Radians
Radian angle:

\[
\frac{\pi}{2}
\]

Arc length:

\[
\frac{\pi}{2}
\]
Radian

180°

360°
Radians

- $360^\circ$
- $180^\circ$
- $90^\circ$
- $360^\circ$
Radians
Radians

An angle is measured in radians. One radian is the angle subtended at the center of a circle by an arc whose length is equal to the radius of the circle. The number of radians in a full circle is $2\pi$. Therefore, one radian is equal to $\frac{180}{\pi}$ degrees. 

$$\text{Angle in radians} = \frac{\text{Arc length}}{\text{Radius}}$$

For example, the angle at the center of a circle subtended by an arc of length equal to the radius of the circle is one radian.

$$\pi \text{ radians} = 180^\circ$$
Radians

\[ 2\pi r \]

\[ r \]
Radians

The diagram illustrates the concept of radians. The angle subtended by an arc length of 1 unit on a circle with radius 1 is defined as 1 radian. The total angle around a circle is $2\pi$ radians, and the arc length for a full circle is $2\pi r$. The diagram shows a circle with a central angle of $\pi$ radians, corresponding to an arc length of $\pi$ units.
Radians

\[ 2\pi \]

\[ 1 \]
Chapter 14: Periodic and trigonometric functions

14.1 Basic trigonometry

Radian
Radians

Angle: $\pi$  arclength: $\pi$
Chapter 14: Periodic and trigonometric functions

14.1 Basic trigonometry

Radian: The angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

$\pi = 180^\circ$

$\frac{\pi}{2} = 90^\circ$

$\frac{\pi}{4} = 45^\circ$

$\frac{\pi}{3} = 60^\circ$

$\frac{\pi}{6} = 30^\circ$

$2\pi = 360^\circ$
Radians

Angle: \( \frac{\pi}{2} \)

Arclength: \( \frac{\pi}{2} \)

\[ \frac{2\pi}{4} = \frac{\pi}{2} \]
Radians and Degrees

\[
\frac{\text{radians}}{2\pi} = \frac{\text{degrees}}{360}
\]
gives the angle as a fraction of the whole circle.

<table>
<thead>
<tr>
<th>radians</th>
<th>degrees</th>
<th>fraction of circle</th>
<th>sketch</th>
</tr>
</thead>
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<td>180</td>
<td>(\frac{1}{2})</td>
<td><img src="https://via.placeholder.com/150" alt="Sketch" /></td>
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<td>(\frac{1}{8})</td>
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<tr>
<td>(\frac{5\pi}{4})</td>
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Radians and Degrees

\[ \frac{\text{radians}}{2\pi} = \frac{\text{degrees}}{360} \]

gives the angle as a fraction of the whole circle

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<td>180</td>
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<td><img src="image1" alt="Sketch" /></td>
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<td><img src="image2" alt="Sketch" /></td>
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<tr>
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<td>225</td>
<td>( \frac{5}{8} )</td>
<td><img src="image4" alt="Sketch" /></td>
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</tbody>
</table>
Unit circle, sines and cosines

unit circle, \( r = 1 \)
Unit circle, sines and cosines

$\cos \theta = \frac{x}{r}$
$\sin \theta = \frac{y}{r}$
$tan \theta = \frac{y}{x}$

$x^2 + y^2 = r^2$

unit circle, $r = 1$
Unit circle, sines and cosines

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Unit circle, sines and cosines

unit circle, $r = 1$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$x^2 + y^2 = 1$$

so

$$\sin^2 \theta + \cos^2 \theta = 1$$
Unit circle, sines and cosines

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y
\]

unit circle, \( r = 1 \)
Unit circle, sines and cosines

The unit circle, $r = 1$

- **Sine**: $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$

- **Cosine**: $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$

The unit circle is defined as $x^2 + y^2 = 1$.
Unit circle, sines and cosines

unit circle, $r = 1$

$x$

$y$

$(x, y)$

$\theta$

$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$

$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x$
Unit circle, sines and cosines

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Unit circle, sines and cosines

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Unit circle, sines and cosines

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- **Sine**: $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y$
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Diagram showing a point $(x, y)$ on the unit circle, with $\theta$ as the angle, and adjacent, opposite, and hypotenuse sides of the triangle formed by the radius, $x$, and $y$. The Pythagorean identity, $\sin^2 \theta + \cos^2 \theta = 1$, is also shown.
Unit circle, sines and cosines

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y \\
\cos \theta &= \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x
\end{align*}
\]

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Unit circle, sines and cosines

\[ \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{opp}}{1} = \text{opp} = y \]

\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{adj}}{1} = \text{adj} = x \]

\[ \tan \theta = \frac{\text{opp}}{\text{adj}} = \]

unit circle, \( r = 1 \)
Unit circle, sines and cosines

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\[
\tan \theta = \frac{\text{opp}_2}{\text{adj}_2} = \frac{\text{opp}_2}{1} = \text{opp}_2
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Unit circle, sines and cosines

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x^2 + y^2 &= 1, \text{ so } \sin^2 \theta + \cos^2 \theta = 1
\end{align*}
\]

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Unit circle, sines and cosines

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Unit circle, sines and cosines

unit circle, $r = 1$

$y = \sin \theta$
Unit circle, sines and cosines

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unit circle, $r = 1$

$x = \cos \theta$
Unit circle, sines and cosines

unit circle, $r = 1$
Chapter 14: Periodic and trigonometric functions

14.1 Basic trigonometry

**Unit circle, sines and cosines**

![Diagram of the unit circle with coordinates (cos θ, sin θ) and an angle θ.]

**unit circle, }r = 1**

**Even function:** $f(-\theta) = f(\theta)$

**Odd function:** $f(-\theta) = -f(\theta)$
Unit circle, sines and cosines

unit circle, \( r = 1 \)

Even function:  \( f(-\theta) = f(\theta) \)

Odd function:  \( f(-\theta) = -f(\theta) \)
Unit circle, sines and cosines

unit circle, $r = 1$

Even function: $f(-\theta) = f(\theta)$

Odd function: $f(-\theta) = -f(\theta)$

Cosine is an even function: $\cos(-\theta) = \cos(\theta)$. 
Unit circle, sines and cosines

unit circle, $r = 1$

Even function: $f(-\theta) = f(\theta)$

Cosine is an even function: $\cos(-\theta) = \cos(\theta)$.

Odd function: $f(-\theta) = -f(\theta)$

Sine is an odd function: $\sin(-\theta) = -\sin(\theta)$. 
Practice

A point \((a, b)\) in the plane is \(r\) centimetres from the origin, at an angle of \(\theta\). It is rotated \(\phi\) radians. What is its new coordinate \((x, y)\)?

Recall \(\cos(A + B) = \cos A \cos B - \sin A \sin B\) and \(\sin(A + B) = \sin A \cos B + \cos A \sin B\).
Practice

A point \((a, b)\) in the plane is \(r\) centimetres from the origin, at an angle of \(\theta\). It is rotated \(\phi\) radians. What is its new coordinate \((x, y)\)?

Recall \(\cos(A + B) = \cos A \cos B - \sin A \sin B\) and \(\sin(A + B) = \sin A \cos B + \cos A \sin B\). link: explanation of 2d rotation

\[
x = r \cos (\theta + \phi) = r (\cos \theta \cos \phi - \sin \theta \sin \phi) \\
= a \cos \phi - b \sin \phi \\
y = r (\theta + \phi) = r \sin (\sin \theta \cos \phi + \cos \theta \sin \phi) \\
= b \cos \theta + a \sin \theta
\]
Sine and cosine are periodic functions, with period $2\pi$.
Sine and cosine are **periodic** functions, with **period** $2\pi$. That is,

$$f(t) = f(t + 2\pi)$$

for every $t$. 
Periodic functions

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![Graph showing the periodic nature of the cosine function](graph.png)
Periodic functions

Sine and cosine are periodic functions, with period $2\pi$. That is,

$$f(t) = f(t + 2\pi)$$

for every $t$. 

![Graph of cosine function with period 2π]
Periodic functions

Sine and cosine have an amplitude of 1, because the range of values they take on is from \((0 - 1)\) to \((0 + 1)\).
Transforming Periodic Functions

\[ y = \sin t \]

\[ y = A \sin(\omega t + \phi) \]

What are \( A \), \( \omega \), and \( \phi \)?

- \( A \) = amplitude
- \( \omega \) = frequency
- \( \phi \) = phase shift
Transforming Periodic Functions

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What are \( A \), \( \omega \), and \( \phi \)?

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Transforming Periodic Functions

$y = \sin t$

$y = A \sin(\omega t + \phi)$

What are $A$, $\omega$, and $\phi$?
Transforming Periodic Functions

\[ y = A \sin(\omega t + \phi) \]

What are \( A \), \( \omega \), and \( \phi \)?

- \( A = 2 \) amplitude
- \( \omega = 3 \) frequency
- \( \phi = 0 \) phase shift
Example 1: Write each function below in the form $y = A \sin(\omega t + \phi)$. 

1. $y_1 = \sin(t - 1)$
2. $y_2 = -\frac{1}{\pi} \sin(\pi t)$ or $y_3 = \frac{1}{\pi} \sin(-\pi t)$
3. $y_4 = \frac{1}{2} \sin(\pi t - \frac{\pi}{2})$ or $y_5 = -\frac{1}{2} \sin(\pi t + \frac{\pi}{2})$
Example 1: Write each function below in the form $y = A \sin(\omega t + \phi)$. 

\[
\begin{align*}
  y &= \sin(t - 1) \\
  y &= -\frac{1}{2} \sin \left( \pi t - \frac{\pi}{2} \right) \\
  y &= \frac{1}{2} \sin \left( \pi t + \frac{\pi}{2} \right)
\end{align*}
\]
Example 1: Write each function below in the form $y = A \sin(\omega t + \phi)$.

$y = \sin(t - 1)$

$y = -\frac{1}{\pi} \sin(\pi t)$ or $y = \frac{1}{\pi} \sin(-\pi t)$
Example 1: Write each function below in the form $y = A \sin(\omega t + \phi)$.

- $y = \sin(t - 1)$
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- $y = \frac{1}{2} \sin(\pi t - \frac{\pi}{2})$ or $y = -\frac{1}{2} \sin(\pi t + \frac{\pi}{2})$
Example 2: Sketch

(a) \( y = \frac{1}{2} \sin(x) \)

(b) \( y = \frac{1}{2} \sin(2\pi x) \)

(c) \( y = \frac{1}{2} \sin(2\pi x + 0.8\pi) \)
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Who wins at the inverse game?

- I’m thinking of an $x$, and $\sin x = 0$. What is my $x$?

- I’m thinking of an $x$, and $\sin x = \frac{1}{\sqrt{2}}$. What is my $x$?
arcsine

Who wins at the inverse game?

- I’m thinking of an $x$, and $\sin x = 0$. What is my $x$?
  
  Could be $x = 0$, $x = \pm \pi$, $x = \pm 2\pi$, etc.

- I’m thinking of an $x$, and $\sin x = \frac{1}{\sqrt{2}}$. What is my $x$?
14.3 Inverse trigonometric functions

arcsine

Who wins at the inverse game?

- I’m thinking of an $x$, and $\sin x = 0$. What is my $x$?
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- I’m thinking of an $x$, and $\sin x = \frac{1}{\sqrt{2}}$. What is my $x$?
  Could be $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{9\pi}{4}$, etc.
Who wins at the inverse game?

- I’m thinking of an $x$, and $\sin x = 0$. What is my $x$?
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Chapter 14: Periodic and trigonometric functions

14.3 Inverse trigonometric functions

arcsine

Who wins at the inverse game?

- I’m thinking of an \( x \), and \( \sin x = 0 \). What is my \( x \)?
  
  Could be \( x = 0 \), \( x = \pm \pi \), \( x = \pm 2\pi \), etc.

- I’m thinking of an \( x \), and \( \sin x = \frac{1}{\sqrt{2}} \). What is my \( x \)?
  
  Could be \( x = \frac{\pi}{4} \), \( x = \frac{3\pi}{4} \), \( x = \frac{9\pi}{4} \), etc.
The function $f(x) = \sin(x)$ is invertible over the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and this is the domain we use to define $\arcsin(x)$.

$\arcsin(x)$ gives the number $y$ such that:

(1) $\sin(y) = x$ and

(2) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
Chapter 14: Periodic and trigonometric functions

14.3 Inverse trigonometric functions

The function \( f(x) = \sin(x) \) is invertible over the domain \( [-\frac{\pi}{2}, \frac{\pi}{2}] \), and this is the domain we use to define \( \arcsin(x) \).

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What is \( \arcsin(\sin 0) \)?

What is \( \arcsin \left( \sin \left(\frac{3\pi}{2}\right)\right) \)?
Chapter 14: Periodic and trigonometric functions

14.3 Inverse trigonometric functions

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\( \arcsin(x) \) gives the number \( y \) such that:

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What is \( \arcsin(\sin 0) \)? \( 0 \)  
What is \( \arcsin \left( \sin \left( \frac{3\pi}{2} \right) \right) \)?
arcsine

The function \( f(x) = \sin(x) \) is invertible over the domain \([-\frac{\pi}{2}, \frac{\pi}{2}]\), and this is the domain we use to define \( \arcsin(x) \).

\( \arcsin(x) \) gives the number \( y \) such that:
(1) \( \sin(y) = x \) and
(2) \(-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\)

What is \( \arcsin(\sin 0) \)? 0

What is \( \arcsin \left( \sin \left( \frac{3\pi}{2} \right) \right) \)? \(-\frac{\pi}{2}\)
The function $f(x) = \cos(x)$ is invertible over the domain $[0, \pi]$, and this is the domain we use to define $\arccos(x)$.

$\arccos(x)$ gives the number $y$ such that:

1. $\cos(y) = x$
2. $0 \leq y \leq \pi$

$\arccos(\cos(\frac{5\pi}{4})) = \arccos(\cos(\frac{3\pi}{4})) = \frac{3\pi}{4}$

$tan(\arccos(x)) = \sqrt{1 - x^2}$ (hint: draw a triangle)
Chapter 14: Periodic and trigonometric functions

14.3 Inverse trigonometric functions

The function \( f(x) = \cos(x) \) is invertible over the domain \([0, \pi]\), and this is the domain we use to define \( \arccos(x) \).

\( \arccos(x) \) gives the number \( y \) such that:

1. \( \cos(y) = x \)
2. \( 0 \leq y \leq \pi \)

\( \arccos(\cos(\frac{3\pi}{4})) = \frac{3\pi}{4} \)

\( \tan(\arccos(x)) = \sqrt{1 - x^2} \) (hint: draw a triangle)
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2. \( 0 \leq y \leq \pi \)

\[
\arccos \left( \cos \left( \frac{5\pi}{4} \right) \right) = \quad\quad \text{(hint: draw a triangle)}
\]

\[
\tan \left( \arccos \left( x \right) \right) = \quad\quad \text{(hint: draw a triangle)}
\]
The function $f(x) = \cos(x)$ is invertible over the domain $[0, \pi]$, and this is the domain we use to define $\arccos(x)$.

$\arccos(x)$ gives the number $y$ such that:

1. $\cos(y) = x$ and
2. $0 \leq y \leq \pi$

$\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right) = \arccos\left(\cos\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$

$tan(\arccos x) =$ (hint: draw a triangle)
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\[
\tan\left(\arccos x\right) = \frac{\sqrt{1 - x^2}}{x} \quad \text{(hint: draw a triangle)}
\]
\[ \tan(\arccos x) \]
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**arccos(x):** an angle whose cosine is \( x \),
\[ \tan(\arccos x) \]

\text{
\textbf{arccos}(x): an angle whose cosine is } x,
}
\[ \tan(\arccos x) \]

\[ \text{adj: } x \]

\[ \text{hyp: } 1 \]

\[ \arccos(x): \text{ an angle whose cosine is } x, \text{ which we can also write as } \frac{x}{1}. \]
**tan(arccos x)**

**Diagram:**
- **adj:** $x$
- **opp:** $\sqrt{1-x^2}$
- **hyp:** 1

**Equation:**
\[ \tan(arccos x) = \frac{\sqrt{1-x^2}}{x} \]

**Explanation:**
- **arccos(x):** an angle whose cosine is $x$, which we can also write as $\frac{x}{1}$. 

(Pythagoras)
\[ \tan(\arccos x) \]

\[ \text{adj: } x \]

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\[
\tan(\arccos x) = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{1 - x^2}}{x}
\]
arctangent

\[
y = \tan(x)
\]
arctangent

\[
\arctan(x) = y \quad \text{means: (1) } \tan(y) = x \quad \text{and (2) } -\frac{\pi}{2} < y < \frac{\pi}{2}
\]
arctangent

\begin{align*}
\text{arctan}(x) &= y \text{ means: } (1) \tan(y) = x \text{ and } (2) -\pi/2 < y < \pi/2
\end{align*}
Chapter 14: Periodic and trigonometric functions

14.3 Inverse trigonometric functions

**Evaluate:**

(a) \( \arctan(0) \)

(b) \( \arctan(1) \)

(c) \( \lim_{x \to \infty} \arctan(x) \)

(d) \( \lim_{x \to -\infty} \arctan(x) \)
arctangent

Evaluate:

(a) \( \arctan(0) = 0 \)

(b) \( \arctan(1) = \frac{\pi}{4} \)

(c) \( \lim_{x \to \infty} \arctan(x) = \frac{\pi}{2} \)

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14.3 Inverse trigonometric functions

$\arctan(x)$

Evaluate:

(a) $\arctan(0) = 0$

(b) $\arctan(1) = \frac{\pi}{4}$

(c) $\lim_{x \to \infty} \arctan(x)$

(d) $\lim_{x \to -\infty} \arctan(x)$
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(a) $\arctan(0) = 0$

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Example 3: Slope Fields

When I make these slides, I draw slope fields using some trigonometry. Suppose at point \((x, y)\), I want to draw a tick with slope \(y'\). I write a horizontal tick, then rotate it \(R\) radians. How can I calculate \(R\)?
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\tan R = \frac{\Delta y}{\Delta x} = y'
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\[R = \arctan(y')\]
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\Delta x \\
\Delta y \\
R
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