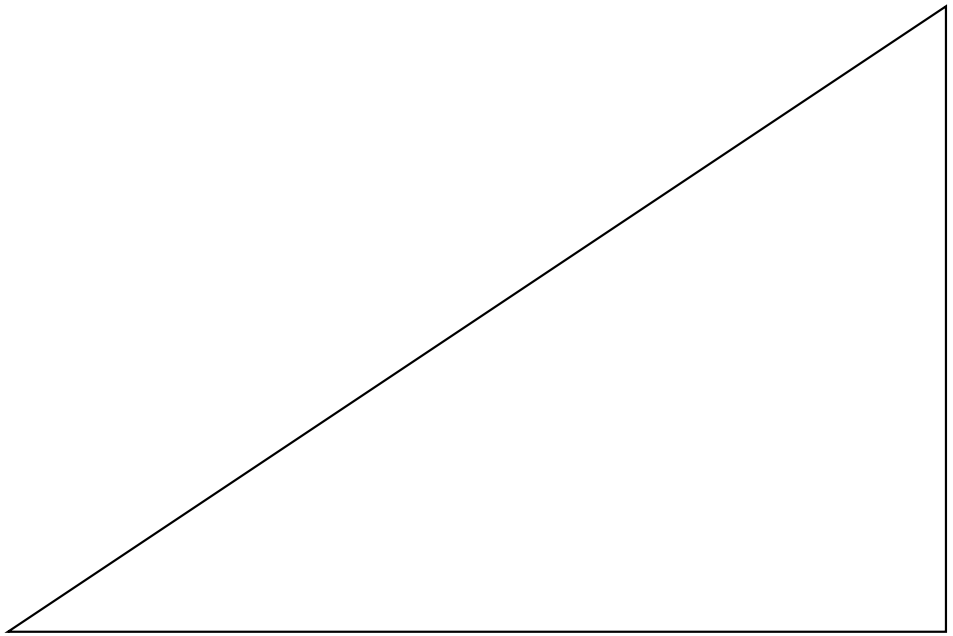
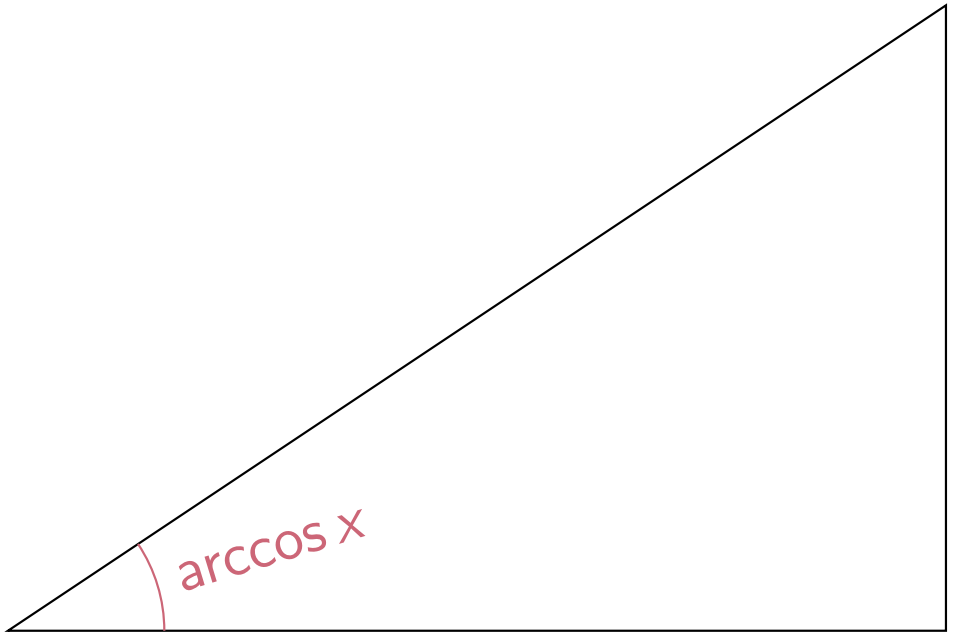


$\arccos(x)$: an angle whose cosine is x,

$\tan(\arccos x)$



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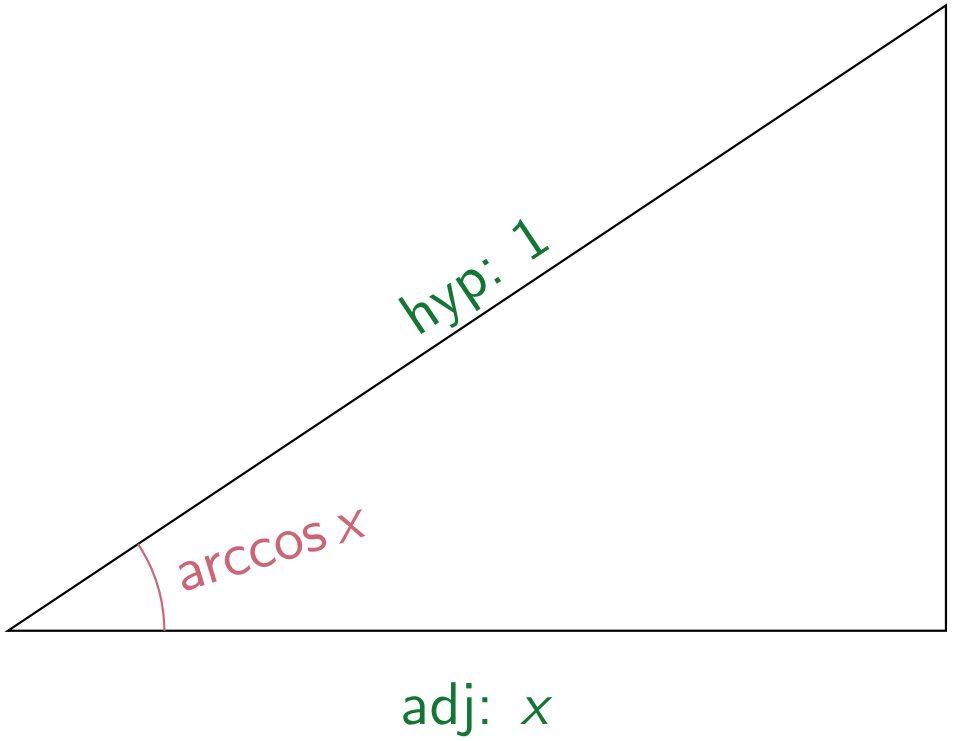
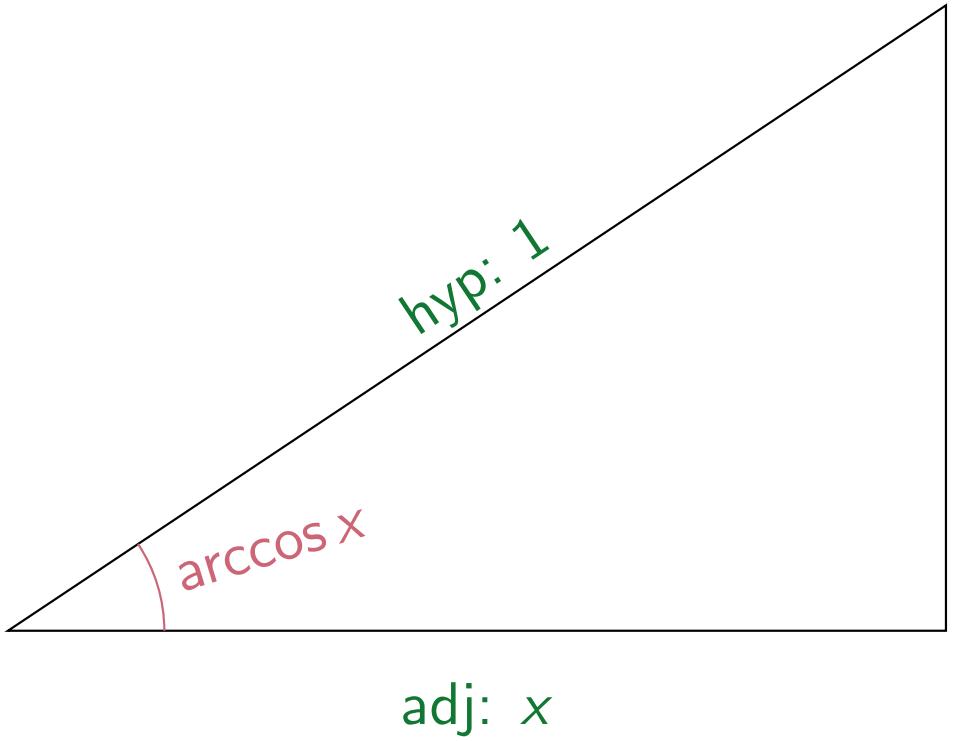


$\arccos(x)$: an angle whose cosine is x, which we can also write as $\frac{x}{1}$.

$\tan(\arccos x)$



(Pythagoras)

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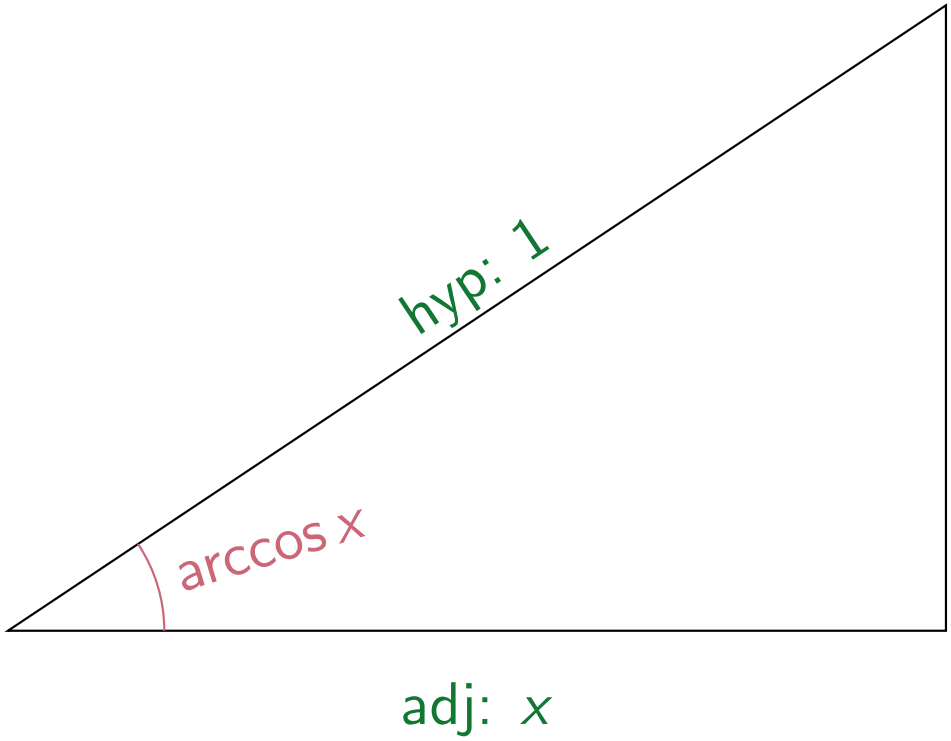

opp: $\sqrt{1-x^2}$
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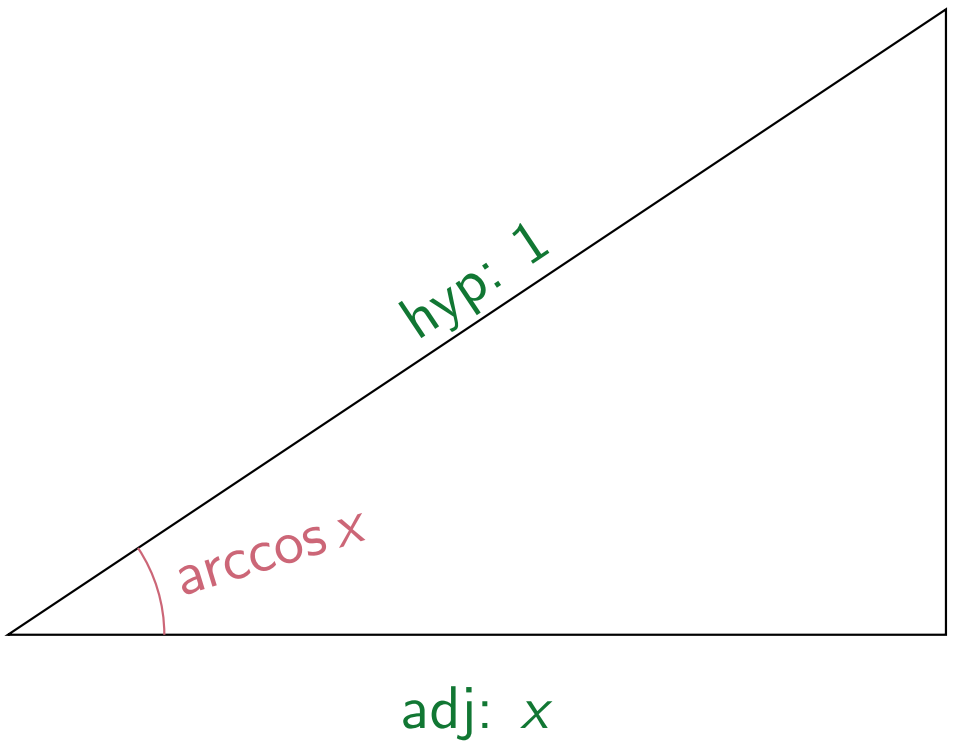

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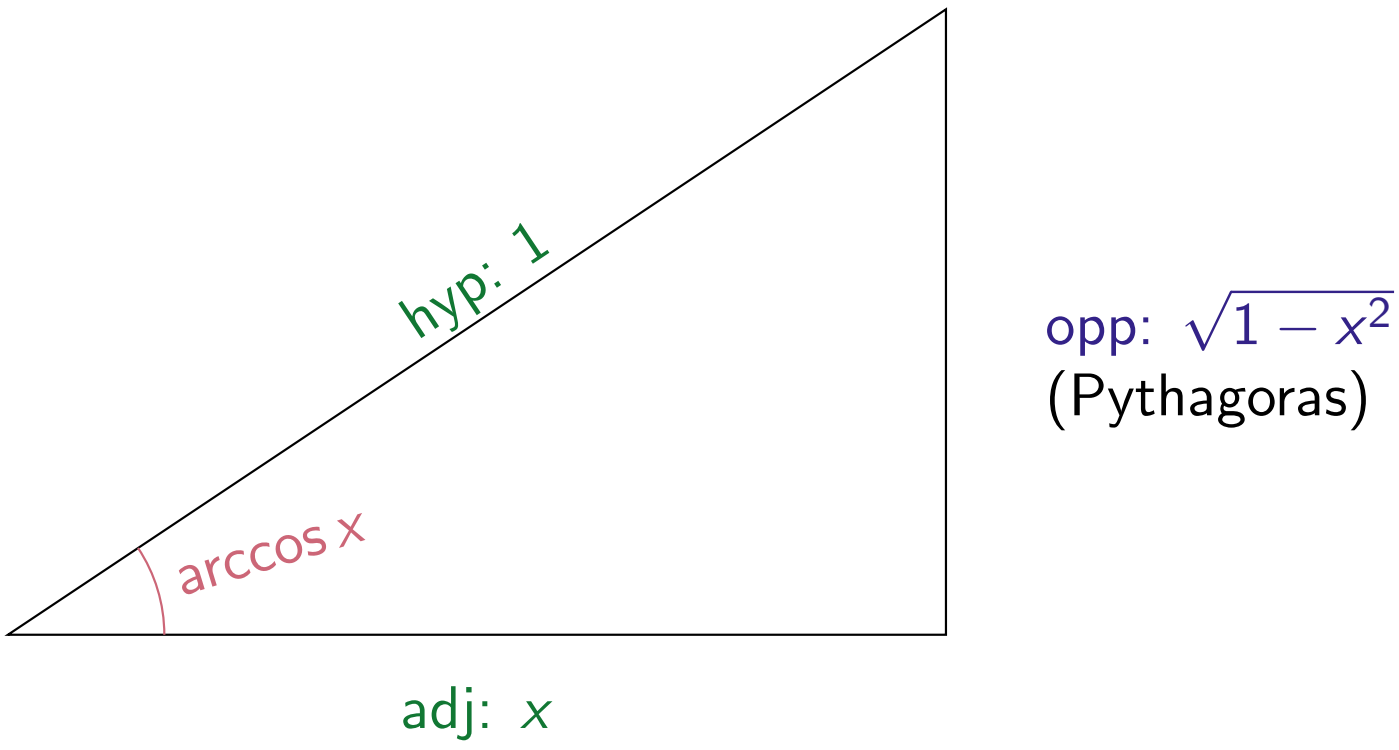

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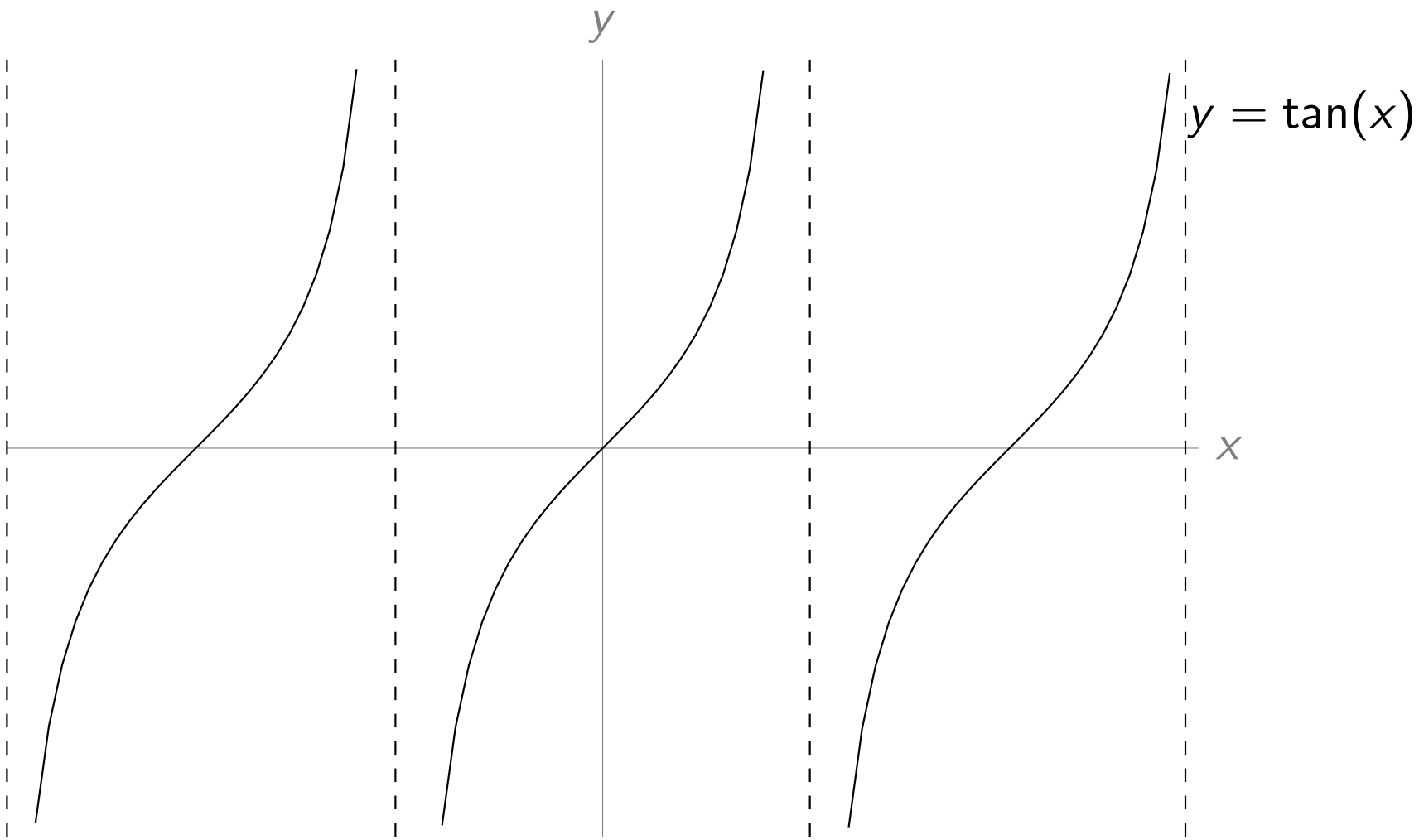
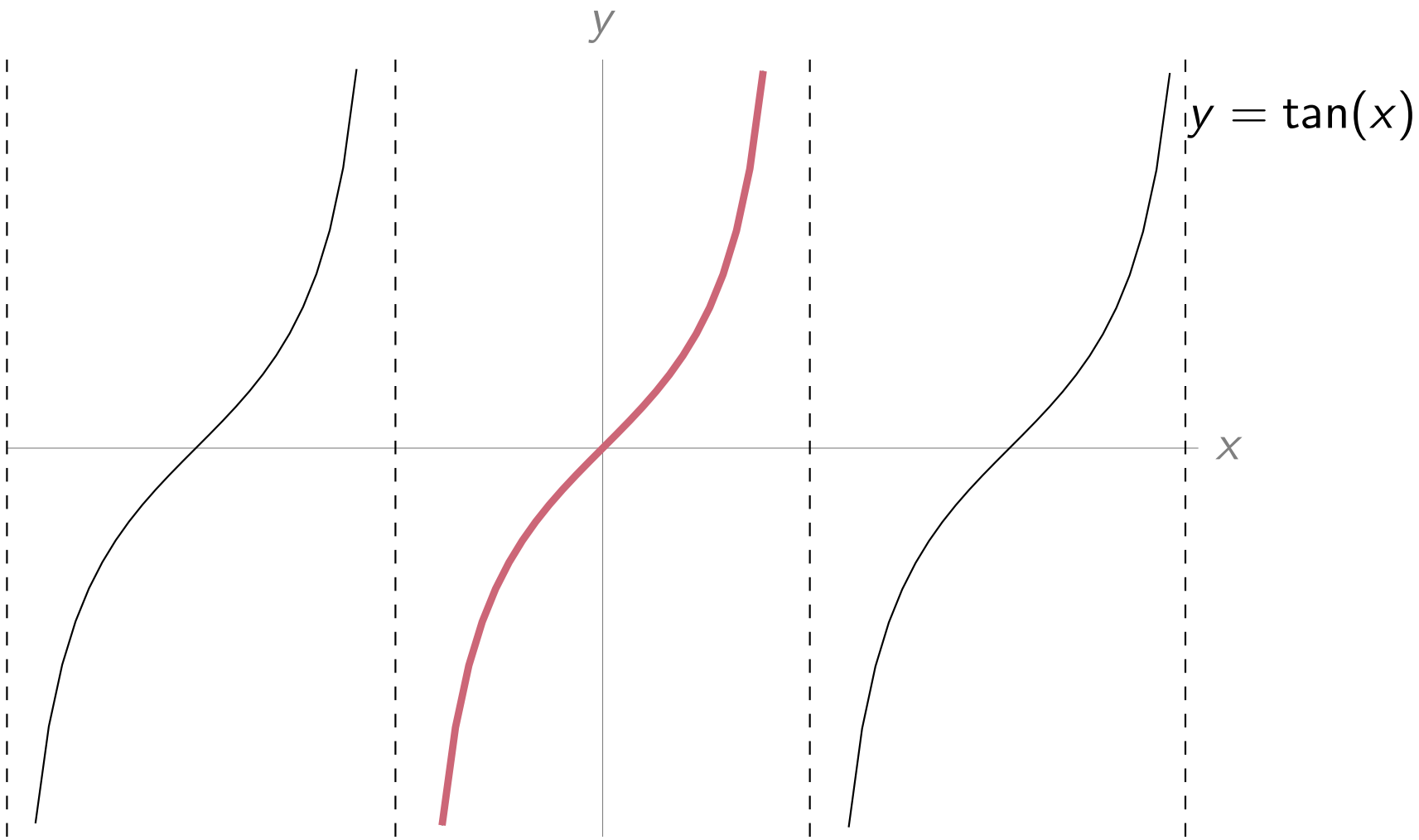
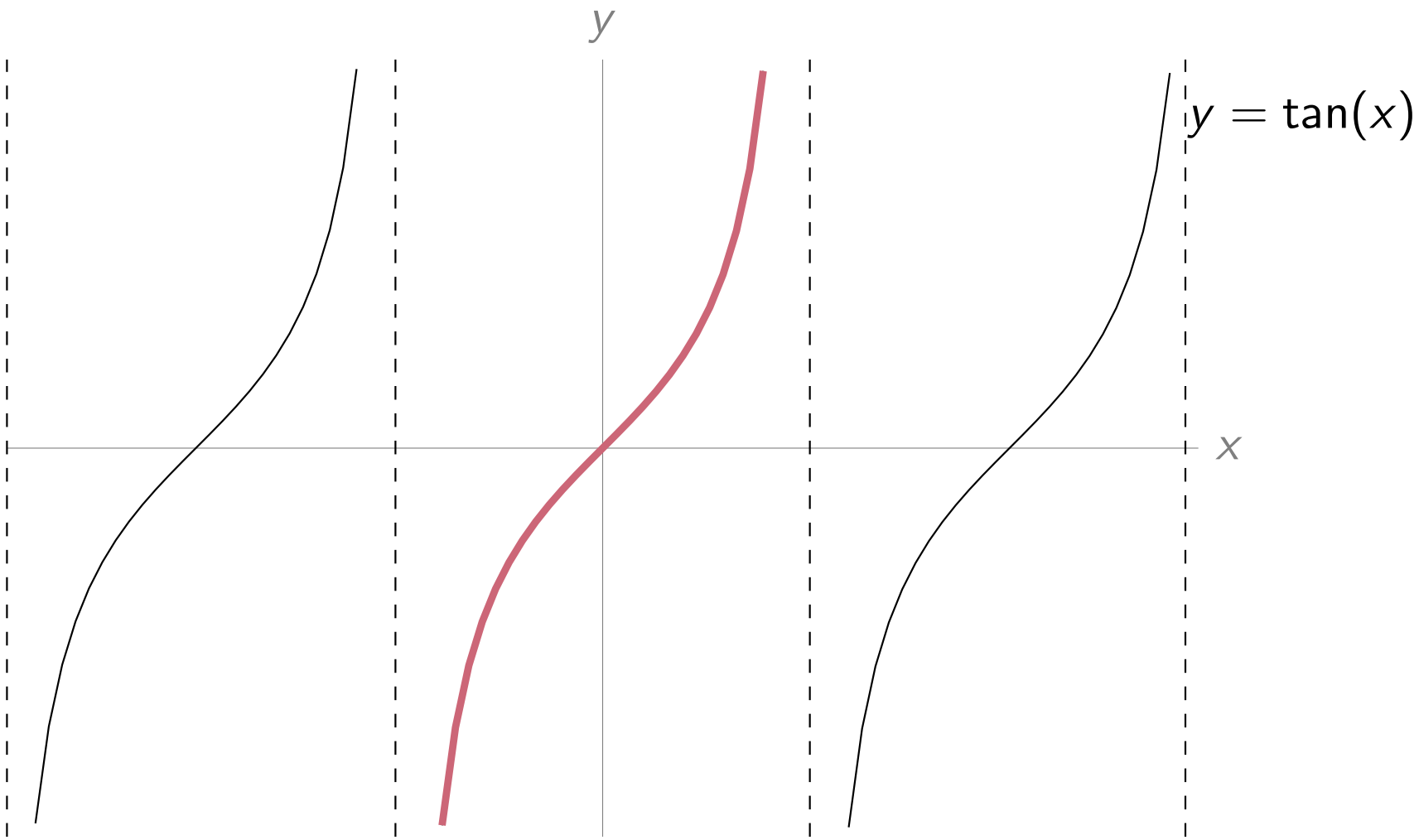
arctangent



arctangent

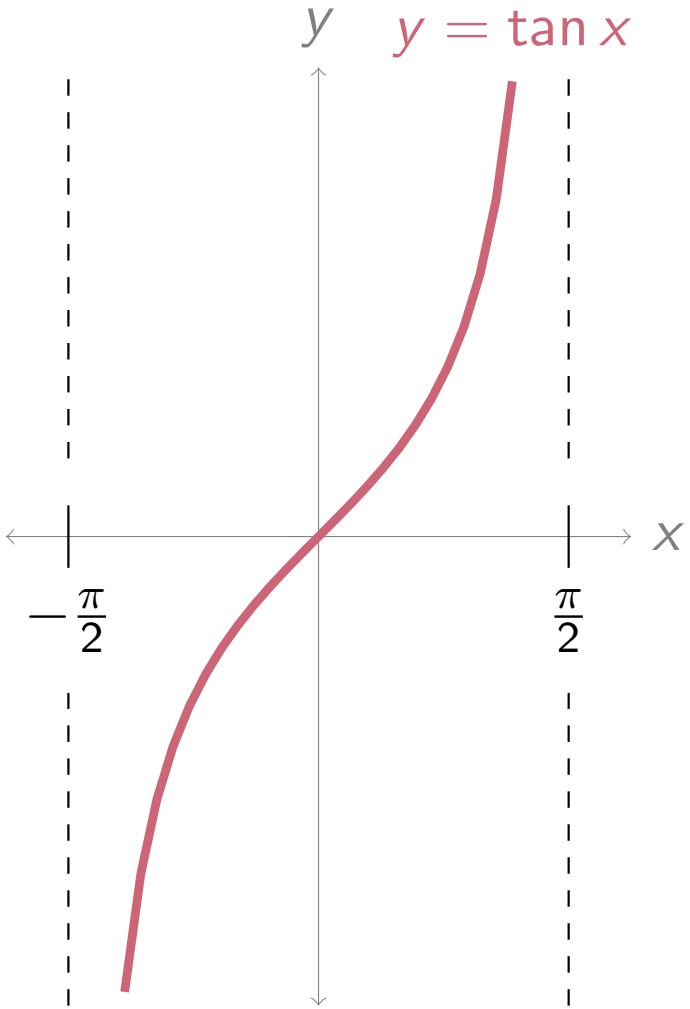


arctangent



$\arctan(x) = y$ means: (1) $\tan(y) = x$ and (2) $-\pi/2 < y < \pi/2$

arctangent



Evaluate:

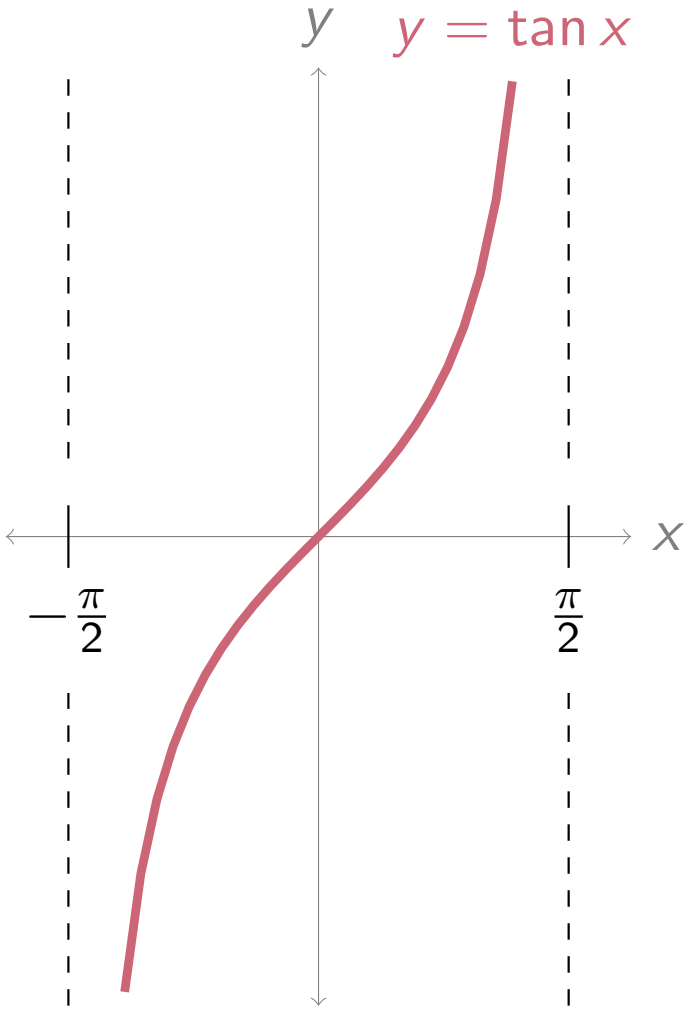
(a) $\arctan(0)$

(b) $\arctan(1)$

(c) $\lim_{x\to\infty} \arctan(x)$

(d) $\lim_{x\to-\infty} \arctan(x)$

arctangent



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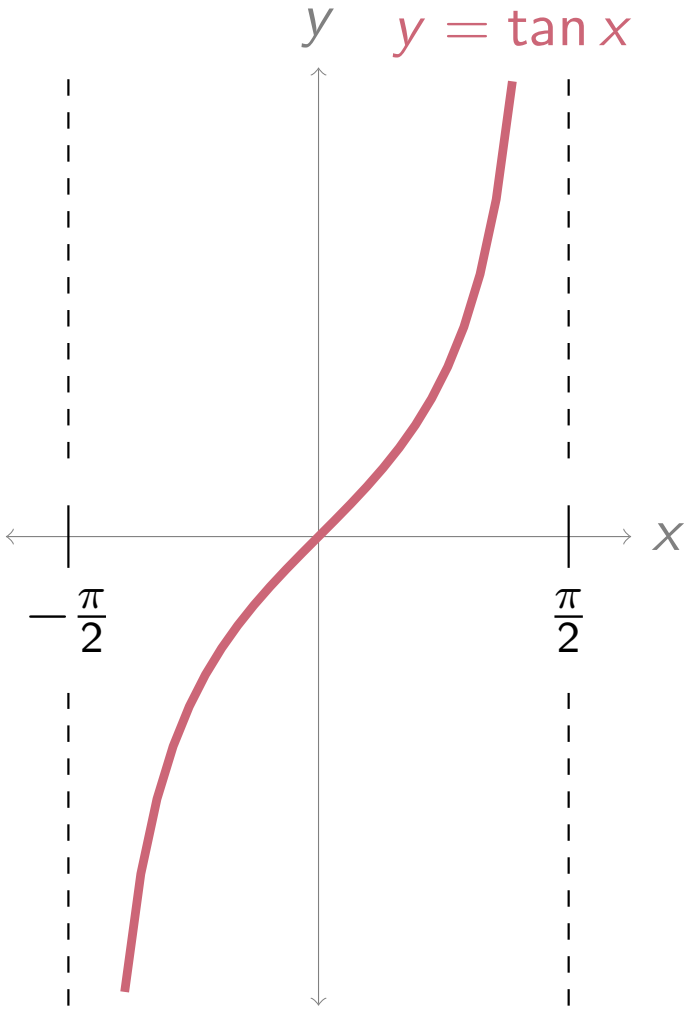

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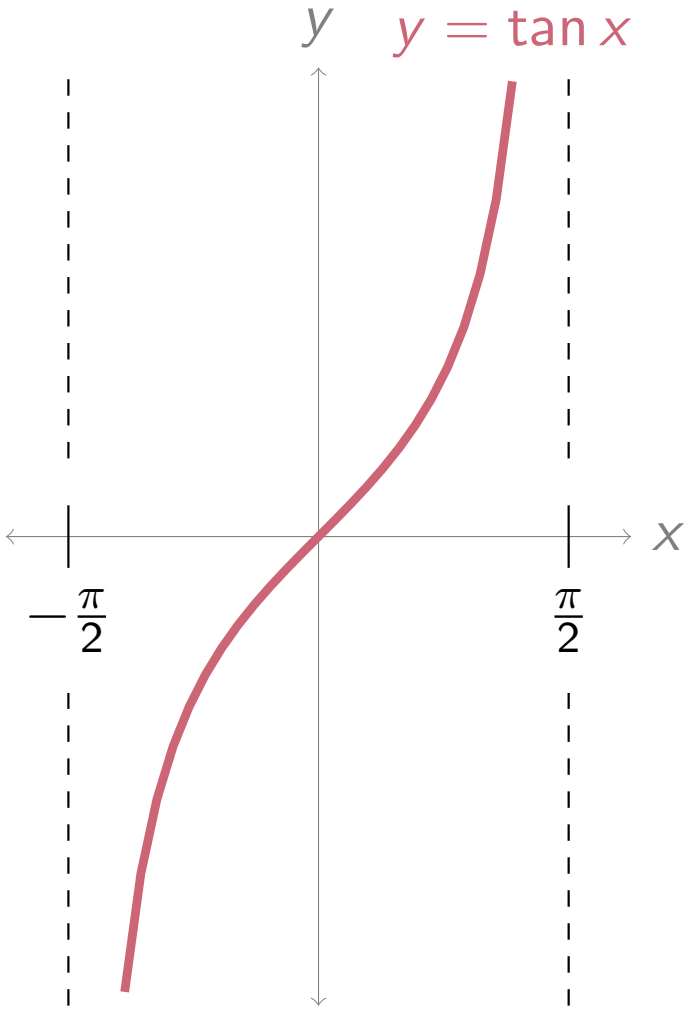

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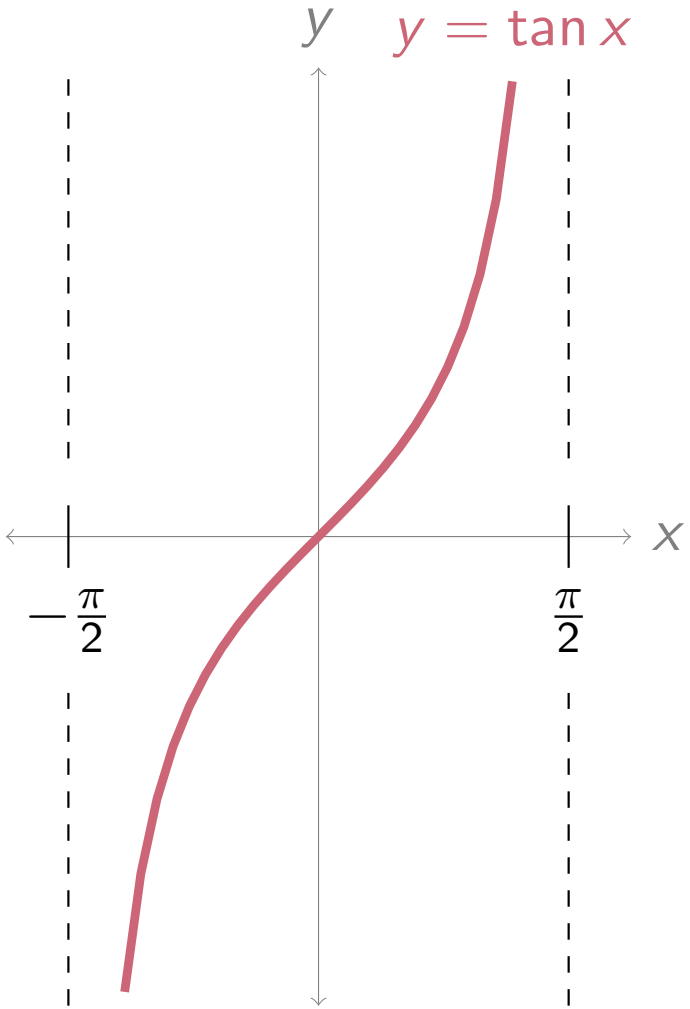

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Example 3: Slope Fields

When I make these slides, I draw slope fields using some trigonometry. Suppose at point (x, y), I want to draw a tick with slope y'. I write a horizontal tick, then rotate it R radians.
How can I calculate R?

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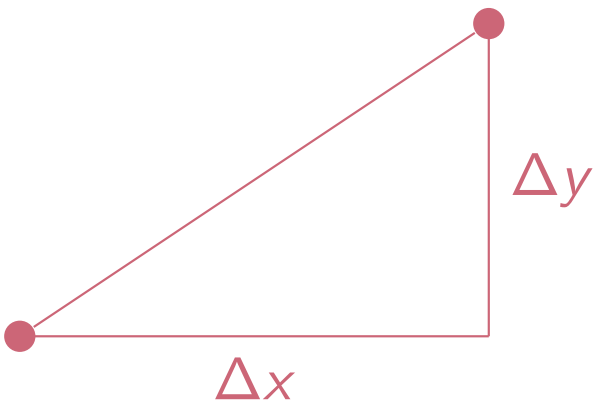

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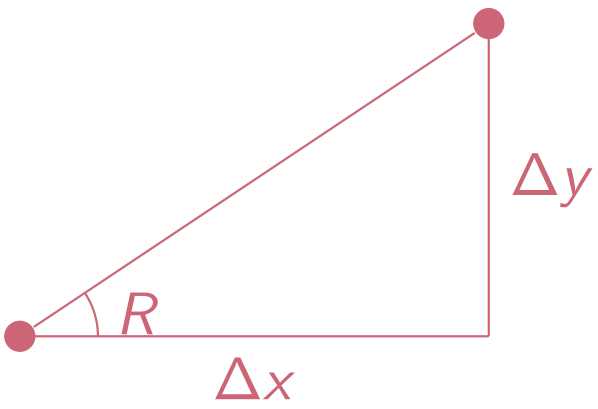

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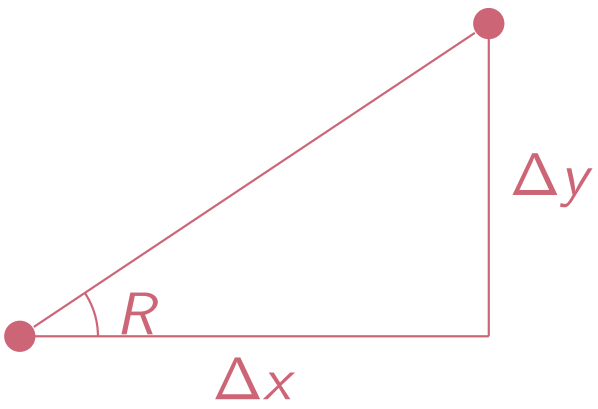

$$\tan R = \frac{\Delta y}{\Delta x} = y'$$

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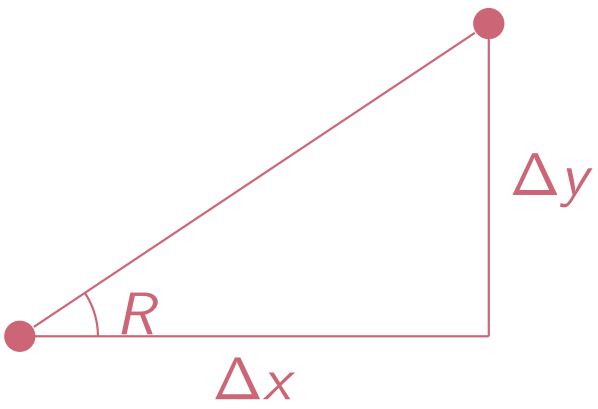

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$$R = \arctan(y')$$