Curve Sketching

Example: Sketch 1

Review: find the domain of the following function.

\[ f(x) = \frac{\sqrt{3 - x^2}}{\ln(x + 1)} \]
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What happens to \( f(x) \) near its other endpoint, \( x = -1 \)?
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https://www.desmos.com/calculator/9funm5gwrt
Good things to check:

- Domain
- Vertical asymptotes: \( \lim_{x \to a} f(x) = \pm \infty \)
- Intercepts: \( x = 0, \ f(x) = 0 \)
- Horizontal asymptotes and end behavior: \( \lim_{x \to \pm \infty} f(x) \)
Curve Sketching

**Example: Sketch 2**

What does the graph of the following function look like?

\[ f(x) = \frac{x - 2}{(x + 3)^2} \]

Remember: domain, vertical asymptotes, intercepts, and horizontal asymptotes
Example: Sketch 2

What does the graph of the following function look like?

\[ f(x) = \frac{x - 2}{(x + 3)^2} \]

Remember: domain, vertical asymptotes, intercepts, and horizontal asymptotes

https://www.desmos.com/calculator/hylz5cyq7i
Curve Sketching

Example: Sketch 3

What does the graph of the following function look like?

\[ f(x) = \frac{(x + 2)(x - 3)^2}{x(x - 5)} \]
Curve Sketching

Example: Sketch 3

What does the graph of the following function look like?

\[ f(x) = \frac{(x + 2)(x - 3)^2}{x(x - 5)} \]

https://www.desmos.com/calculator/ploa0q7bxn
First Derivative

Example: Sketch 4

Add complexity: Increasing/decreasing, critical and singular points.
First Derivative

Example: Sketch 4

Add complexity: Increasing/decreasing, critical and singular points.

\[ f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2 \]
Chapter 3.6: Sketching Graphs

3.6.2: First Derivative: Increasing or Decreasing

First Derivative

Example: Sketch 4

Add complexity: Increasing/decreasing, critical and singular points.

\[ f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2 \]

- **Domain:** all real numbers
- **Intercepts:** \((0, 0)\) jumps out; we can factor \(f(x) = x^2(\frac{1}{2}x^2 - \frac{4}{3}x - 15)\) then use quadratic formula to find y-intercepts at \(x = \frac{4\pm\sqrt{286}}{3}\), so \(x \approx 7\) and \(x \approx -4.3\).
- **As** \(x\) **goes to** positive or negative infinity, function goes to infinity
- **\(f'(x) = 2x^3 - 4x^2 - 30x = 2x(x^2 - 2 - 15) = 2x(x - 5)(x + 3)\)** so critical points are \(x = 0\), \(x = -3\), and \(x = 5\). No singular points.

<table>
<thead>
<tr>
<th>(x \approx -4.3)</th>
<th>(x &lt; -3)</th>
<th>(x = -3)</th>
<th>(-3 &lt; x &lt; 0)</th>
<th>(x = 0)</th>
<th>(0 &lt; x &lt; 5)</th>
<th>(x = 5)</th>
<th>(x &gt; 5)</th>
<th>(x \approx 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x) = 0)</td>
<td>(f' &lt; 0)</td>
<td>(CP)</td>
<td>(f' &gt; 0)</td>
<td>(CP)</td>
<td>(f' &lt; 0)</td>
<td>(CP)</td>
<td>(f' &gt; 0)</td>
<td>(f(x) = 0)</td>
</tr>
<tr>
<td>intercept</td>
<td>decr</td>
<td>l. min</td>
<td>incr</td>
<td>l. max</td>
<td>decr</td>
<td>l. min</td>
<td>incr</td>
<td>intercept</td>
</tr>
</tbody>
</table>

https://www.desmos.com/calculator/lxdlglmhns1
Example: Sketch 5

What does the following function look like?

\[ f(x) = \frac{1}{3} x^3 + 2x^2 + 4x + 24 \]
What does the following function look like?

\[ f(x) = \frac{1}{3}x^3 + 2x^2 + 4x + 24 \]

- **Domain:** all real numbers. No VA. Goes to \( \pm \infty \).
- **\( f(0) = 24 \):** \( f(x) = \frac{1}{3}x^2(x + 6) + 4(x + 6) = (\frac{1}{3}x^2 + 4)(x + 6) \), so only one root: \( f(-6) = 0 \).
- **\( f'(x) = x^2 + 4x4 = (x + 2)^2 \):** only one critical point, at \( x = -2 \), and increasing everywhere else.
- **So, at the left, comes from negative infinity; levels crosses x-axis at \( x = -6 \); levels out at \( x = -2 \); crosses y-axis at \( y = 24 \); carries on to infinity**

https://www.desmos.com/calculator/xum0mstmiv
What does the graph of the following function look like?

\[ f(x) = e^{\frac{x+1}{x-1}} \]
What does the graph of the following function look like?

\[ f(x) = e^{\frac{x+1}{x-1}} \]

- **Domain:** \( x \neq 1 \)
- **VA:** something weird happens at \( x = 1 \). Check out limits:
  
  \[
  \lim_{x \to 1^-} \frac{x + 1}{x - 1} = -\infty \text{ and } \lim_{x \to 1^+} \frac{x + 1}{x - 1} = \infty, \quad \text{so } \lim_{x \to 1^-} f(x) = lim_{A \to -\infty} e^A = 0 \text{ while } \]
  
  \[
  \lim_{x \to 1^+} f(x) = lim_{A \to \infty} e^A = \infty.
  \]

- **Horizontal asymptotes:** \( \lim_{x \to \pm \infty} f(x) = e \)

- **Intercepts:** the function is never zero; \( f(0) = \frac{1}{e} \).

- **Derivative:** \( f'(x) = e^{\frac{x+1}{x-1}} \left( \frac{-2}{(x-1)^2} \right) \); so the function is always decreasing (when it's defined!)

- **So,** on either end, it gets extremely close to \( e \); as we move left to right, it dips to \( \frac{1}{e} \) at the \( y \)-axis; gets nearly to the \( x \)-axis at 1; then has a VA from the right only at 1; then dips back to very close to \( e \).

https://www.desmos.com/calculator/x0cccy1ggj
Concavity

Slopes are increasing:

\[ f''(x) > 0 \]

"concave up"

tangent line below curve

Slopes are decreasing:

\[ f''(x) < 0 \]

"concave down"

tangent line above curve
Concavity

Slopes are increasing

$f''(x) > 0$

"concave up" tangent line below curve

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"concave down" tangent line above curve
Concavity

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tangent line above curve
Concavity

Slopes are increasing: $f''(x) > 0$ "concave up" tangent line below curve

Slopes are decreasing: $f''(x) < 0$ "concave down" tangent line above curve
Concavity

Slopes are increasing if $f''(x) > 0$, which makes the graph "concave up". The tangent line is below the curve.

Slopes are decreasing if $f''(x) < 0$, which makes the graph "concave down". The tangent line is above the curve.
Concavity

Slopes are increasing: \( f''(x) > 0 \) "concave up" tangent line below curve

Slopes are decreasing: \( f''(x) < 0 \) "concave down" tangent line above curve
Concavity

Slopes are increasing
\[ f''(x) > 0 \]
"concave up"
tangent line below curve

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"concave down"
tangent line above curve
Concavity

Slopes are increasing
Concavity

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Concavity

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tangent line below curve

Slopes are decreasing
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tangent line above curve
Mnemonic

+ + - -
Concavity
Concavity

Concave up

Inflection point

$f''(x)$ changes sign
Concavity

Concave up

Concave down

Inflection point

$f''(x)$ changes sign
Concavity
Concavity
Concavity

concave up

inflection point

concave down

inflection point

concave up
Concavity

Inflection point

$f''(x)$ changes sign

Concave up  Concave down  Concave up
Poll Questions

Describe the concavity of the function \( f(x) = e^x \).

A. concave up
B. concave down
C. concave up for \( x < 0 \); concave down for \( x > 0 \)
D. concave down for \( x < 0 \); concave up for \( x > 0 \)
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Is it possible to be concave up and decreasing?

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B. No  
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From Last Time

Example: Sketch 6.5

\[ f(x) = \frac{1}{2} x^4 - \frac{4}{3} x^3 - 15x^2 \]
From Last Time

Example: Sketch 6.5

\[ f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2 \]

\[ f''(x) = 6x^2 - 8x - 30 = 2(x - 3)(3x + 5) \]
3.6.3: Second derivative: concavity

From Last Time

Example: Sketch 6.5

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Example: Sketch 7

Sketch:

\[ f(x) = x^5 - 15x^3 \]
Example: Sketch 7

Sketch:

\[ f(x) = x^5 - 15x^3 \]

Symmetry!
Example: Sketch 7

Sketch:

\[ f(x) = x^5 - 15x^3 \]

Symmetry!

- Defined and differentiable for all real numbers.
- Roots: \( x = 0, x = \pm \sqrt{15} \approx 4 \)
- Goes to \( \pm \infty \) as \( x \) goes to \( \pm \infty \)
- CP: \( x = 0, x = \pm 3 \). Increasing on \((-\infty, -3)\), decreasing \((-3, 0)\) and \((0, 3)\), decreasing \((3, \infty)\)
- So, local max at \( x = -3 \) and local min at \( x = 3 \)
- \( f''(x) = 0 \) for \( x = 0 \) and \( x = \pm \frac{3}{\sqrt{2}} \approx \pm 2 \). All of these are inflection points; concave down \((-\infty, -\frac{3}{\sqrt{2}})\), concave up \((\frac{3}{\sqrt{2}}, 0)\), concave down \((0, \frac{3}{\sqrt{2}})\), and concave up \((\frac{3}{\sqrt{2}}, \infty)\).
- \( f(3) = -162, f(-3) = -162, f(-3/\sqrt{2}) \approx 100, f(3/\sqrt{2}) \approx -100 \)

https://www.desmos.com/calculator/uoii6nmgr8
Even and Odd Functions

\[ f(x) = x^5 - 15x^3 \]
Even and Odd Functions

\[ f(x) = x^5 - 15x^3 \]

odd function
Even and Odd Functions

\[ f(x) = x^5 - 15x^3 \]
Even and Odd Functions

even function
Even Functions

**Even Function**

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$
Even Functions

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$

Suppose $f(3) = -1$. Then $f(-3) = -1$ also. Suppose $f(6) = 1$. Then $f(-6) = 1$ also.
Even Functions

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Examples:

$f(x) = x^2$

$f(x) = x^4$

$f(x) = \cos(x)$
Even Functions

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**Examples:**

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- $f(x) = x^4$
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- $f(x) = \cos(x)$
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$$f(-x) = f(x)$$

Examples:
- $f(x) = x^2$
- $f(x) = x^4$
- $f(x) = \cos(x)$
- $f(x) = \frac{x^4 + \cos(x)}{x^{16} + 7}$
Odd Functions

Suppose $f(1) = 2$. Then $f(-1) = -2$.

Suppose $f(3) = -2$. Then $f(-3) = 2$.
Odd Functions

Suppose \( f(1) = 2 \).

Even Function

A function \( f(x) \) is odd if, for all \( x \) in its domain,

\[
  f(-x) = -f(x)
\]
Odd Functions

Suppose $f(1) = 2$. Then $f(-1) =$
### Odd Functions

Suppose $f(1) = 2$. Then $f(-1) = -2$. 

**Odd function**
Odd Functions

Suppose $f(1) = 2$. Then $f(-1) = -2$.
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odd function
Odd Functions

Suppose \( f(1) = 2 \). Then \( f(-1) = -2 \).
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Even Function

A function \( f(x) \) is **odd** if, for all \( x \) in its domain,
Odd Functions

Suppose \( f(1) = 2 \). Then \( f(-1) = -2 \).
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Even Functions

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Examples:
Even Functions

**Even Function**
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$$f(-x) = -f(x)$$

Examples:
$f(x) = x$
Even Functions

Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

Examples:

- $f(x) = x$
- $f(x) = x^3$
Even Functions

Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

Examples:

- $f(x) = x$
- $f(x) = x^3$
- $f(x) = \sin(x)$
### Even Functions

**Even Function**

A function $f(x)$ is **odd** if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

**Examples:**

- $f(x) = x$
- $f(x) = x^3$
- $f(x) = \sin(x)$
- $f(x) = \frac{x(1 + x^2)}{x^2 + 5}$
Poll Time

Pick out the **odd** function.

A: 

B: 

C: 

D: 
Poll Tiiime

Pick out the **odd** function.

A:  

B:  

C:  

D:  
Poll Time

Pick out the **even** function.

A:  

B:  

C:  

D:  
Poll Time

Pick out the **even** function.

A:  

B:  

C:  

D:
Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$
B. $f(0) = -f(0)$
C. $f(0) = 0$
D. all of the above are true
E. none of the above are necessarily true
Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

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E. none of the above are necessarily true
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Suppose \( f(x) \) is an odd function, continuous, defined for all real numbers. What is \( f(0) \)?

Pick the best answer.

A. \( f(0) = f(-0) \) ← true but uninteresting, for all functions
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Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

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B. $f(0) = -f(0)$ — only possible for $f(0) = 0$
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A. $f(0) = f(-0)$ — true but uninteresting, for all functions
B. $f(0) = -f(0)$ — only possible for $f(0) = 0$
C. $f(0) = 0$ — this is equivalent to the choice above
D. all of the above are true
E. none of the above are necessarily true

Even more Poll tiiiiiime
Even more and more Poll tiiiiime

Suppose $f(x)$ is an even function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

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Suppose $f(x)$ is an even function, continuous, defined for all real numbers. What is $f(0)$?
Pick the best answer.

A. $f(0) = f(-0)$
B. $f(0) = -f(0)$
C. $f(0) = 0$
D. all of the above are true
E. none of the above are necessarily true
Suppose $f(x)$ is an even function, differentiable for all real numbers. What can we say about $f'(x)$?

A. $f'(x)$ is also even
B. $f'(x)$ is odd
C. $f'(x)$ is constant
D. all of the above are true
E. none of the above are necessarily true
Suppose \( f(x) \) is an even function, differentiable for all real numbers. What can we say about \( f'(x) \)?

A. \( f'(x) \) is also even
B. \( f'(x) \) is odd
C. \( f'(x) \) is constant
D. all of the above are true
E. none of the above are necessarily true
Periodicity

Periodic

A function is periodic with period $P$ if

$$f(x) = f(x + P)$$

whenever $x$ and $x + P$ are in the domain of $f$, and $P$ is the smallest such (positive) number.

Examples: $\sin(x)$, $\cos(x)$ both have period $2\pi$; $\tan(x)$ has period $\pi$. 
Example: Sketch 8

\[ f(x) = \sin(\sin x) \]

(ignoring concavity)
Example: Sketch 8

\[ f(x) = \sin(\sin x) \]

(ignore concavity)

Example: Sketch 9

\[ g(x) = \sin(2\pi \sin x) \]
Example: Sketch 10

\[ f(x) = (x^2 - 64)^{1/3} \]
Let’s Graph

Example: Sketch 10

\[ f(x) = (x^2 - 64)^{1/3} \]

\[ f'(x) = \frac{2x}{3(x^2 - 64)^{2/3}}; \]

\[ f''(x) = \frac{-2\left(\frac{1}{3}x^2 + 64\right)}{3(x^2 - 64)^{5/3}} \]
Let’s Graph

Example: Sketch 11

\[ f(x) = \frac{x^2 + x}{(x + 1)(x^2 + 1)^2} \]
Let’s Graph

Example: Sketch 11

\[ f(x) = \frac{x^2 + x}{(x + 1)(x^2 + 1)^2} \]

Note for \( x \neq -1 \), \( f(x) = \frac{x(x + 1)}{(x + 1)(x^2 + 1)^2} = \frac{x}{(x^2 + 1)^2} \]
Let’s Graph

Example: Sketch 11

\[ f(x) = \frac{x^2 + x}{(x + 1)(x^2 + 1)^2} \]

Note for \( x \neq -1 \), \( f(x) = \frac{x(x + 1)}{(x + 1)(x^2 + 1)^2} = \frac{x}{(x^2 + 1)^2} \)

Example: Sketch 12

\[ g(x) := \frac{x}{(x^2 + 1)^2} \]
### Example: Sketch 11

Let's Graph

\[ f(x) = \frac{x^2 + x}{(x + 1)(x^2 + 1)^2} \]

Note for \( x \neq -1 \), \( f(x) = \frac{x(x + 1)}{(x + 1)(x^2 + 1)^2} = \frac{x}{(x^2 + 1)^2} \)

### Example: Sketch 12

\[ g(x) := \frac{x}{(x^2 + 1)^2} \]

\[ g'(x) = \frac{1 - 3x^2}{(x^2 + 1)^3}; \quad g''(x) = \frac{12x(x^2 - 1)}{(x^2 + 1)^4} \]
Let’s Graph

Example: Sketch 13

\[ f(x) = x(x - 1)^{2/3} \]
Match the Function to its Graph

A. \( f(x) = \frac{x - 1}{(x + 1)(x + 2)} \)
B. \( f(x) = \frac{(x - 1)^2}{(x + 1)(x + 2)} \)
C. \( f(x) = \frac{x - 1}{(x + 1)^2(x + 2)} \)
D. \( f(x) = \frac{(x - 1)^2}{(x + 1)^2(x + 2)} \)
Match the Function to its Graph

A. \( f(x) = \frac{x - 1}{(x + 1)(x + 2)} \)

B. \( f(x) = \frac{(x - 1)^2}{(x + 1)(x + 2)} \)

C. \( f(x) = \frac{x - 1}{(x + 1)^2(x + 2)} \)

D. \( f(x) = \frac{(x - 1)^2}{(x + 1)^2(x + 2)} \)
Match the Function to its Graph

A. \( f(x) = x^3(x + 2)(x - 2) = x^5 - 4x^3 \)
B. \( f(x) = x(x + 2)^3(x - 2) = x^5 + 4x^4 - 16x^2 - 16x \)
C. \( f(x) = x(x + 2)(x - 2)^3 = x^5 - 4x^4 + 16x^2 - 16x \)
Match the Function to its Graph

A. \( f(x) = x^3(x + 2)(x - 2) = x^5 - 4x^3 \)
B. \( f(x) = x(x + 2)^3(x - 2) = x^5 + 4x^4 - 16x^2 - 16x \)
C. \( f(x) = x(x + 2)(x - 2)^3 = x^5 - 4x^4 + 16x^2 - 16x \)
Match the Function to its Graph

A. \( f(x) = |x|^e \)  
B. \( f(x) = e^{|x|} \)  
C. \( f(x) = e^{x^2} \)  
D. \( f(x) = e^{x^4-x} \)

BLACK  
ORANGE  
BLUE 1  
PURPLE  
BLUE 2  
RED
Match the Function to its Graph

A. $f(x) = x^5 + 15x^3$
B. $f(x) = x^5 - 15x^3$
C. $f(x) = x^5 - 15x^2$
D. $f(x) = x^3 - 15x$
E. $f(x) = x^7 - 15x^4$
Match the Function to its Graph

A. \( f(x) = x^5 + 15x^3 \)
B. \( f(x) = x^5 - 15x^3 \)
C. \( f(x) = x^5 - 15x^2 \)
D. \( f(x) = x^3 - 15x \)
E. \( f(x) = x^7 - 15x^4 \)