Rates of Change

Suppose the population of a small country was 1 million individuals in 1990, and is growing at a steady rate of 20,000 individuals per year.

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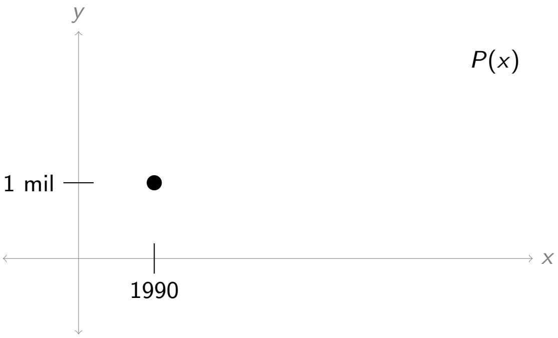

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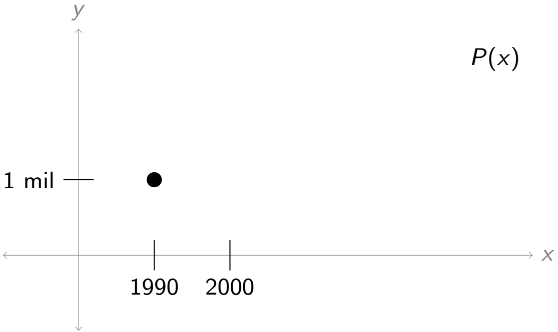

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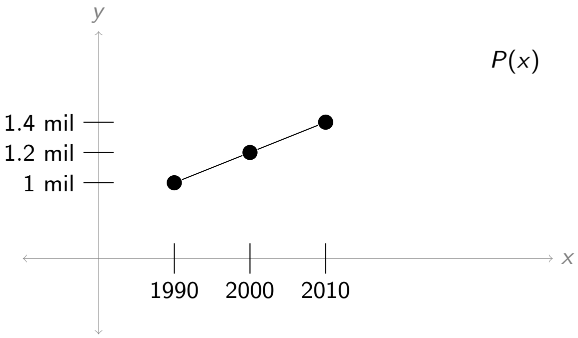

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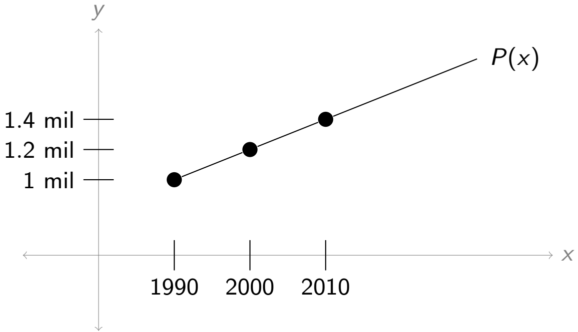
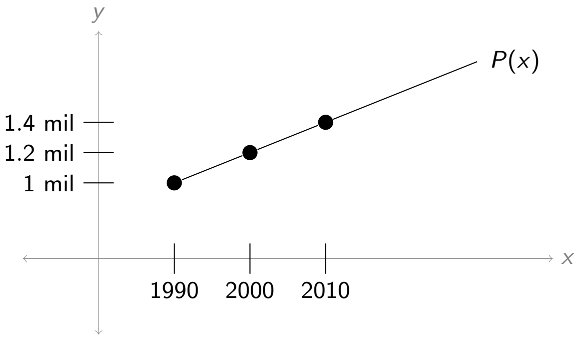


Definition

The **slope** of a line that passes through the points (x_1, y_1) and (x_2, y_2) is "rise over run"

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

This is also called the **rate of change** of the function.
If a line has equation $y = mx + b$, its slope is m.



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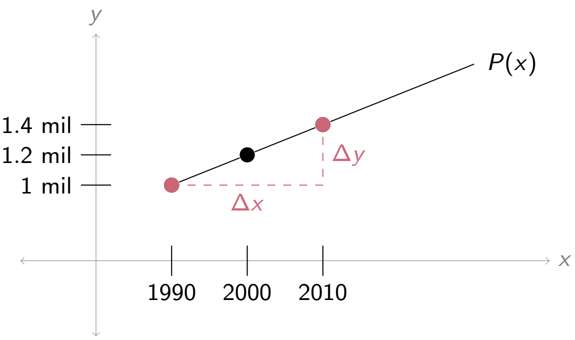

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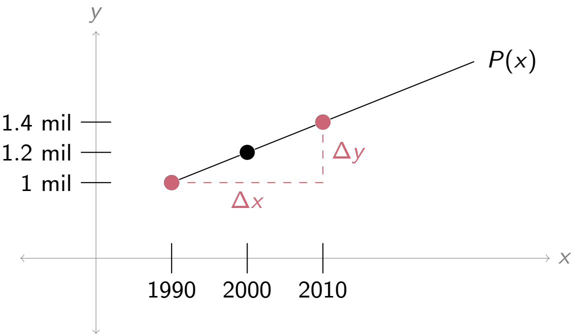

Rate of change: $\dfrac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \dfrac{\text{people}}{\text{year}}$

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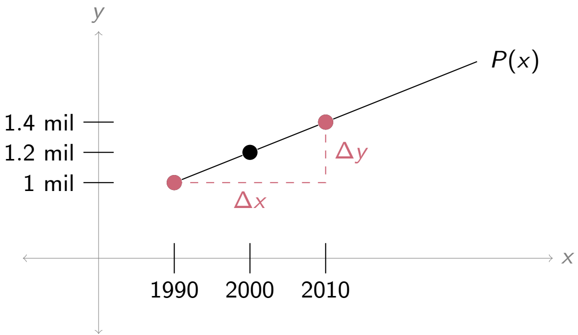

Rate of change: $\dfrac{400,000 \text{ people}}{20 \text{ years}} = 20,000 \dfrac{\text{people}}{\text{year}}$ (doesn't depend on the year)

Rates of Change

Suppose the population of a small country is given in the chart below.

Rates of Change

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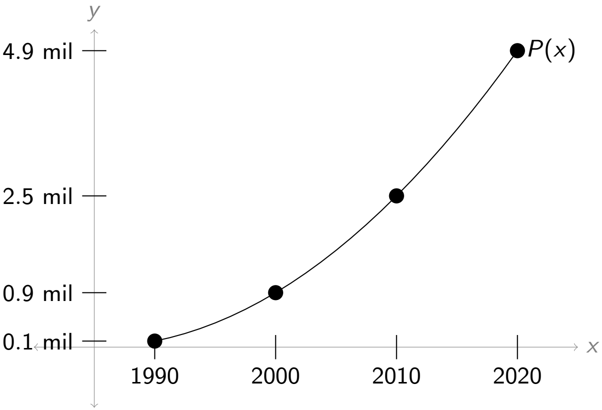

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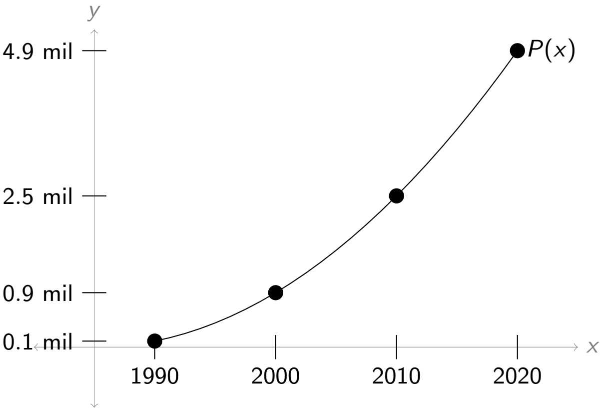
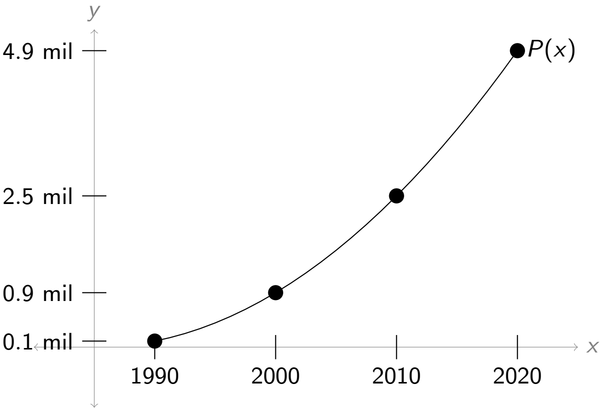


Rate of change: $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$

Rates of Change

Suppose the population of a small country is given in the chart below.



Rate of change: $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

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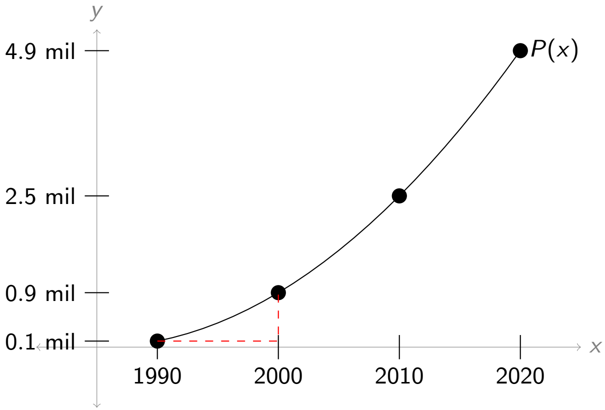

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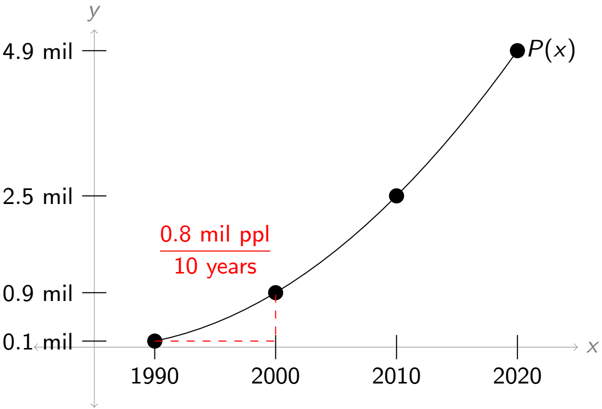

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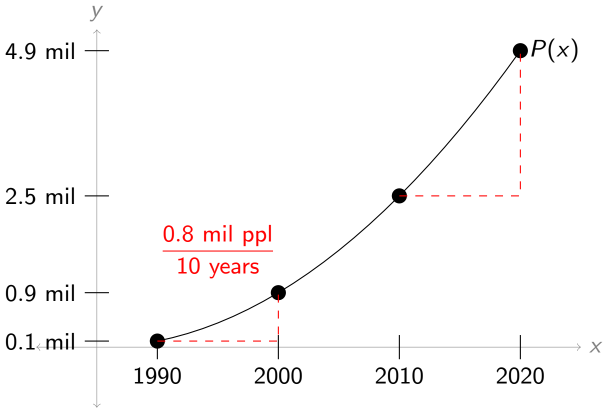

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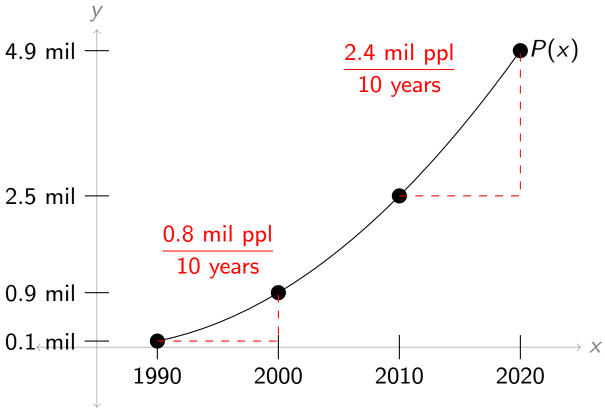

Rate of change: $\dfrac{\Delta \text{ pop}}{\Delta \text{ time}}$ depends on time interval

Definition

Let $y = f(x)$ be a curve that passes through (x_1, y_1) and (x_2, y_2). Then the **average rate of change** of $f(x)$ when $x_1 \leq x \leq x_2$ is

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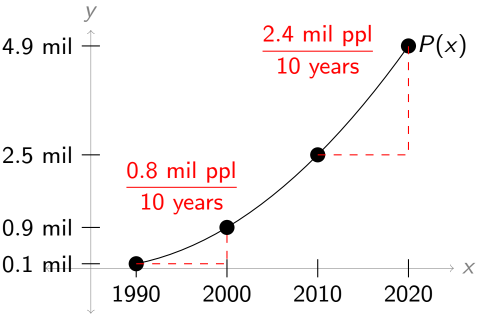

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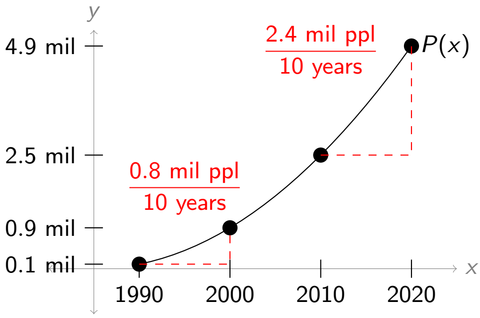

Average rate of change from 1990 to 2000: $80,000$ people per year.

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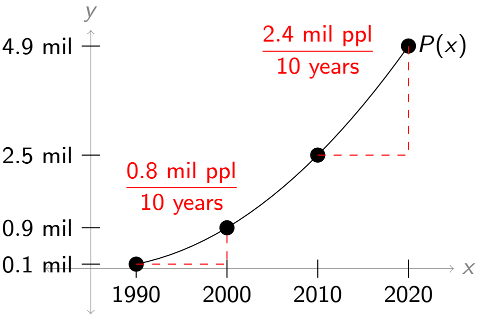

Average rate of change from 1990 to 2000: $80,000$ people per year.
Average rate of change from 2010 to 2020: $240,000$ people per year.

Average Rate of Change and Slope

What is the difference between average rate of change and slope?

Average Rate of Change and Slope

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Average Rate of Change and Slope

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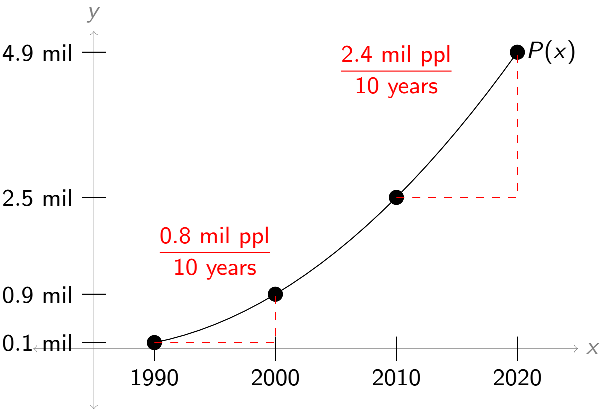
The average rate of change for a straight line is always the same, regardless of the interval we choose. We call it the slope of the line. If a curve is not a straight line, its average rate of change will differ over different intervals.

Rates of Change

How fast is the population growing in the year 2010?



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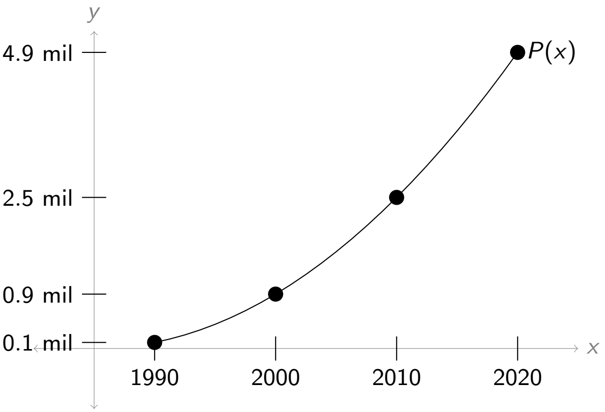

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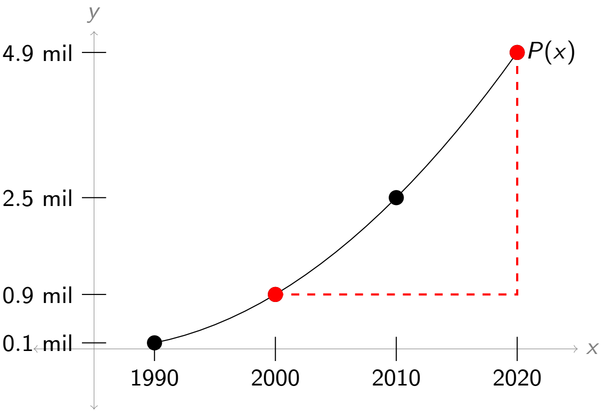

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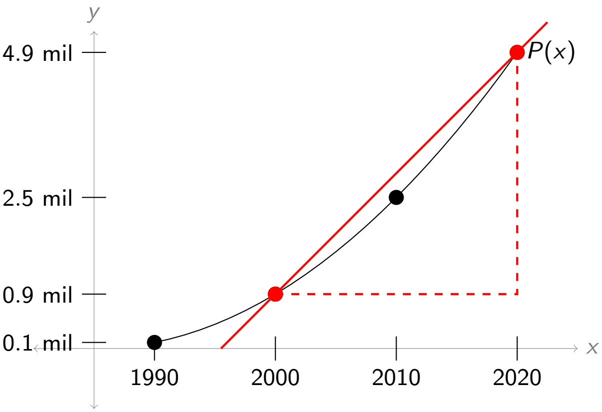

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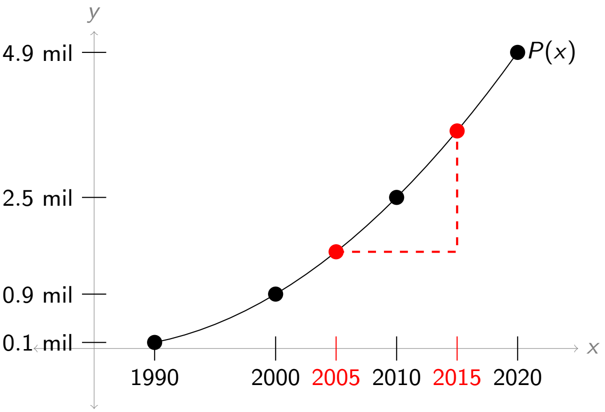

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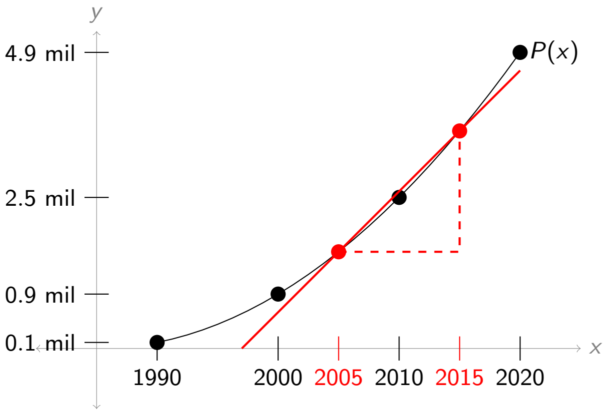

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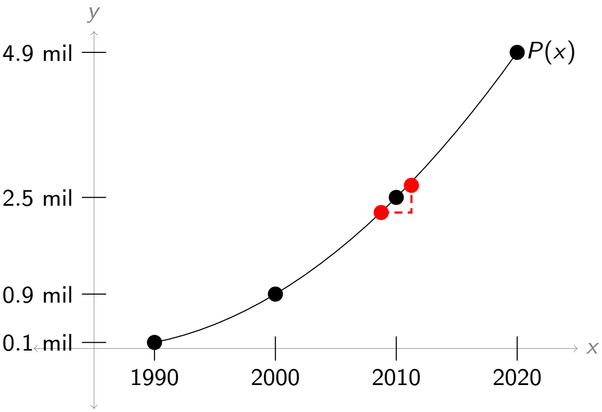

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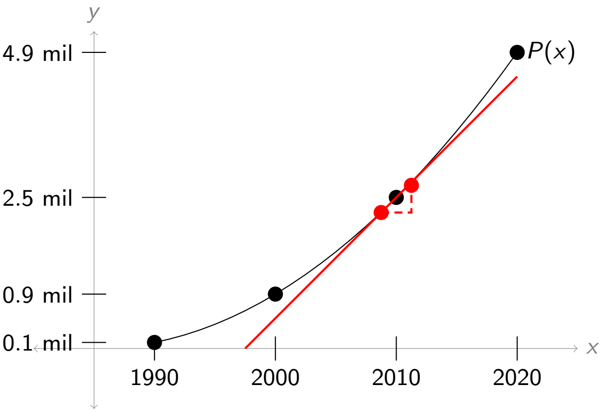

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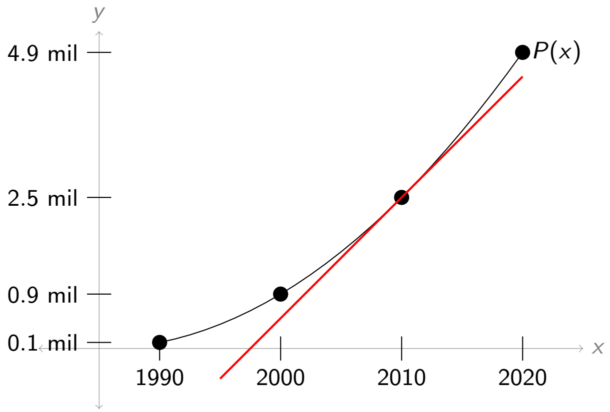

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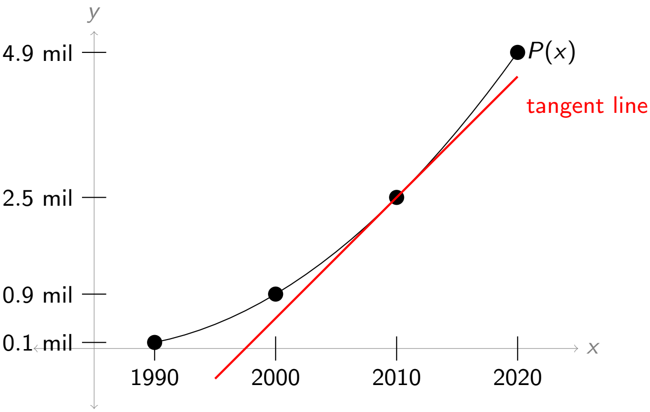

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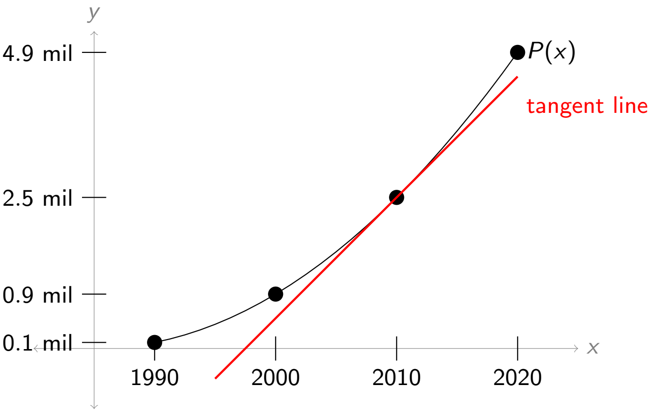

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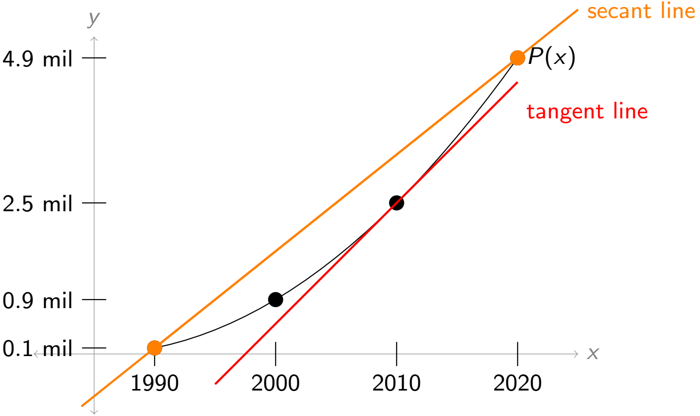
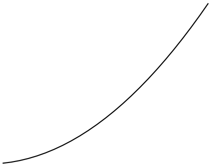


Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.



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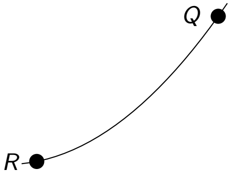

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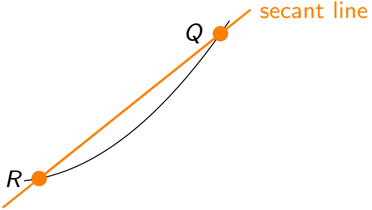
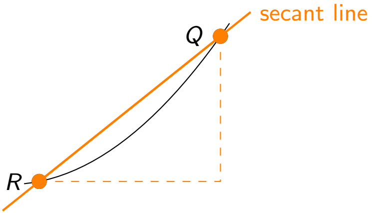


Definition

The **secant line** to the curve $y = f(x)$ through points R and Q is a line that passes through R and Q.
We call the slope of the secant line the **average rate of change of $f(x)$ from R to Q.**



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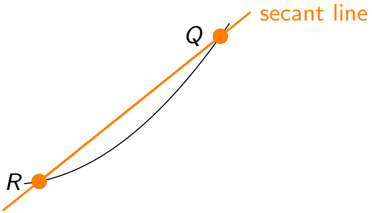

Definition

The **tangent line** to the curve $y = f(x)$ at point P is a line that

- passes through P and
- has the same slope as $f(x)$ at P.

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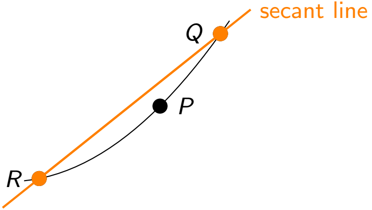

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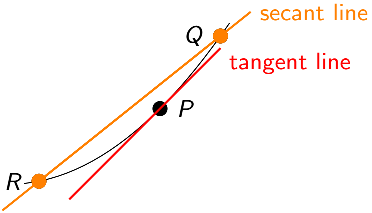

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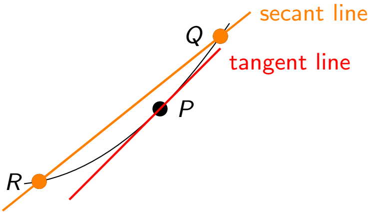


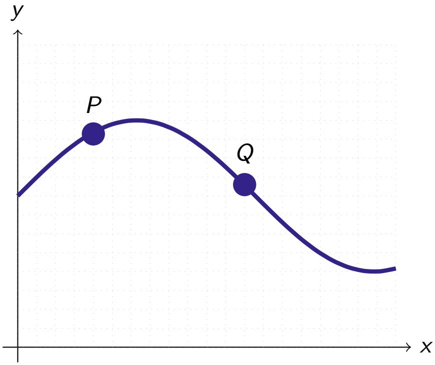
Definition

The **tangent line** to the curve $y = f(x)$ at point P is a line that

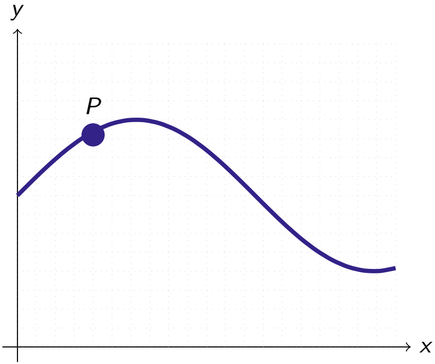
- passes through P and
- has the same slope as $f(x)$ at P.

We call the slope of the tangent line the **instantaneous rate of change of $f(x)$ at P.**

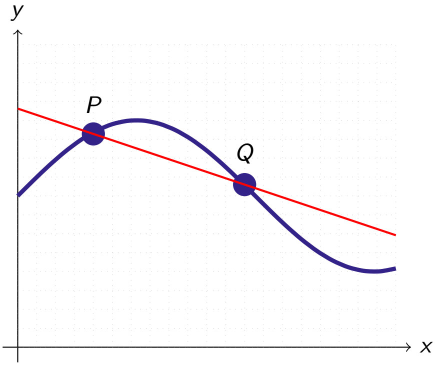
On the graph below, draw the secant line to the curve through points P and Q.



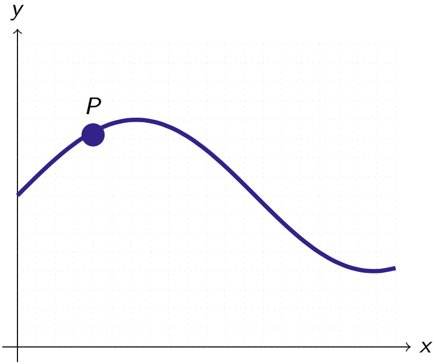
On the graph below, draw the tangent line to the curve at point P.



On the graph below, draw the secant line to the curve through points P and Q.



On the graph below, draw the tangent line to the curve at point P.



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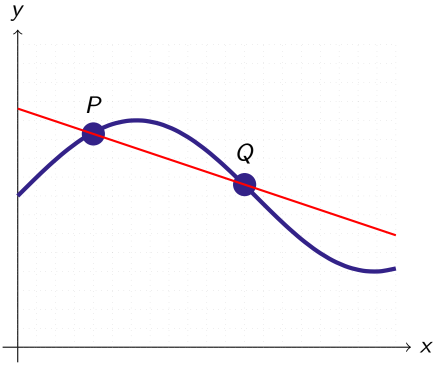
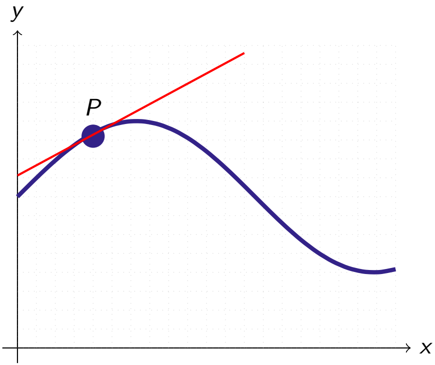
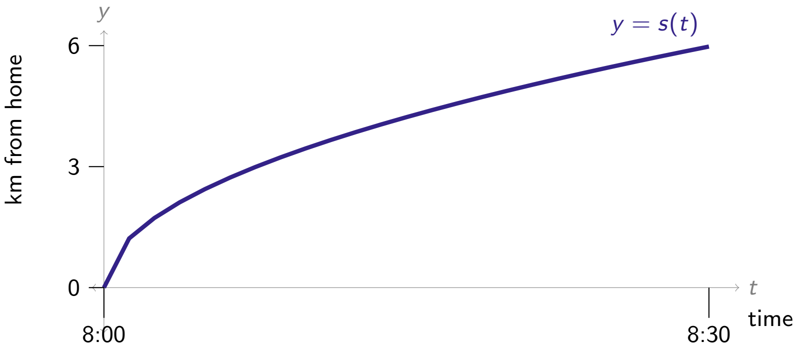
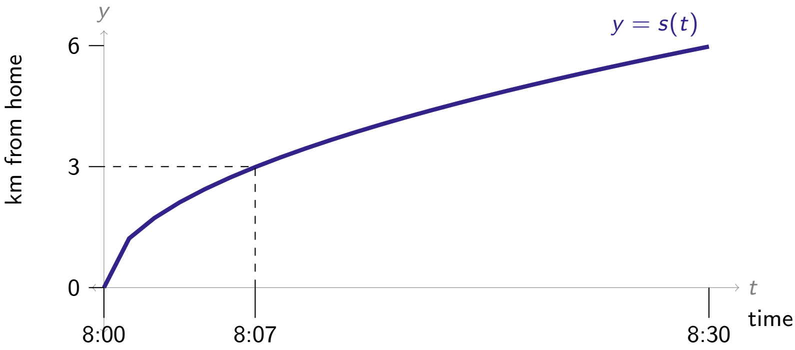
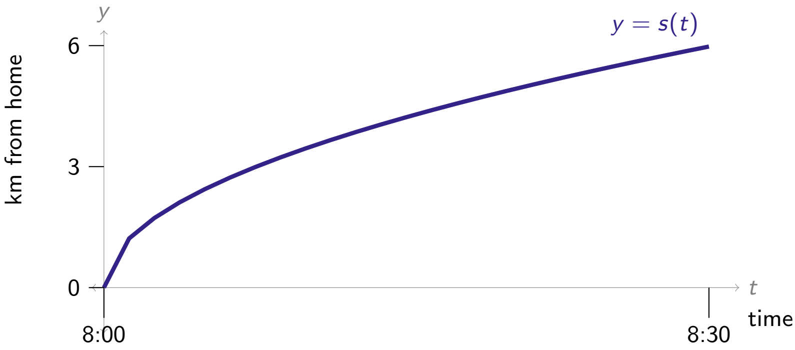


On the graph below, draw the tangent line to the curve at point P.




y
y = s(t)
6
3
0
km from home
t
time
8:00
8:30


y
$y = s(t)$
6
3
0
km from home
t
time
8:00
8:07
8:30



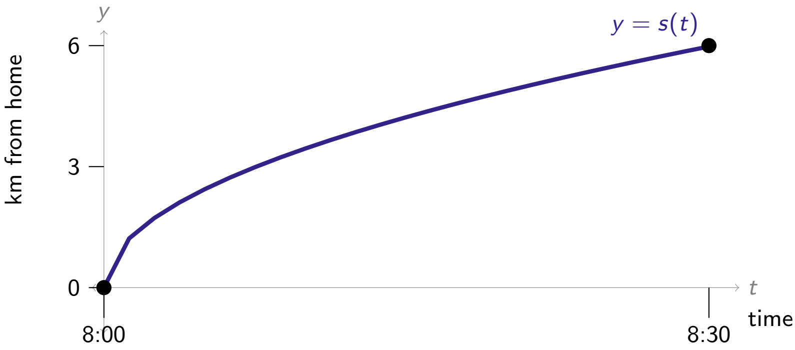
It takes me half an hour to bike 6 km. So, 12 kph represents the:

A. secant line to $y = s(t)$ from $t = 8:00$ to $t = 8:30$

B. slope of the secant line to $y = s(t)$ from $t = 8:00$ to $t = 8:30$

C. tangent line to $y = s(t)$ at $t = 8:30$

D. slope of the tangent line to $y = s(t)$ at $t = 8:30$



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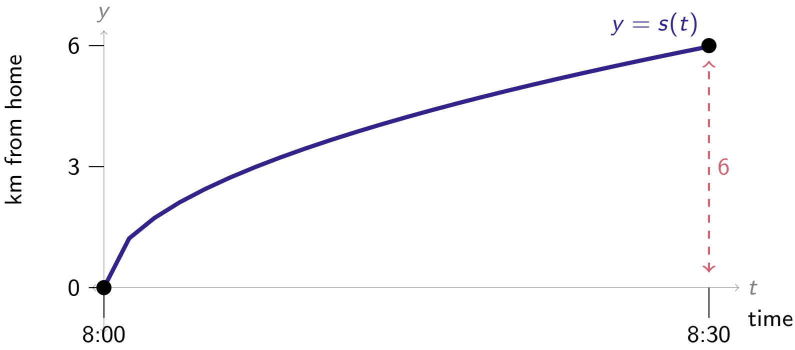

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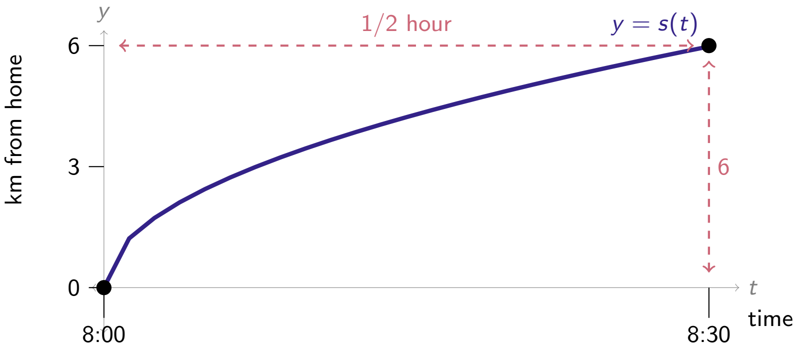

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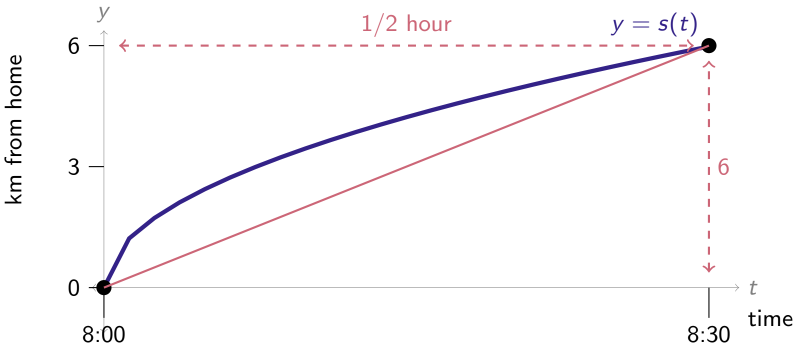

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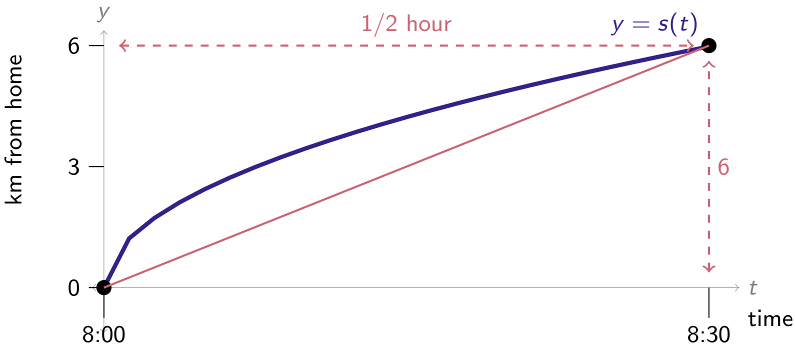

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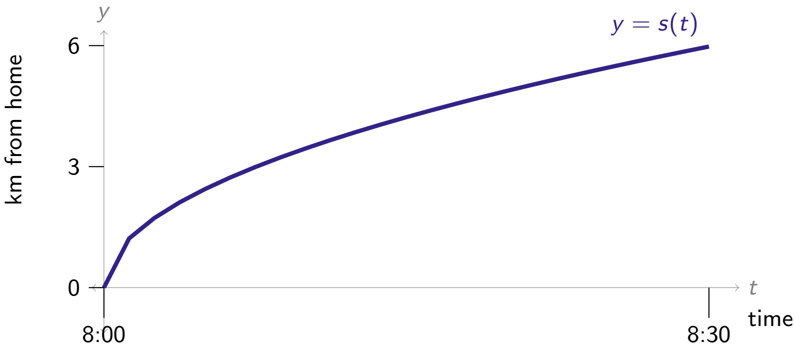

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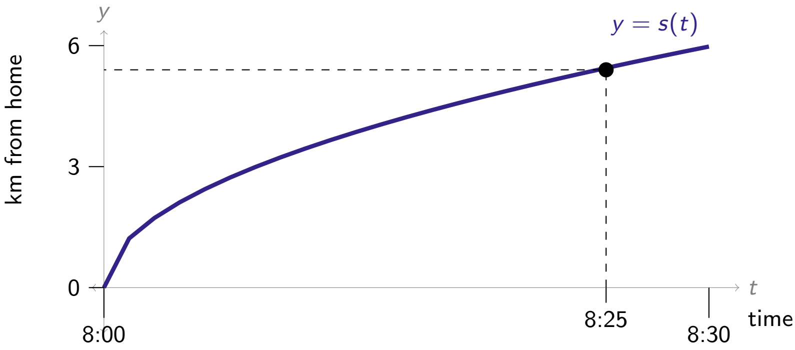

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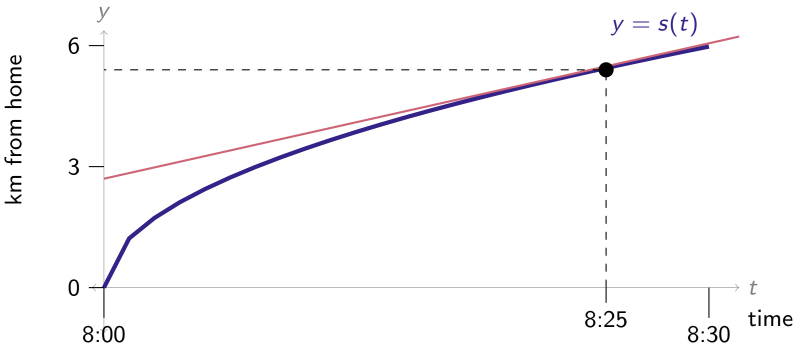

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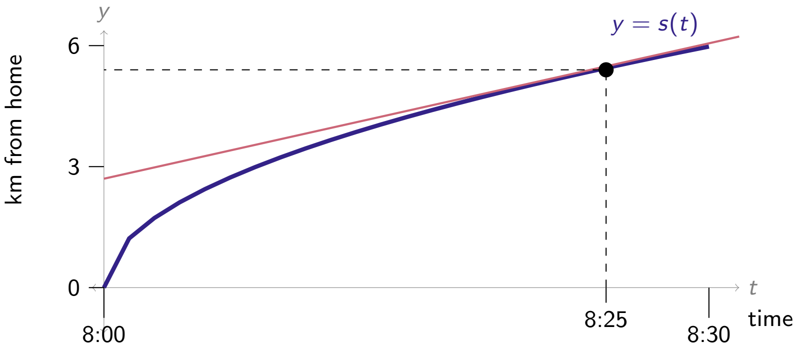

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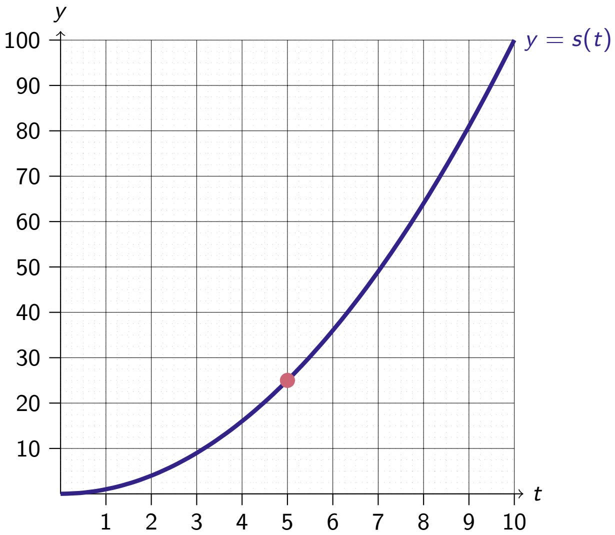

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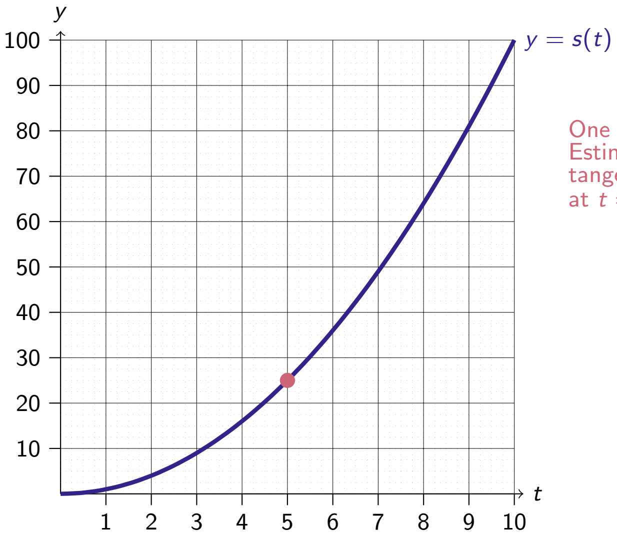

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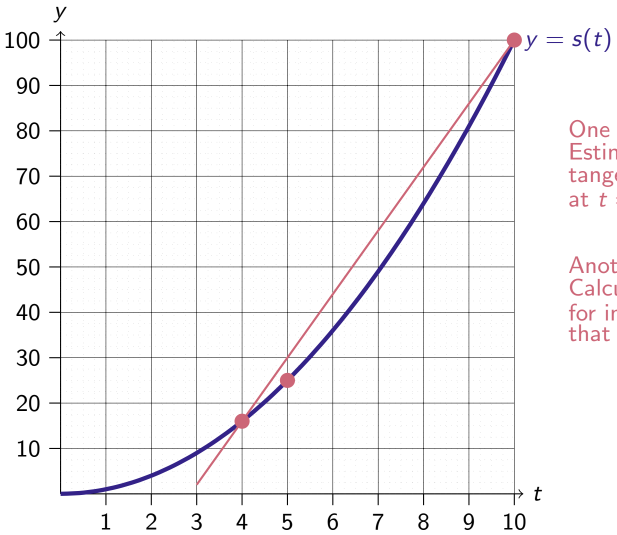

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Another way:
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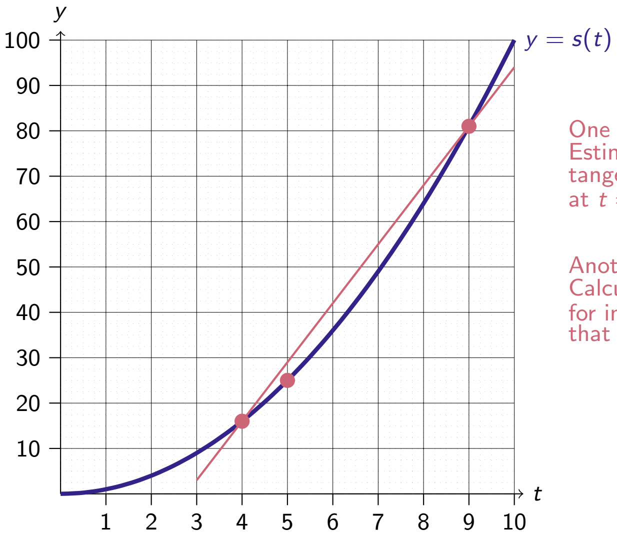

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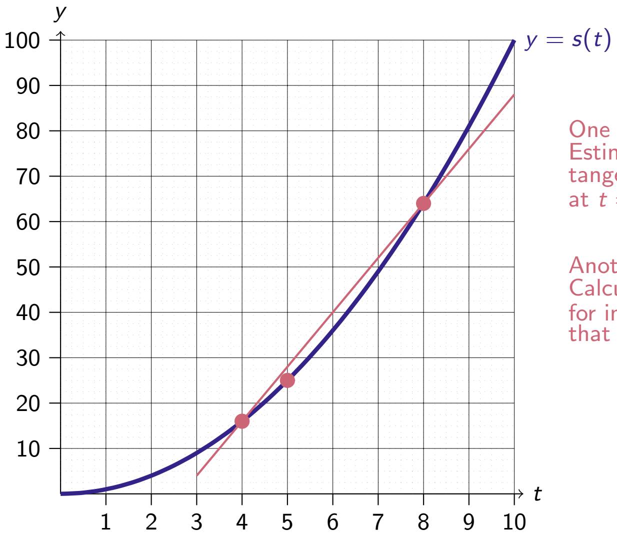

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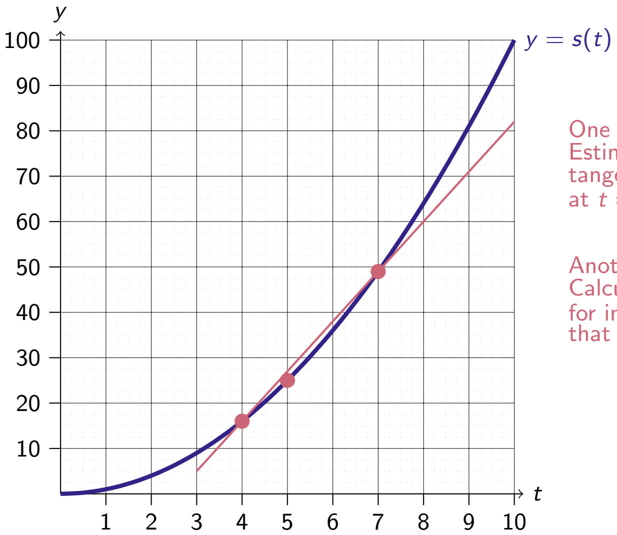

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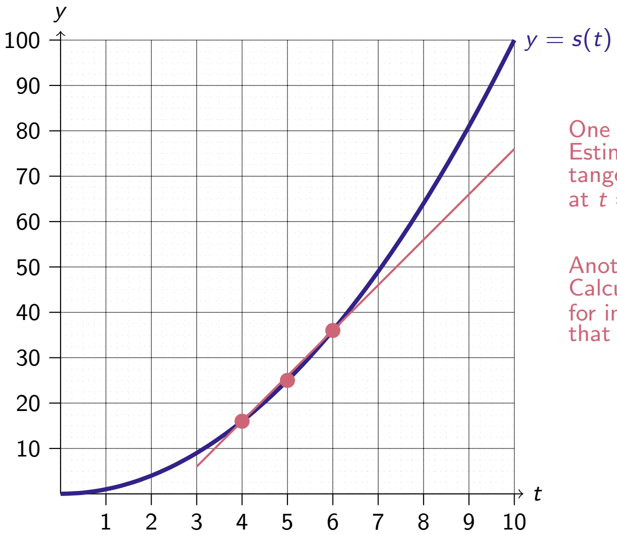

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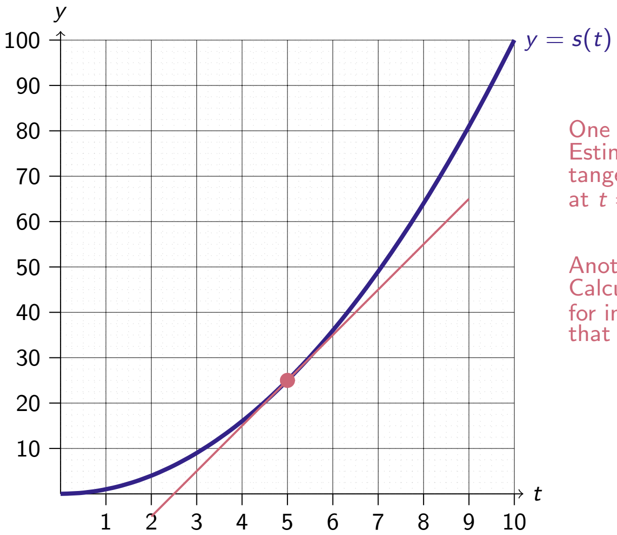

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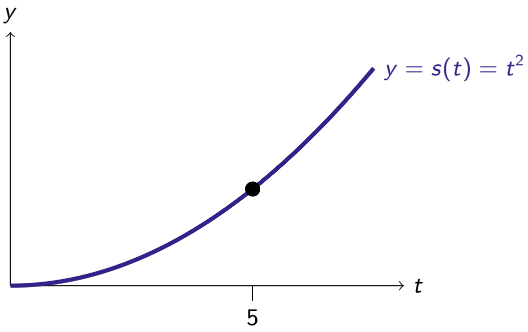

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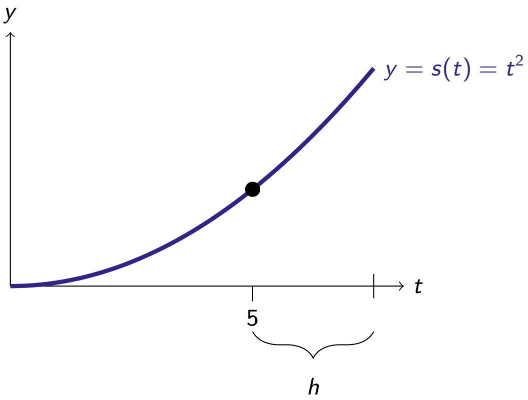

Suppose the interval [5,] has length h. What is the right endpoint of the interval?

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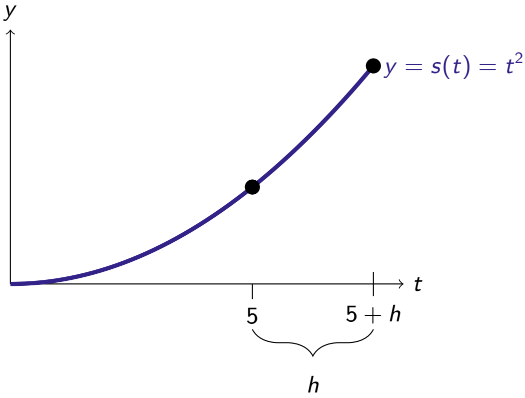

Write the equation for the average (vertical) velocity from $t = 5$ to $t = 5 + h$.

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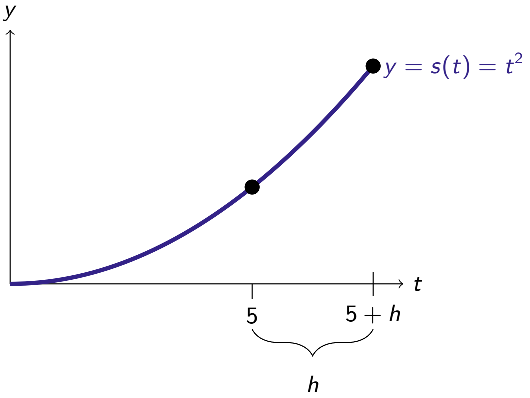

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$$vel = \frac{\Delta \text{ height}}{\Delta \text{ time}} = \frac{s(5+h) - s(5)}{(5+h) - 5} = \frac{(5+h)^2 - 5^2}{h}$$

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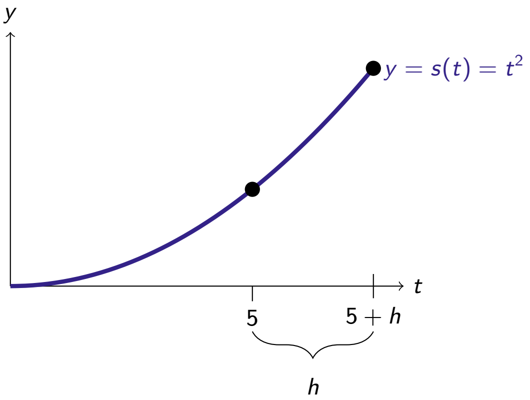

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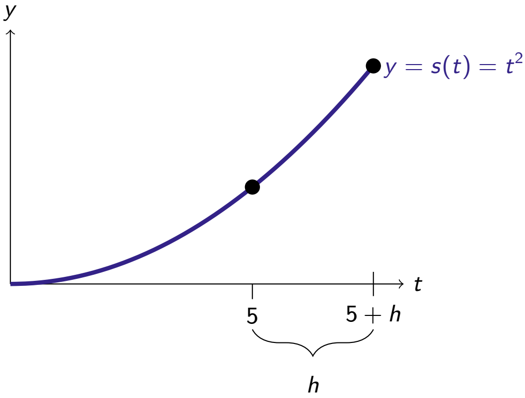
What happens to the equation above when h is very, very small?

Let's look for an algebraic way of determining the velocity of the balloon when $t = 5$.



What do you think is the slope of the *tangent* line to the graph when $t = 5$?

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Average Velocity, $t = 5$ to $t = 5 + h$:

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As h gets extremely close to 0, $(10 + h)$ gets extremely close to 10.

Definition

$$\lim_{x \to a} f(x) = L$$

where a and L are real numbers
We read the above as "the limit as x goes to a of $f(x)$ is L."
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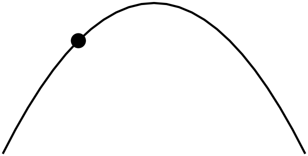

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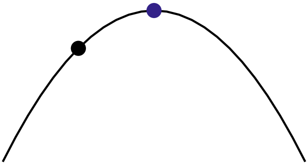

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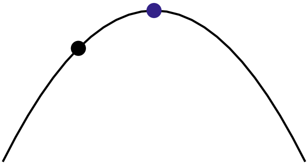

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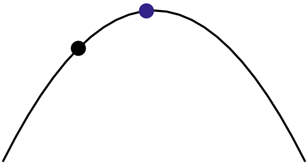

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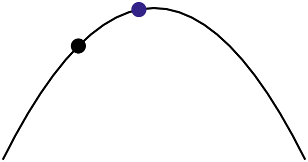

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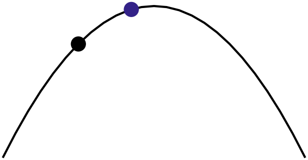

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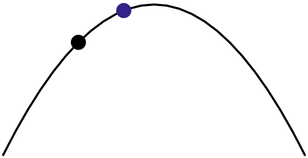

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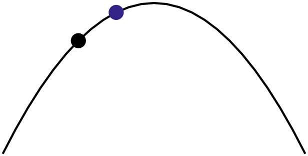
If the position of an object at time t is given by $s(t)$, then its instantaneous velocity is given by $\displaystyle\lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$.

Definition

$$\lim_{x \to a} f(x) = L$$

where a and L are real numbers
We read the above as "the limit as x goes to a of $f(x)$ is L."
Its meaning is: as x gets very close to (but not equal to) a, $f(x)$ gets very close to L.

We DESPERATELY NEED limits to find slopes of tangent lines.



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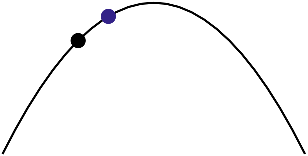

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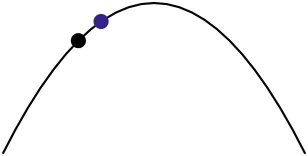

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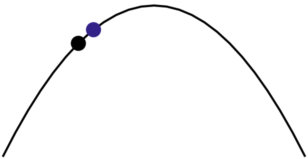

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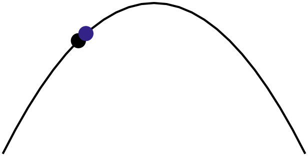

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We want to evaluate $\lim_{x \to 1} f(x)$.

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x	$f(x)$
0.9	3.61
0.99	3.9601
0.999	3.99600
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1.1	4.41
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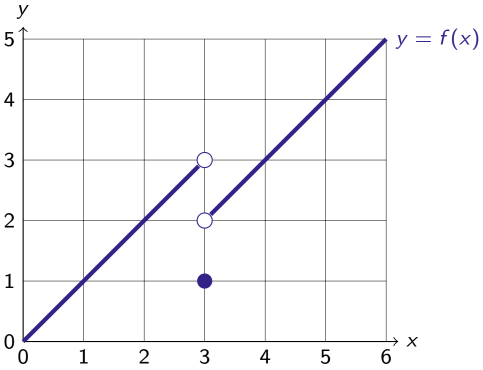

What do you think should be the value of $\lim_{x \to 3} f(x)$?

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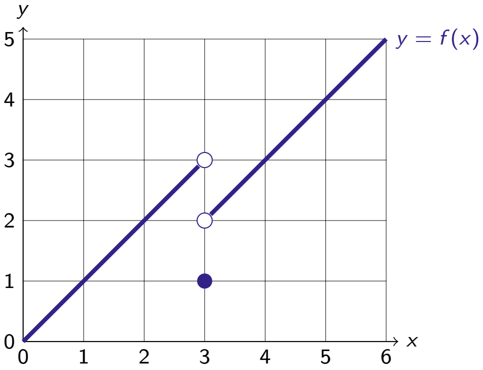

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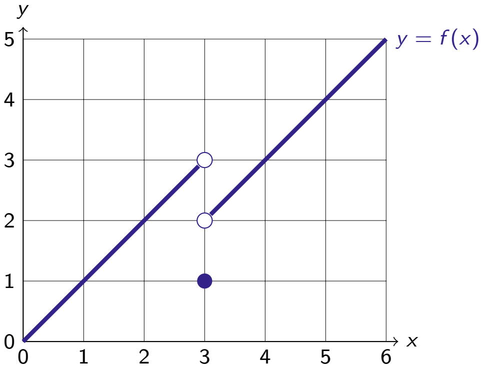

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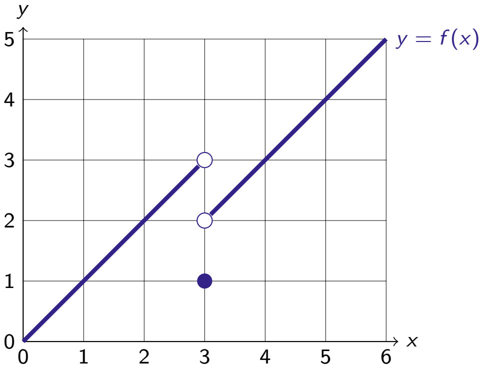

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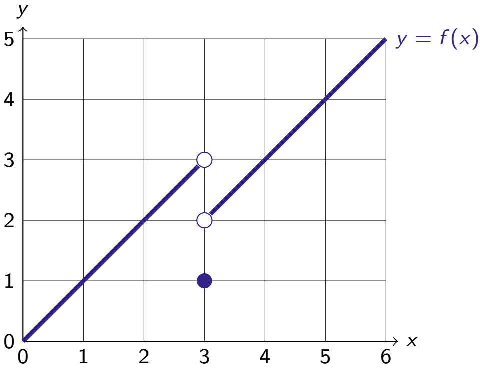

What do you think should be the value of $\lim_{x\to 3} f(x)$? DNE

Evaluate: $\underbrace{\lim_{x\to 3^-} f(x)}_{\text{from the left}} = 3$ $\underbrace{\lim_{x\to 3^+} f(x)}_{\text{from the right}} = 2$

Definition

The limit as x goes to a **from the left** of $f(x)$ is written

$$\lim_{x \to a^-} f(x)$$

We only consider values of x that are *less than a*.

Definition

The limit as x goes to a **from the right** of $f(x)$ is written

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Theorem

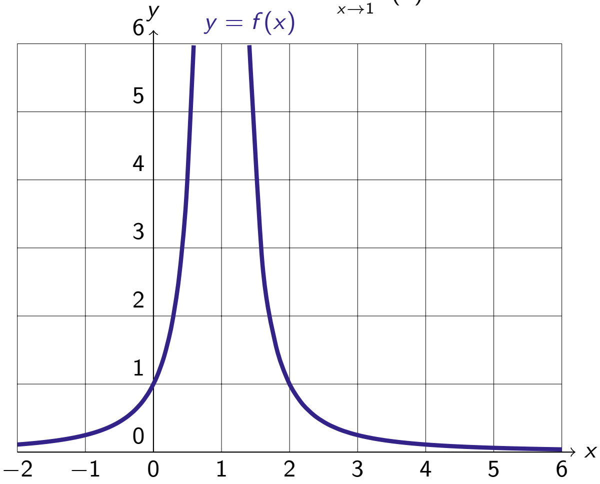
In order for $\lim_{x \to a} f(x)$ to exist, both one-sided limits must exist and be equal.

Consider the function $f(x) = \dfrac{1}{(x-1)^2}$.
For what value of x is $f(x)$ *not* defined?

Consider the function $f(x) = \dfrac{1}{(x-1)^2}$.

Based on the graph below, what would you like to write for:

$$\lim_{x \to 1} f(x) =$$



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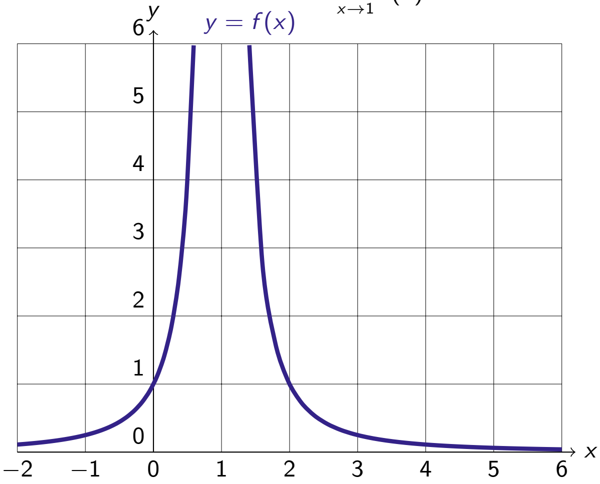

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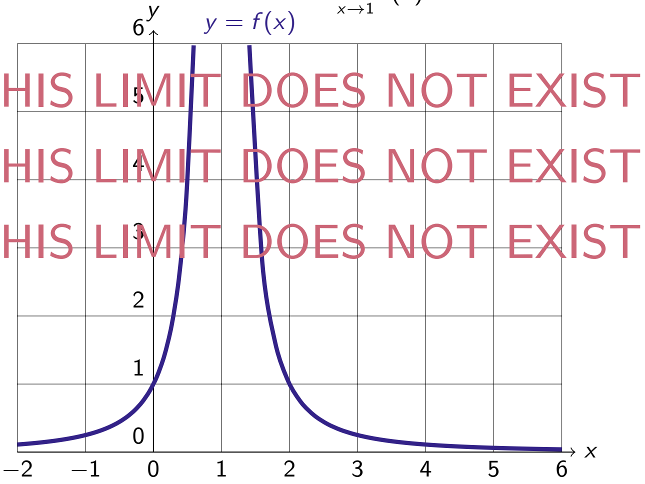
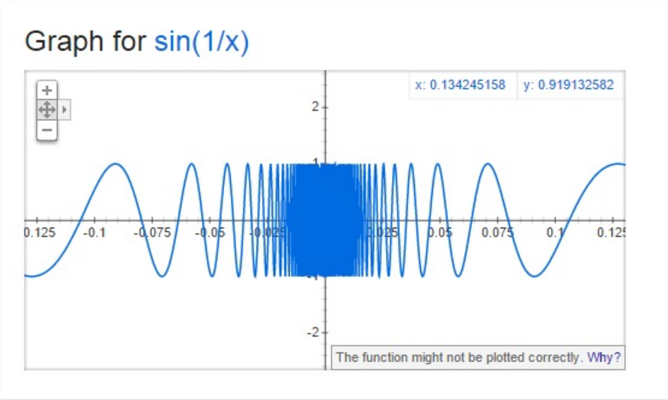


Let $f(x) = \sin\left(\frac{1}{x}\right)$. Graphed by Google:



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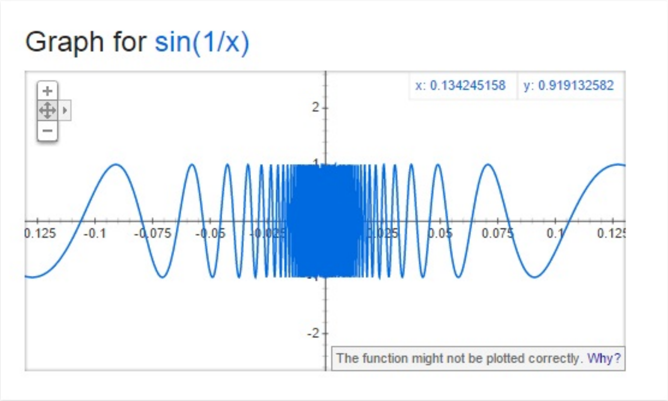

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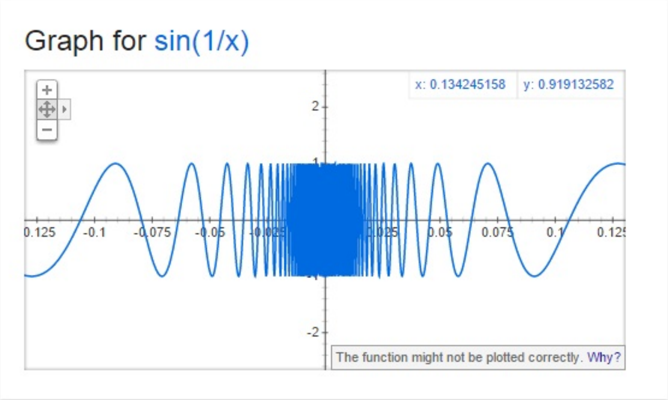
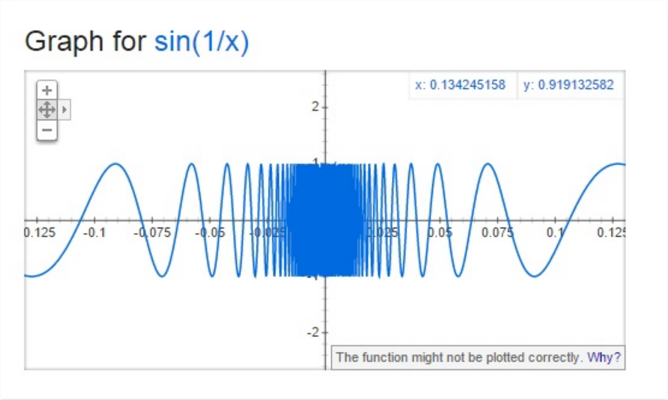


To understand why it looks this way, first evaluate: $\lim\limits_{x \to 0^+} \frac{1}{x} = \infty$

Now, what should the graph of $f(x)$ look like when x is near 0?

Let $f(x) = \sin\left(\frac{1}{x}\right)$. Graphed by Google:



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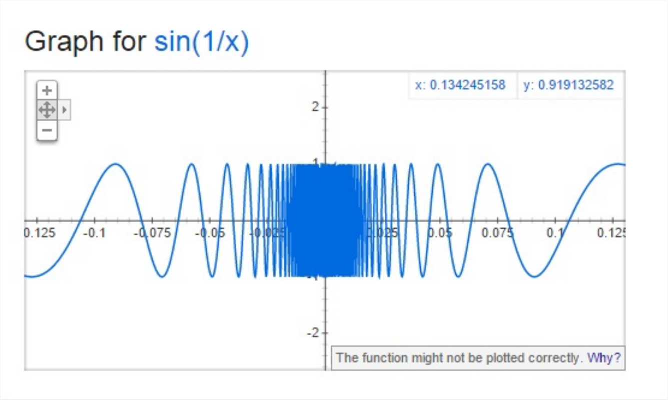

What is $\lim_{x \to 0} f(x)$? DNE. This is sometimes called "infinite wiggling"

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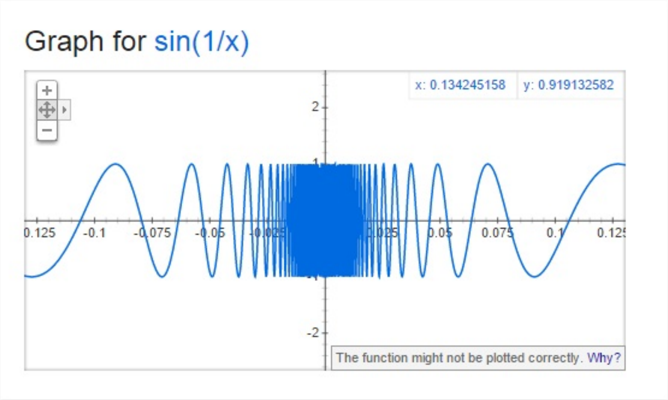
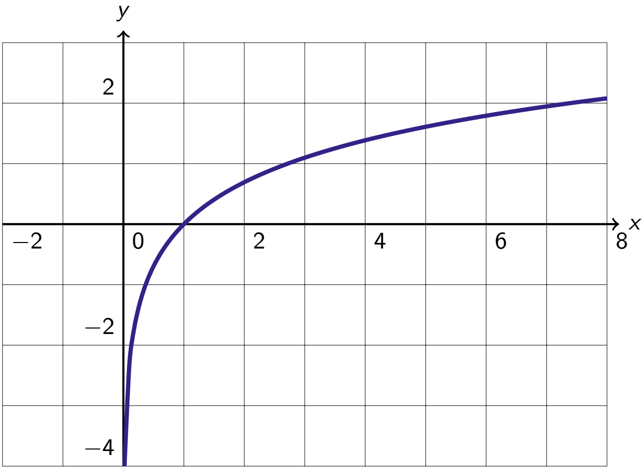


What is $\lim_{x \to 0} f(x)$? DNE. This is sometimes called "infinite wiggling"

Does a limit exist for other points?

Suppose $f(x) = \log(x)$.



Where is $f(x)$ defined, and where is it not defined?

What can you say about the limit of $f(x)$ near 0?

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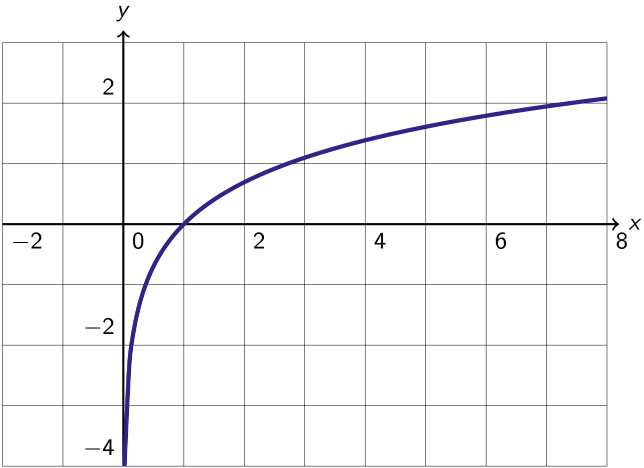
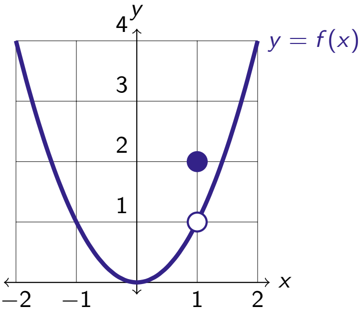


Where is $f(x)$ defined, and where is it not defined?

What can you say about the limit of $f(x)$ near 0? $\lim_{x \to 0^+} \log(x) = -\infty$

$$f(x) = \begin{cases} x^2 & x \neq 1 \\ 2 & x = 1 \end{cases}$$



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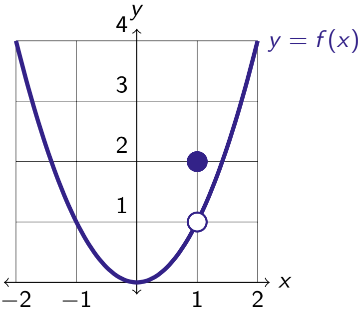

What is $\lim_{x\to 1} f(x)$?

A. $\lim_{x\to 1} f(x) = 2$

B. $\lim_{x\to 1} f(x) = 1$

C. $\lim_{x\to 1} f(x)$ DNE

D. none of the above

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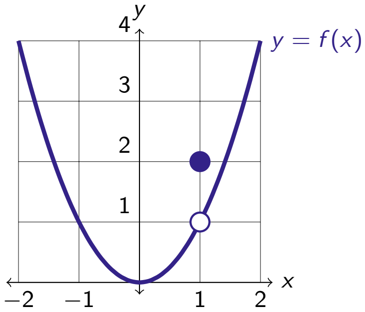

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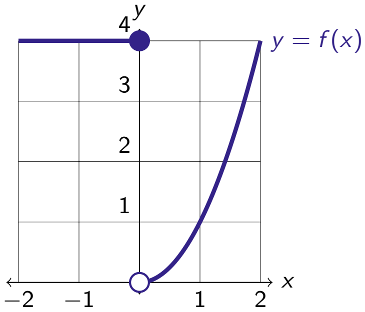

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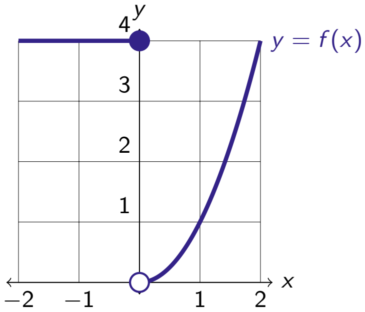

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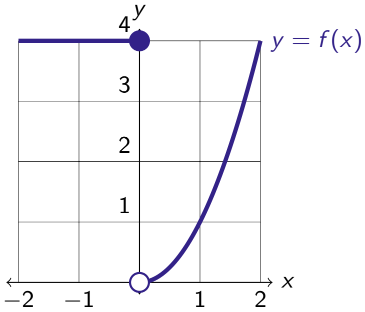

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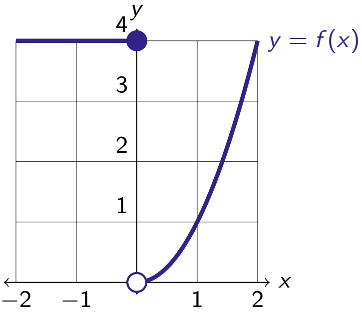

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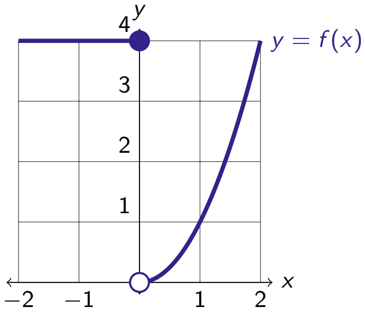

$$\lim_{x \to 0^-} f(x) = 4$$

"the limit as x approaches 0 from the left is 4"

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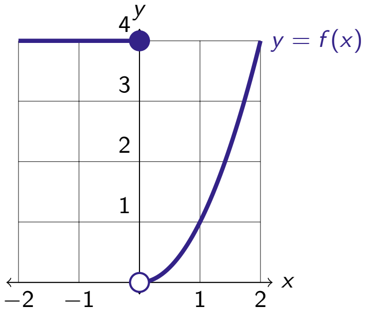

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$$\lim_{x \to 0^-} f(x) = 4$$

"the limit as x approaches 0 from the left is 4"

$$\lim_{x \to 0^+} f(x) = 0$$

"the limit as x approaches 0 from the right is 0"

Suppose $\lim_{x \to 3^-} f(x) = 1$ and $\lim_{x \to 3^+} f(x) = 1.5$. Does $\lim_{x \to 3} f(x)$ exist?

Suppose $\lim_{x \to 3^-} f(x) = 1$ and $\lim_{x \to 3^+} f(x) = 1.5$. Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist.

B. No, never, because the limit from the left is not the same as the limit from the right.

C. Can't tell. For some functions is might exist, for others not.

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Suppose $\lim_{x \to 3^-} f(x) = 22 = \lim_{x \to 3^+} f(x)$. Does $\lim_{x \to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist and are equal to each other.
B. No, never, because we only talk about one-sided limits when the actual limit doesn't exist.
C. Can't tell. We need to know the value of the function at $x = 3$.

Suppose $\lim_{x\to 3^-} f(x) = 1$ and $\lim_{x\to 3^+} f(x) = 1.5$. Does $\lim_{x\to 3} f(x)$ exist?

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Suppose $\lim_{x\to 3^-} f(x) = 22 = \lim_{x\to 3^+} f(x)$. Does $\lim_{x\to 3} f(x)$ exist?

A. Yes, certainly, because the limits from both sides exist and are equal to each other.
B. No, never, because we only talk about one-sided limits when the actual limit doesn't exist.
C. Can't tell. We need to know the value of the function at $x = 3$.

Calculating Limits in Simple Situations

Direct Substitution

If $f(x)$ is a polynomial or rational function, and a is in the domain of f, then:

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Calculate: $\lim_{x \to 3} \left(\frac{x^2 - 9}{x - 3} \right)$ Can't do it the same way: 3 not in domain

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Algebra with Limits

Suppose $\lim_{x \to a} f(x) = F$ and $\lim_{x \to a} g(x) = G$, where F and G are both real numbers. Then:

- $\lim_{x \to a}(f(x) + g(x)) = F + G$
- $\lim_{x \to a}(f(x) - g(x)) = F - G$
- $\lim_{x \to a}(f(x)g(x)) = FG$
- $\lim_{x \to a}(f(x)/g(x)) = F/G$ PROVIDED $G \neq 0$

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Calculate: $\lim\limits_{x\to 1}\left[\dfrac{2x+4}{x+2}+13\left(\dfrac{x+5}{3x}\right)\left(\dfrac{x^2}{2x-1}\right)\right]$

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Direct Substitution

Algebra with Limits

Calculate: $\lim_{x\to 1}\left[\frac{2x+4}{x+2}+13\left(\frac{x+5}{3x}\right)\left(\frac{x^2}{2x-1}\right)\right]$

$$= \lim_{x\to 1}\left(\frac{2x+4}{2x+2}\right)+\left(\lim_{x\to 1} 13\right)\left(\lim_{x\to 1}\frac{x+5}{3x}\right)\left(\frac{x^2}{2x-1}\right)$$

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$$= \left(\frac{2(1)+4}{2(1)+2} \right) + (13) \left(\frac{(1)+5}{3(1)} \right) \left(\frac{1^2}{2(1)-1} \right)$$

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$$= (2)+13(2)(1) \qquad = 28$$

Powers and Roots of Limits

Which of the following gives a real number?

A. $4^{1/2}$ B. $(-4)^{1/2}$ C. $4^{-1/2}$ D. $(-4)^{-1/2}$

Powers and Roots of Limits

Which of the following gives a real number?

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If n is a positive integer, and $\lim\limits_{x\to a} f(x) = F$ (where F is a real number), then:

$$\lim_{x\to a} (f(x))^n = F^n.$$

Furthermore,

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UNLESS n is even and F is negative.

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Cautionary Tales

$$\lim_{x \to 0} \frac{(5+x)^2 - 25}{x}$$

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$$\lim_{x \to 0} \frac{(5+x)^2 - 25}{x} \to \frac{0}{0}; \text{ NEED ANOTHER WAY}$$

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$$\lim_{x\to 0} \frac{(5+x)^2-25}{x} \to \frac{0}{0}\text{; NEED ANOTHER WAY}$$

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$$\lim_{x\to 0} \frac{32}{x}$$

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$$\lim_{x\to 5} \left(x^2+2\right)^{1/3}$$

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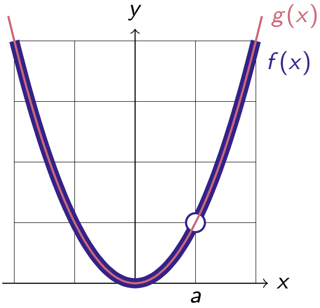
$$\lim_{x \to 5} \left(x^2 + 2\right)^{1/3} = (5^2 + 2)^{1/3} = \sqrt[3]{27} = 3$$

Functions that Differ at a Single Point

Suppose $\lim_{x \to a} g(x)$ exists, and $f(x) = g(x)$ when x is close to a (but not necessarily equal to a).

Then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$.



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Evaluate $\lim_{x \to 1} \dfrac{x^3 + x^2 - x - 1}{x - 1}$.

$$\begin{aligned} \frac{x^3 + x^2 - x - 1}{x - 1} &= \frac{(x+1)^2(x-1)}{x-1} \\ &= (x+1)^2 \text{ whenever } x \neq 1 \end{aligned}$$

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So, $\lim_{x\to 1} \dfrac{x^3 + x^2 - x - 1}{x - 1} = \lim_{x\to 1} (x+1)^2 = 4$

Evaluate $\lim\limits_{x \to 5} \dfrac{\sqrt{x+20} - \sqrt{4x+5}}{x-5}$

Evaluate $\lim\limits_{x\to 5} \dfrac{\sqrt{x+20}-\sqrt{4x+5}}{x-5}$

$$\begin{aligned}
\frac{\sqrt{x+20}-\sqrt{4x+5}}{x-5} &= \frac{\sqrt{x+20}-\sqrt{4x+5}}{x-5}\left(\frac{\sqrt{x+20}+\sqrt{4x+5}}{\sqrt{x+20}+\sqrt{4x+5}}\right)\\
&= \frac{(x+20)-(4x+5)}{(x-5)(\sqrt{x+20}+\sqrt{4x+5})}\\
&= \frac{-3x+25}{(x-5)(\sqrt{x+20}+\sqrt{4x+5})}\\
&= \frac{-3}{\sqrt{x+20}+\sqrt{4x+5}}
\end{aligned}$$

So,

$$\begin{aligned}
\lim_{x\to 5} \frac{\sqrt{x+20}-\sqrt{4x+5}}{x-5} &= \lim_{x\to 5} \frac{-3}{\sqrt{x+20}+\sqrt{4x+5}}\\
&= \frac{-3}{\sqrt{5+20}+\sqrt{4(5)+5}} = \frac{-3}{10}
\end{aligned}$$

A Few Strategies for Calculating Limits

First, hope that you can directly substitute (plug in). If your function is made up of the sum, difference, product, quotient, or power of ploynomials, you can do this PROVIDED the function exists where you're taking the limit.

$$\lim_{x\to 1}\left(\sqrt{35+x^5}+\frac{x-3}{x^2}\right)^3 =$$

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If you have a function as described above and the point where you're taking the limit is NOT in its domain, if you think the limit exists, try to simplify and cancel.

$$\lim_{x\to 0}\frac{x+7}{\frac{1}{x}-\frac{1}{2x}}$$

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$$\lim_{x\to 0}\frac{x+7}{\frac{1}{x}-\frac{1}{2x}}=\lim_{x\to 0}\frac{x+7}{\frac{2}{2x}-\frac{1}{2x}}=\lim_{x\to 0}\frac{x+7}{\frac{1}{2x}}=\lim_{x\to 0}2x(x+7)=0$$

Otherwise, you can try graphing the function, or making a table of values, to get a better picture of what is going on.

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As x gets close to zero, the denominator gets very tiny, and the numberator is close to 1. Since a tiny, tiny , tiny number "goes into" 1 many, many, many times, the value of $\frac{x-1}{x}$ will get very large–but might be positive or negative depending on the sign of x.

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$$\lim_{x \to 0} \frac{x-1}{x^2} =$$

$$\lim_{x \to -4^-} \frac{-3}{\sqrt{x^2} - 4} =$$

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Suppose a lemonade stand diversifies to sell lemonade, raspberry juice, and raspberry lemonade. The prices of ingredients fluctuate, but the sale price of raspberry lemonade is always between the sale prices of lemonade and raspberry juice. One day, as a promotion, the stand sells lemonade and raspberry juice each for \$1 a cup. How much do they sell raspberry lemonade for on that day?

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Always: lemonade $\leq$ raspberry lemonade $\leq$ raspberry juice

Today: lemonade $=$ \$1 $=$ raspberry juice

Suppose a lemonade stand diversifies to sell lemonade, raspberry juice, and raspberry lemonade. The prices of ingredients fluctuate, but the sale price of raspberry lemonade is always between the sale prices of lemonade and raspberry juice. One day, as a promotion, the stand sells lemonade and raspberry juice each for \$1 a cup. How much do they sell raspberry lemonade for on that day?

Always: lemonade $\leq$ raspberry lemonade $\leq$ raspberry juice

Today: lemonade $= \$1 =$ raspberry juice

So, today also raspberry lemonade $= \$1$

Squeeze Theorem

Suppose, when x is near (but not necessarily equal to) a, we have functions $f(x)$, $g(x)$, and $h(x)$ so that

$$f(x) \leq g(x) \leq h(x)$$

and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x)$. Then $\lim_{x \to a} g(x) = \lim_{x \to a} f(x)$.

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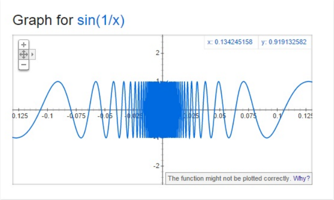

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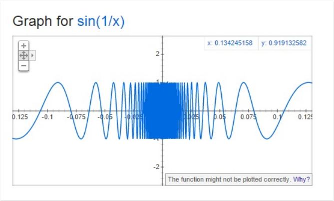

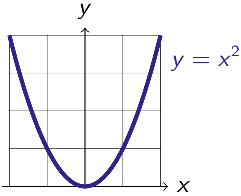

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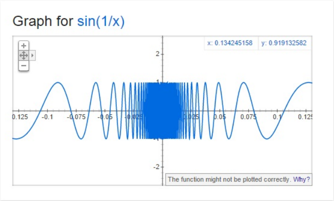

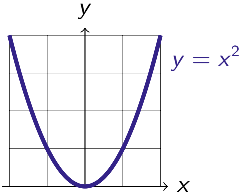

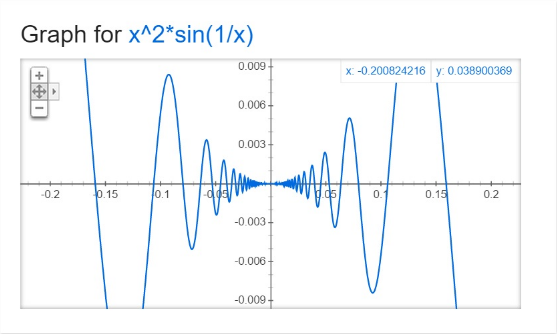

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 & -1 & \leq & \sin\left(\frac{1}{x}\right) & \leq & 1 \\
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\text{and also} & \lim\limits_{x\to 0} -x^2 & = & 0 & = & \lim\limits_{x\to 0} x^2
\end{array}$$

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Therefore, by the Squeeze Theorem, $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

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Squeeze Theorem

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Limits at Infinity

End Behavior

We write:

$$\lim_{x \to \infty} f(x) = L$$

to express that, as x grows larger and larger, $f(x)$ approaches L.
Similarly, we write:

$$\lim_{x \to -\infty} f(x) = L$$

to express that, as x grows more and more strongly negative, $f(x)$ approaches L.
If L is a number, we call $y = L$ a horizontal asymptote of $f(x)$.

Common Limits at Infinity

$$\lim_{x\to\infty} 13 =$$

$$\lim_{x\to-\infty} 13 =$$

$$\lim_{x\to\infty} \frac{1}{x} =$$

$$\lim_{x\to-\infty} \frac{1}{x} =$$

$$\lim_{x\to\infty} x^2 =$$

$$\lim_{x\to-\infty} x^2 =$$

$$\lim_{x\to\infty} x^3 =$$

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$$\lim_{x\to-\infty} x^{5/3} =$$

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Arithmetic with Limits at Infinity

$$\lim_{x \to \infty} \left(x + \frac{x^2}{10} \right) =$$

$$\lim_{x \to \infty} \left(x - \frac{x^2}{10} \right) =$$

$$\lim_{x \to -\infty} \left(x^2 + x^3 + x^4 \right) =$$

$$\lim_{x \to -\infty} (x + 13) \left(x^2 + 13 \right)^{1/3} =$$

Arithmetic with Limits at Infinity

$$\lim_{x\to\infty}\left(x+\frac{x^2}{10}\right)=\infty$$

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$$\lim_{x\to-\infty}\left(x^2+x^3+x^4\right)=\lim_{x\to-\infty}x^4\left(\frac{1}{x^2}+\frac{1}{x}+1\right)$$

$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=$$

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$$\lim_{x\to-\infty}(x+13)\left(x^2+13\right)^{1/3}=-\infty$$

Calculating Limits at Infinity

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Trick: factor out largest power of denominator.

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$$\begin{aligned}\lim_{x\to\infty} \frac{x^2+2x+1}{x^3} &= \lim_{x\to\infty} \frac{x^2+2x+1}{x^3}\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)\\ &= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^2}+\frac{1}{x^3}}{1}\end{aligned}$$

Now, you can do algebra

$$\begin{aligned}&= \frac{\lim\limits_{x\to\infty}\frac{1}{x}+\lim\limits_{x\to\infty}\frac{2}{x^2}+\lim\limits_{x\to\infty}\frac{1}{x^3}}{\lim\limits_{x\to\infty} 1}\\ &= \frac{0+0+0}{1} = 0\end{aligned}$$

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$$\lim_{x \to -\infty} (x^{7/3} - x^{5/3})$$

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Again: factor out largest power of x.

$$(x^{7/3} - x^{5/3}) = x^{7/3}\left(1 - \frac{1}{x^{2/3}}\right)$$

$$\lim_{x \to -\infty} x^{7/3} = -\infty$$

$$\lim_{x \to -\infty} \left(1 - \frac{1}{x^{2/3}}\right) = 1$$

Calculating Limits at Infinity

$$\lim_{x \to -\infty} (x^{7/3} - x^{5/3})$$

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$$\lim_{x \to -\infty} x^{7/3} = -\infty$$

$$\lim_{x \to -\infty} \left(1 - \frac{1}{x^{2/3}}\right) = 1$$

So, $\lim_{x \to -\infty} (x^{7/3} - x^{5/3}) = -\infty$

Calculating Limits at Infinity

Suppose the height of a bouncing ball is given by $h(t) = \frac{\sin(t)+1}{t}$, for $t \geq 1$. What happens to the height over a long period of time?

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$$\begin{array}{ccccc} 0 & \leq & \dfrac{\sin(t)+1}{t} & \leq & \dfrac{2}{t} \\ \lim\limits_{t\to\infty} 0 & = & 0 & = & \lim\limits_{t\to\infty} \dfrac{2}{t} \end{array}$$

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So, by the Squeeze Theorem,

$$\lim_{t\to\infty} \frac{\sin(t)+1}{t} = 0$$

Tough One

$$\lim_{x\to\infty} \sqrt{x^4 + x^2 + 1} - \sqrt{x^4 + 3x^2}$$

Tough One

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Multiply function by conjugate:

$$\left(\sqrt{x^4+x^2+1} - \sqrt{x^4+3x^2}\right)\left(\frac{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}\right)$$

$$= \frac{-2x^2+1}{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}$$

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Factor out highest power: x^2 (same as $\sqrt{x^4}$)

$$\frac{-2x^2+1}{\sqrt{x^4+x^2+1} + \sqrt{x^4+3x^2}}\left(\frac{1/x^2}{1/\sqrt{x^4}}\right)$$
$$= \frac{-2+\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x^4}} + \sqrt{1+\frac{3}{x^2}}}$$

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$$\lim_{x\to\infty} \frac{-2+\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x^4}} + \sqrt{1+\frac{3}{x^2}}} = \frac{-2+0}{\sqrt{1+0+0}+\sqrt{1+0}} = \frac{-2}{2} = -1$$

Evaluate $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{3 + x^2}}{3x}$

Evaluate $\lim_{x\to-\infty} \frac{\sqrt{3+x^2}}{3x}$

We factor out the largest power of the denominator, which is is x.

$$\lim_{x\to-\infty} \frac{\sqrt{3+x^2}}{3x}\left(\frac{1/x}{1/x}\right) = \lim_{x\to-\infty} \frac{\frac{\sqrt{3+x^2}}{x}}{3}$$

When $x<0$, $\sqrt{x^2} = |x| = -x$

$$\begin{aligned} &= \lim_{x\to-\infty} \frac{1}{3}\frac{\sqrt{3+x^2}}{-\sqrt{x^2}} \\ &= \lim_{x\to-\infty} -\frac{1}{3}\sqrt{\frac{3+x^2}{x^2}} \\ &= \lim_{x\to-\infty} -\frac{1}{3}\sqrt{\frac{3}{x^2}+1} \\ &= -\frac{1}{3} \end{aligned}$$

Continuity

Definition

A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.

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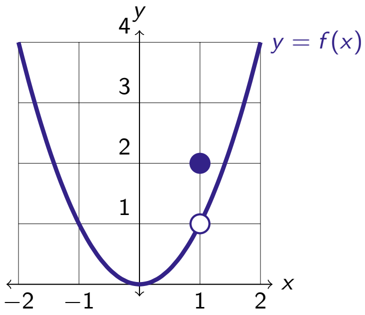

Is $f(x)$ continuous at 0?
Is $f(x)$ continuous at 1?

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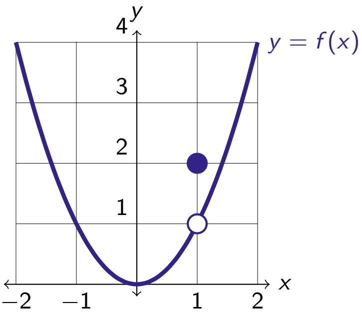

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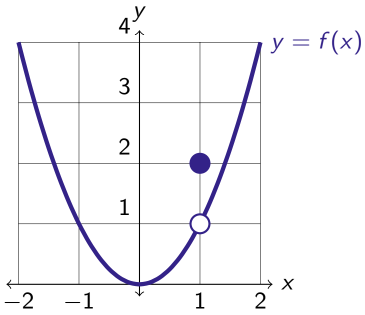

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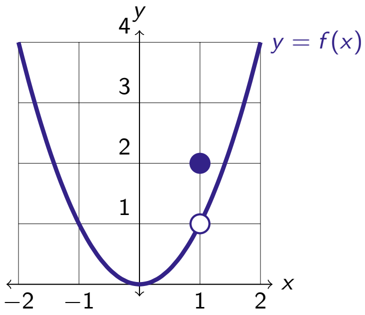

Is $f(x)$ continuous at 0? Yes.
Is $f(x)$ continuous at 1? No.

This kind of discontinuity is called removable.

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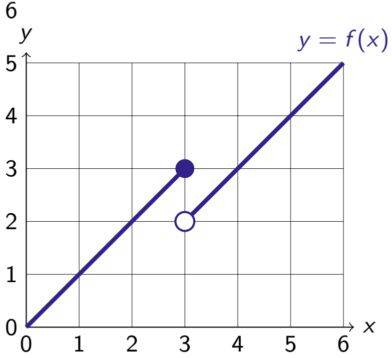
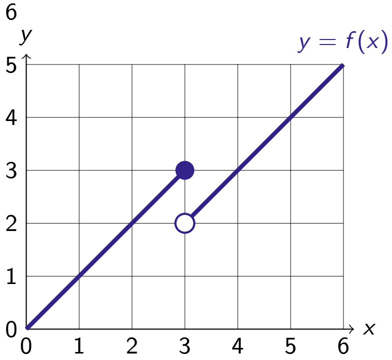


Is $f(x)$ continuous at 3?

Continuity

Definition

A function $f(x)$ is continuous at a point a if $\lim_{x \to a} f(x)$ exists AND is equal to $f(a)$.



Is $f(x)$ continuous at 3? No.

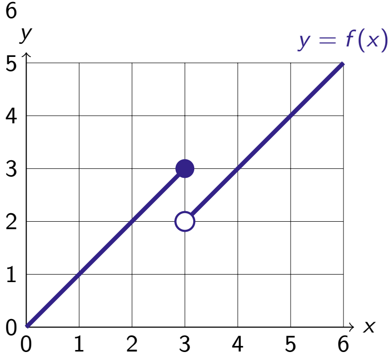
This kind of discontinuity is called a jump.

Continuity

Definition

A function $f(x)$ is continuous at a point a if $\lim_{x\to a} f(x)$ exists AND is equal to $f(a)$.

A function $f(x)$ is continuous from the left at a point a if $\lim_{x\to a^-} f(x)$ exists AND is equal to $f(a)$.



Is $f(x)$ continuous at 3? No.

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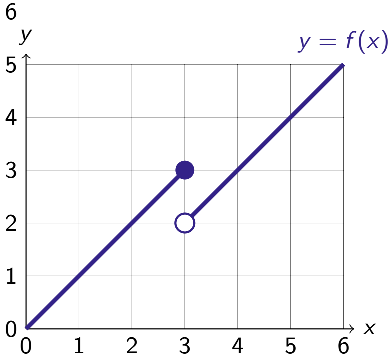

Is $f(x)$ continuous at 3? No.

Is $f(x)$ continuous from the left at 3?

Is $f(x)$ continuous from the right at 3?

This kind of discontinuity is called a jump.

Continuity

Definition

A function $f(x)$ is continuous at a point a if $\lim\limits_{x \to a} f(x)$ exists AND is equal to $f(a)$.

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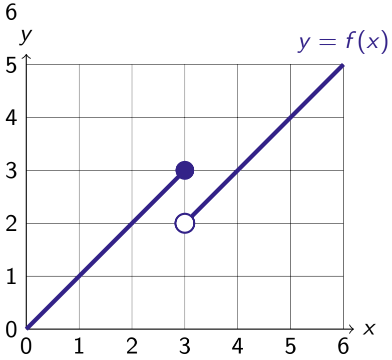

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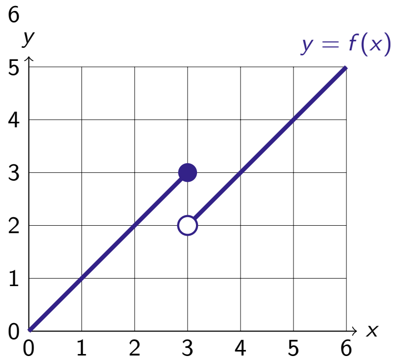

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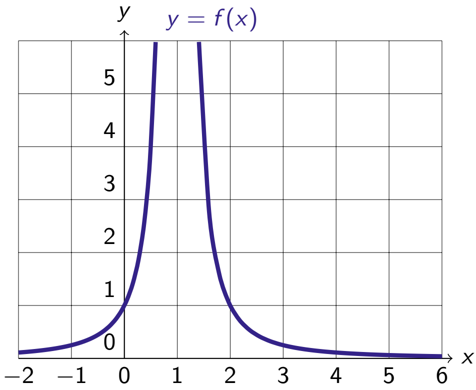

Since no one-sided limits exist at 1, there's no hope for continuity.

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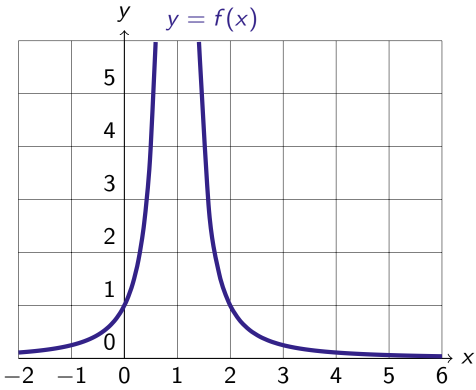

Since no one-sided limits exist at 1, there's no hope for continuity.

This is called an infinite discontinuity.

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$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Is $f(x)$ continuous at 0?

Continuity

Definition

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$$\lim_{x \to a} f(x) = f(a)$$

Functions made by adding, subtracting, multiplying, dividing, and taking appropriate powers of polynomials are continuous for every point in their domain.

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$$f(x) = \frac{x^2}{2x-10} - \left(\frac{x^2 + 2x - 1}{x - 1} + \frac{\sqrt[5]{25 - x} - \frac{1}{x}}{x + 2} \right)^{1/3}$$

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We say $f(x)$ is continuous over (a, b) if it is continuous at every point in (a, b). So, $f(x)$ is continuous over its domain, $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, 5) \cup (5, \infty)$.

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Common Functions

Functions of the following types are continuous over their domains:

- polynomials and rationals
- roots and powers
- trig functions and their inverses
- exponential and logarithm
- The products, sums, differences, quotients, powers, and compositions of continuous functions

Where is the following function continuous?

$$f(x) = \left(\frac{\sin x}{(x-2)(x+3)} + e^{\sqrt{x}} \right)^3$$

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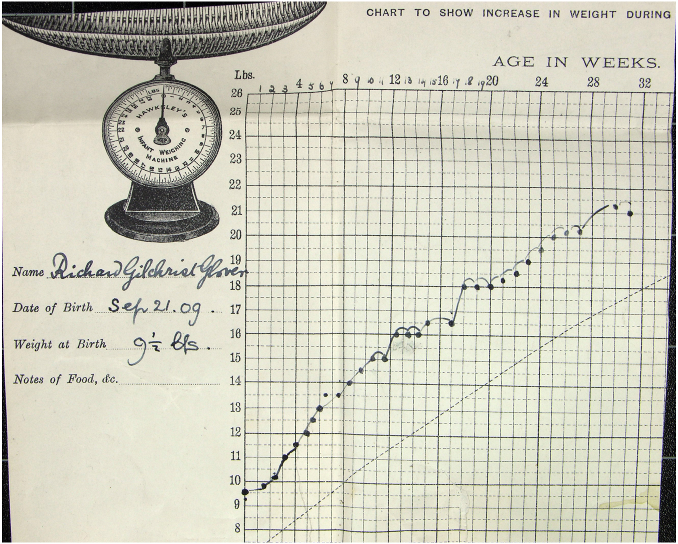
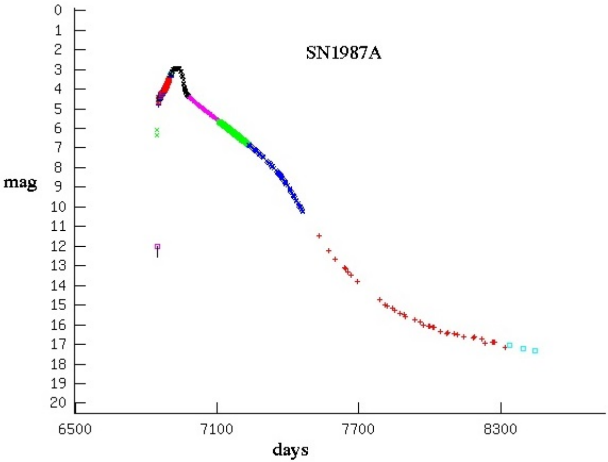
Lots of examples in notes.

Continuity in Nature



Baby weight chart, 1909
Source: http://gallery.nen.gov.uk/asset668105-.html

Continuity in Nature



Graph of luminosity of a star over time, after star explodes. Data from 1987. Source: http://abyss.uoregon.edu/ ∼js/ast122/lectures/ lec18.html

A Technical Point

Definition

A function $f(x)$ is continuous on the closed interval $[a, b]$ if:

- $f(x)$ is continuous over (a, b), and
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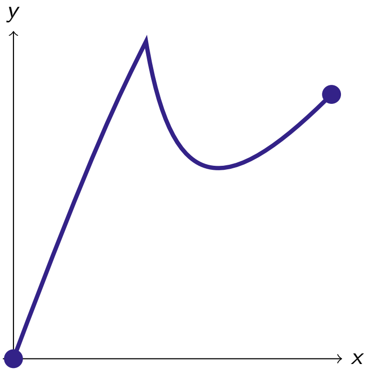
Intermediate Value Theorem (IVT)

Theorem:

Let $a < b$ and let $f(x)$ be continuous over $[a, b]$. If y is any number between $f(a)$ and $f(b)$, then there exists c in (a, b) such that $f(c) = y$.



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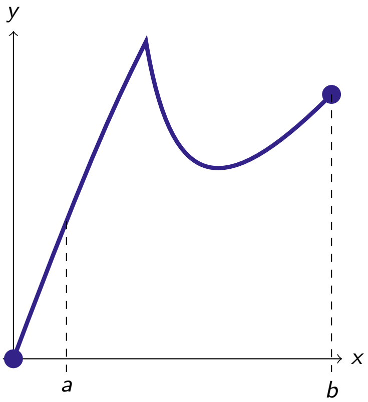

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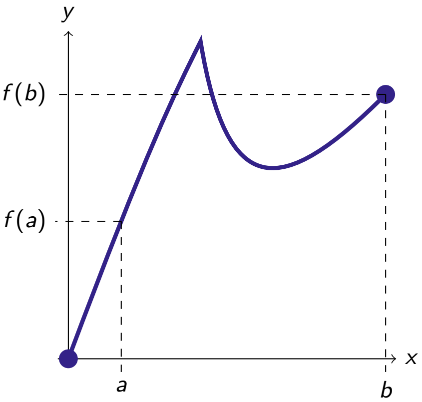

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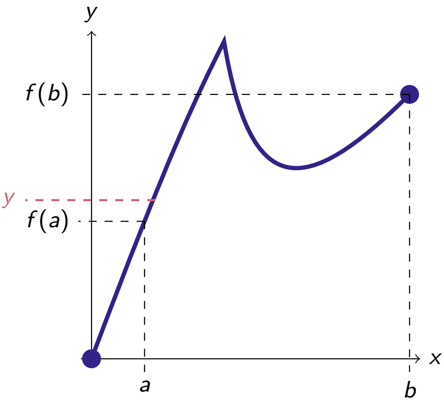

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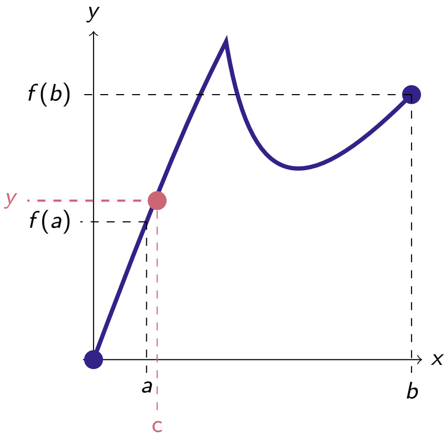

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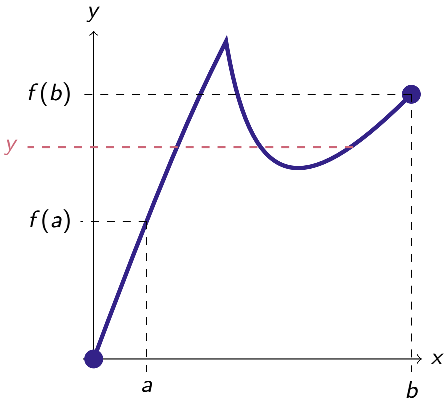

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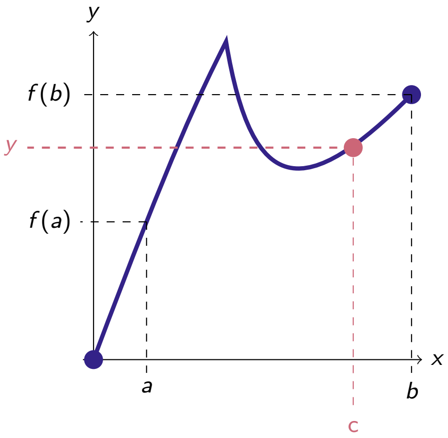

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Suppose your favorite number is 45.54. At noon, your car is parked, and at 1pm you're driving 100kph. By the Intermediate Value Theorem, at some point between noon and 1pm you were going exactly 45.54 kph.

Using IVT to Find Roots: "Bisection Method"

Let $f(x) = x^5 - 2x^4 + 2$. Find any value x for which $f(x) = 0$.

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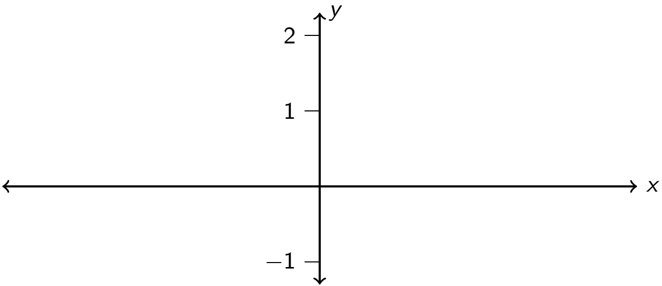

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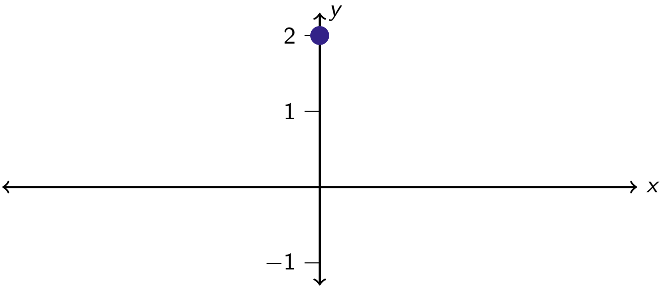
$$f(0) = 2$$



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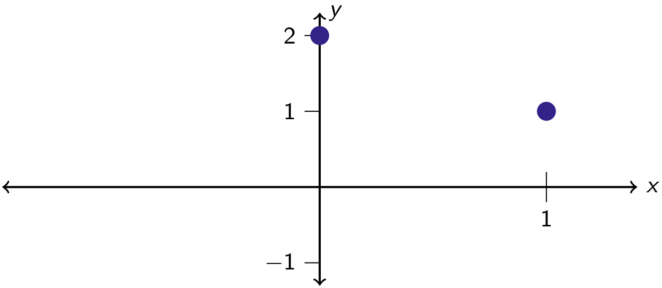

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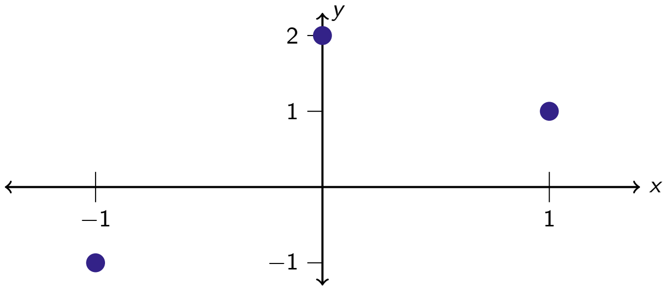

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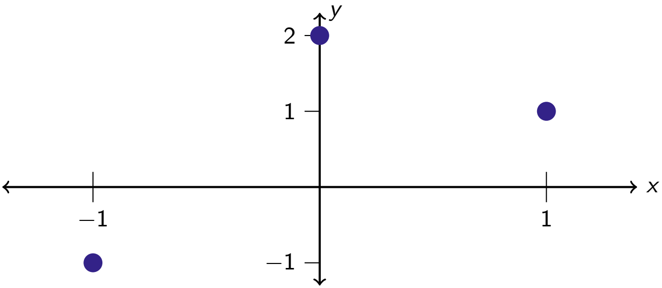

So, there has to be a root between $x = -1$ and $x = 0$.

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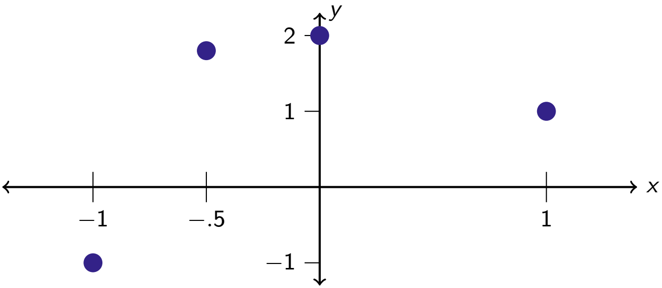

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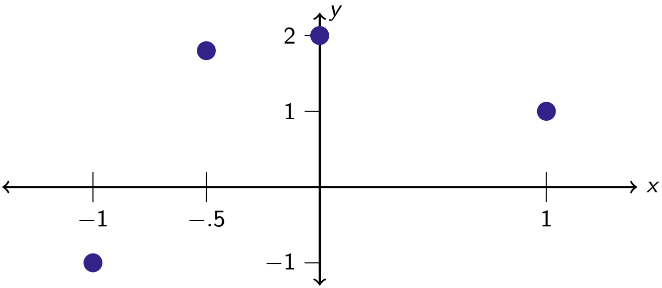

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$$f(0) = 2 \qquad f(1) = 1 \qquad f(-1) = -1$$
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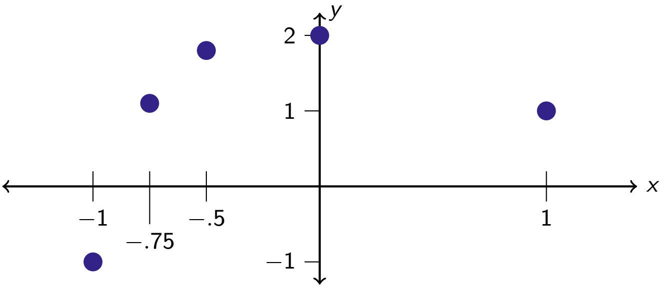

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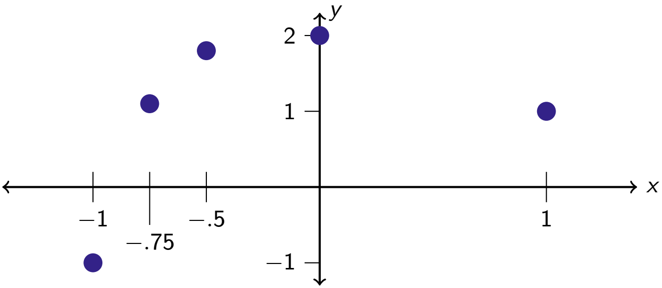

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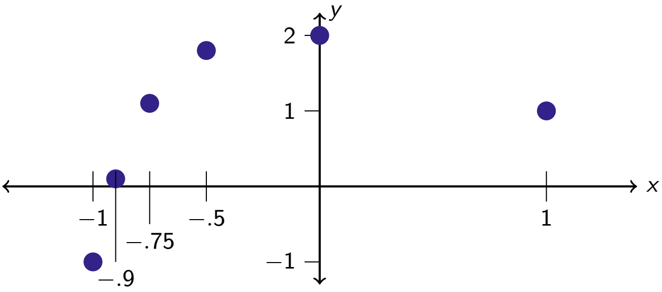

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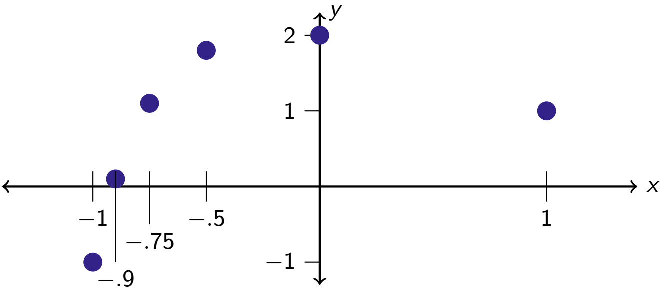

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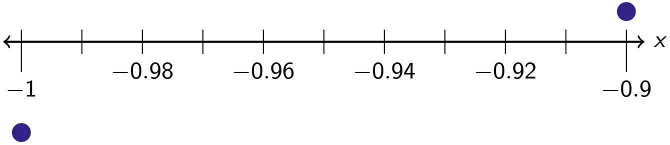

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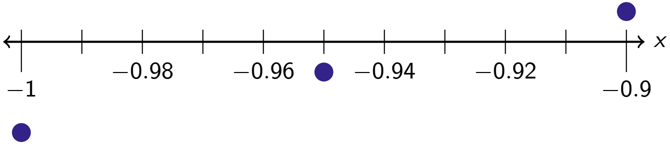

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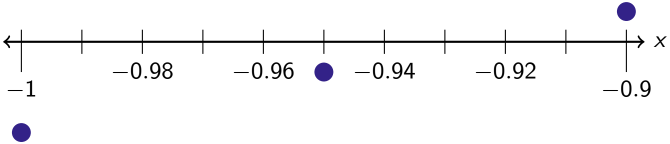

So, there has to be a root between $x = -0.95$ and $x = -0.9$.
We can say there is a root at approximately $x = -0.9$

Don't use a calculator for these problems: use values that you can easily calculate.

Use the Intermediate Value Theorem to show that there exists some solution to the equation $\ln x \cdot e^x = 4$ and give a reasonable interval where that solution might occur.

Use the Intermediate Value Theorem to give a reasonable interval where the following is true: $e^x = \sin(x)$

Is there any value of x so that $\sin x = \cos(2x) + \frac{1}{4}$?

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- The function $f(x) = \ln x \cdot e^x$ is continuous over its domain, which is $(0, \infty)$. In particular, then, it is continuous over the interval $(1, e)$.
- $f(1) = \ln(1)e = 0 \cdot e = 0$ and $f(e) = \ln(e) \cdot e^e = e^e$. Since $e > 2$, we know $f(e) = e^e > 2^2 = 4$.
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Use the Intermediate Value Theorem to give a reasonable interval where the following is true: $e^x = \sin(x)$

We can rearrange this: let $f(x) = e^x - \sin(x)$, and note $f(x)$ has roots exactly when the above equation is true.

- The function $f(x) = e^x - \sin x$ is continuous over its domain, which is all real numbers. In particular, then, it is continuous over the interval $\left(-\frac{3\pi}{2}, e\right)$.
- $f(0) = e^0 - \sin 0 = 1 - 0 = 0$ and $f\left(-\frac{3\pi}{2}\right) = e^{-\frac{3\pi}{2}} - \sin\left(\frac{-3\pi}{2}\right) = e^{-\frac{3\pi}{2}} - 1 < e^0 - 1 = 1 - 1 = 0$.
- Then 0 is between $f(0)$ and $f\left(-\frac{3\pi}{2}\right)$.
- By the Intermediate Value Theorem, $f(c) = 0$ for some c in $\left(-\frac{3\pi}{2}, 0\right)$.
- Therefore, $e^c = \sin c$ for some c in $\left(-\frac{3\pi}{2}, 0\right)$.

Is there any value of x so that $\sin x = \cos(2x) + \frac{1}{4}$?

Don't use a calculator for these problems: use values that you can easily calculate.

Use the Intermediate Value Theorem to show that there exists some solution to the equation $\ln x \cdot e^x = 4$ and give a reasonable interval where that solution might occur.

- The function $f(x) = \ln x \cdot e^x$ is continuous over its domain, which is $(0, \infty)$. In particular, then, it is continuous over the interval $(1, e)$.
- $f(1) = \ln(1)e = 0 \cdot e = 0$ and $f(e) = \ln(e) \cdot e^e = e^e$. Since $e > 2$, we know $f(e) = e^e > 2^2 = 4$.
- Then 4 is between $f(1)$ and $f(e)$.
- By the Intermediate Value Theorem, $f(c) = 4$ for some c in $(1, e)$.

Use the Intermediate Value Theorem to give a reasonable interval where the following is true: $e^x = \sin(x)$

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- Therefore, $e^c = \sin c$ for some c in $\left(-\frac{3\pi}{2}, 0\right)$.

Is there any value of x so that $\sin x = \cos(2x) + \frac{1}{4}$?

Yes, somewhere between 0 and $\frac{\pi}{2}$.

Is the following reasoning correct?

- $f(x) = \tan x$ is continuous over its domain, because it is a trigonometric function.
- In particular, $f(x)$ is continuous over the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.
- $f\left(\frac{\pi}{4}\right) = 1$, and $f\left(\frac{3\pi}{4}\right) = -1$.
- Since $f\left(\frac{3\pi}{4}\right) < 0 < f\left(\frac{\pi}{4}\right)$, by the Intermediate Value Theorem, there exists some number c in the interval $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ such that $f(c) = 0$.

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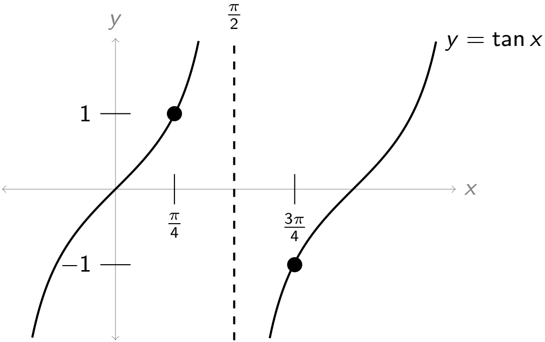

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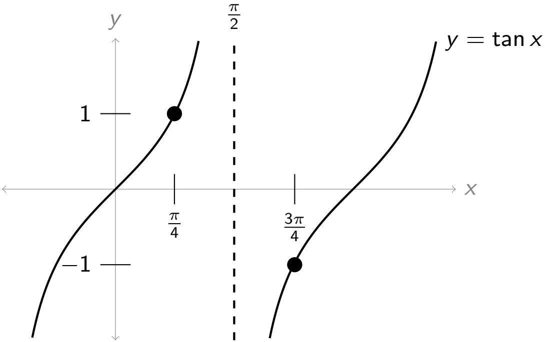

Let's Review

Suppose $f(x)$ is continuous at $x = 1$. Does $f(x)$ have to be defined at $x = 1$?

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Suppose $f(x)$ is continuous at $x = 1$ and $\lim_{x \to 1^-} f(x) = 30$.
True or false: $\lim_{x \to 1^+} f(x) = 30$.

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True or false: $\lim_{x\to 1^+} f(x) = 30$.
True. Since $f(x)$ is continuous at $x = 1$, $\lim_{x\to 1} f(x) = f(1)$, so $\lim_{x\to 1} f(x)$ must exist. That means both one-sided limits exist, and are equal to each other.

Suppose $f(x)$ is continuous at $x = 1$. Does $f(x)$ have to be defined at $x = 1$?
Yes. Since $f(x)$ is continuous at $x = 1$, $\lim\limits_{x \to 1} f(x) = f(1)$, so $f(1)$ must exist.

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Suppose $\lim_{x \to 1} f(x) = 2$. Must it be true that $f(1) = 2$?

Suppose $f(x)$ is continuous at $x = 1$. Does $f(x)$ have to be defined at $x = 1$?
Yes. Since $f(x)$ is continuous at $x = 1$, $\lim\limits_{x\to1} f(x) = f(1)$, so $f(1)$ must exist.

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$22 = f(1) = \lim\limits_{x\to1} f(x)$.

Suppose $\lim\limits_{x\to1} f(x) = 2$. Must it be true that $f(1) = 2$?
No. In order to determine the limit as x goes to 1, we ignore $f(1)$. Perhaps every $f(x)$ is not defined at 1.

$$f(x) = \begin{cases} ax^2 & x \geq 1 \\ 3x & x < 1 \end{cases}$$

For which value(s) of a is $f(x)$ continuous?

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For which value(s) of a is $f(x)$ continuous?

We need $ax^2 = 3x$ when $x = 1$, so $a = 3$.

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = -\sqrt{3}$?
For which value(s) of a is $f(x)$ continuous at $x = \sqrt{3}$?

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = -\sqrt{3}$?
By the definition of continuity, if $f(x)$ is continuous at $x = -\sqrt{3}$, then $f(-\sqrt{3}) = \lim_{x \to -\sqrt{3}} f(x)$. Note $f(-\sqrt{3}) = a$, and when x is close to (but not equal to) $-\sqrt{3}$, then $f(x) = \frac{\sqrt{3}x+3}{x^2-3}$.

$$\begin{aligned} f(-\sqrt{3}) &= \lim_{x \to -\sqrt{3}} f(x) \\ a &= \lim_{x \to -\sqrt{3}} \frac{\sqrt{3}x+3}{x^2-3} = \lim_{x \to -\sqrt{3}} \frac{\sqrt{3}(x+\sqrt{3})}{(x+\sqrt{3})(x-\sqrt{3})} \\ &= \lim_{x \to -\sqrt{3}} \frac{\sqrt{3}}{x-\sqrt{3}} = \frac{\sqrt{3}}{-\sqrt{3}-\sqrt{3}} = -\frac{1}{2} \end{aligned}$$

So we can use $a = -\frac{1}{2}$ to make $f(x)$ continuous at $x = -\sqrt{3}$.
For which value(s) of a is $f(x)$ continuous at $x = \sqrt{3}$?

$$f(x) = \begin{cases} \frac{\sqrt{3}x+3}{x^2-3} & x \neq \pm\sqrt{3} \\ a & x = \pm\sqrt{3} \end{cases}$$

For which value(s) of a is $f(x)$ continuous at $x = -\sqrt{3}$?
For which value(s) of a is $f(x)$ continuous at $x = \sqrt{3}$?
By the definition of continuity, if $f(x)$ is continuous at $x = \sqrt{3}$, then $f(\sqrt{3}) = \lim\limits_{x \to \sqrt{3}} f(x)$.

When x is close to (but not equal to) $\sqrt{3}$, then $f(x) = \frac{\sqrt{3}x+3}{x^2-3}$. However, as x approaches $\sqrt{3}$, the denominator of this expression gets closer and closer to zero, while the top gets closer and closer to 6. So, this limit does not exist. Therefore, no value of a will make $f(x)$ continuous at $x = \sqrt{3}$.

For the next batch of slides, see
`http://www.math.ubc.ca/~elyse/100/2016/Deriv_Conceptual.pdf`