Secant and Tangent Lines

http://www.mathscoop.com/applets/tangent-line-applet.php

Slope of Secant and Tangent Line

Slope

Recall, the slope of a line is given by any of the following:

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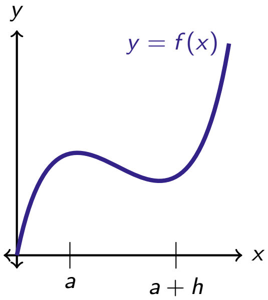

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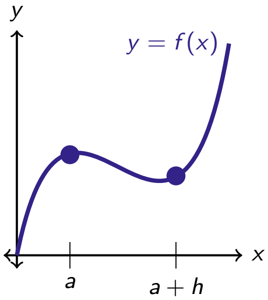

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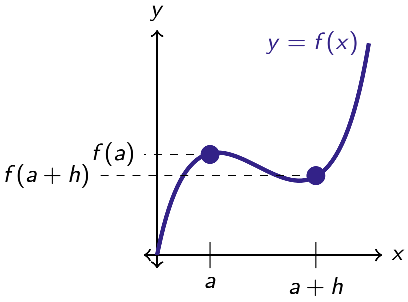

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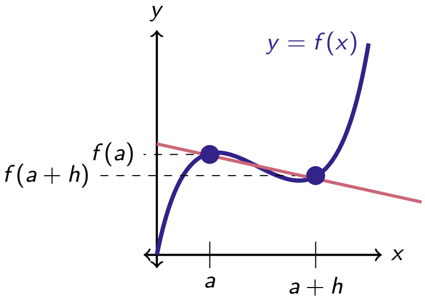

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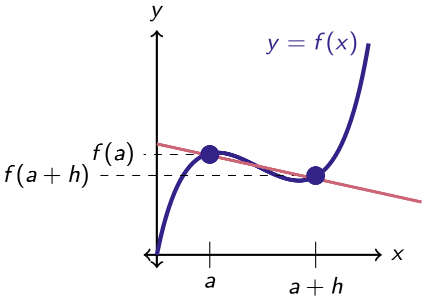

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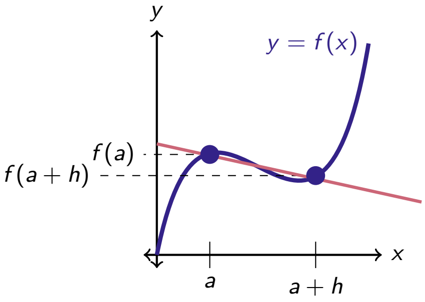

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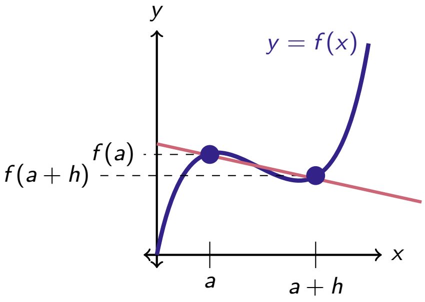

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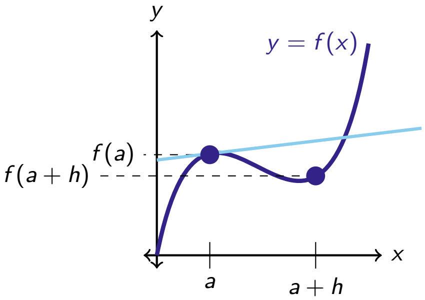

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Derivative at a Point

Definition

Given a function $f(x)$ and a point a, the slope of the tangent line to $f(x)$ at a is the derivative of f at a, written $f'(a)$.

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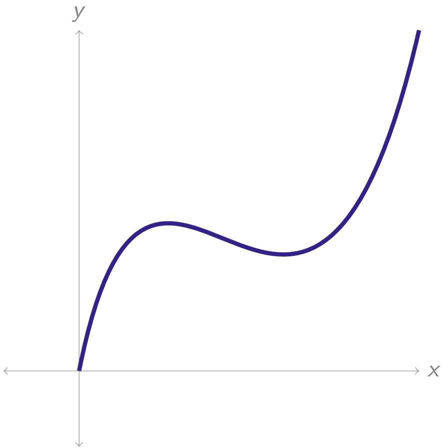
If $f'(a) = 0$, then f looks constant at a. It looks flat.

Practice: Increasing and Decreasing



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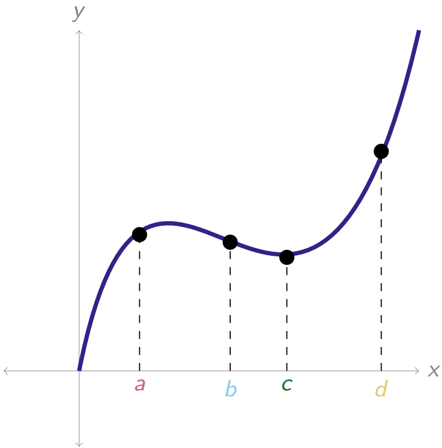

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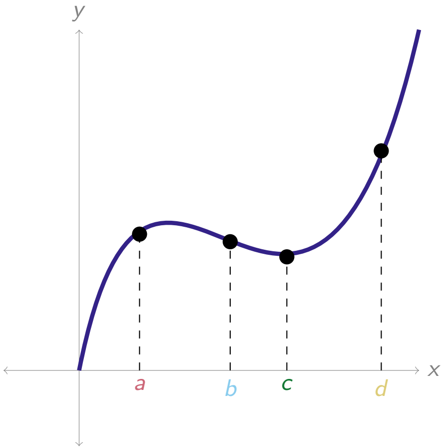

Where is $f'(x) < 0$?

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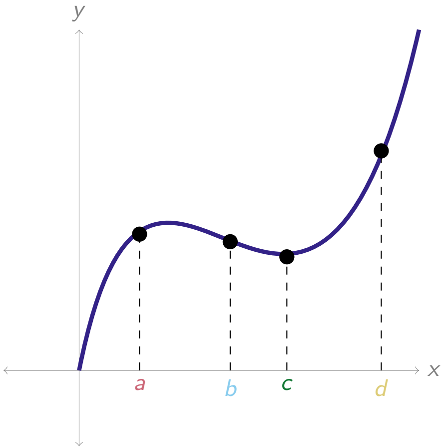
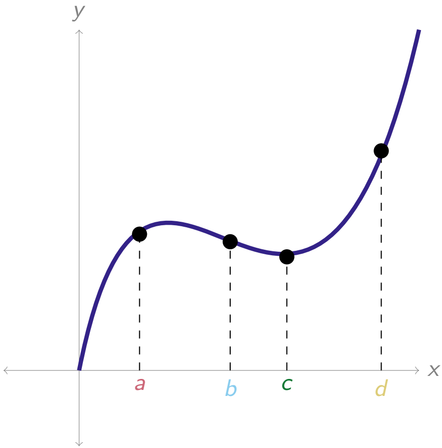


Where is $f'(x) < 0$? $f'(b) < 0$

Practice: Increasing and Decreasing



Where is $f'(x) > 0$?

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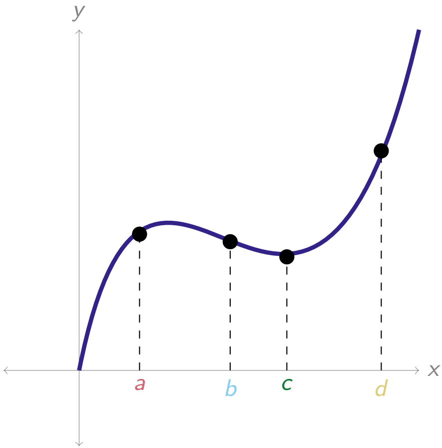

Where is $f'(x) > 0$? $f'(a) > 0$ and $f'(d) > 0$

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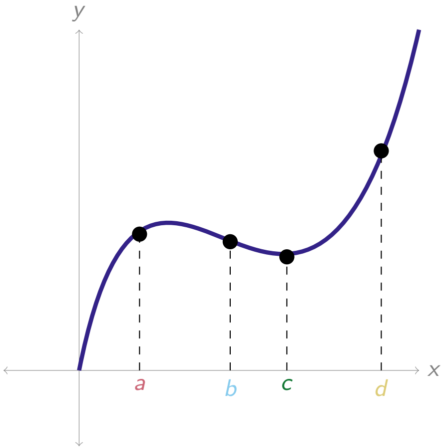
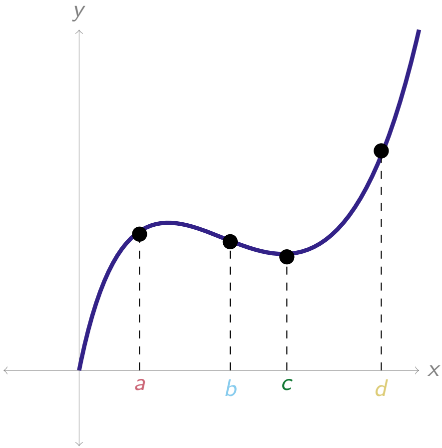


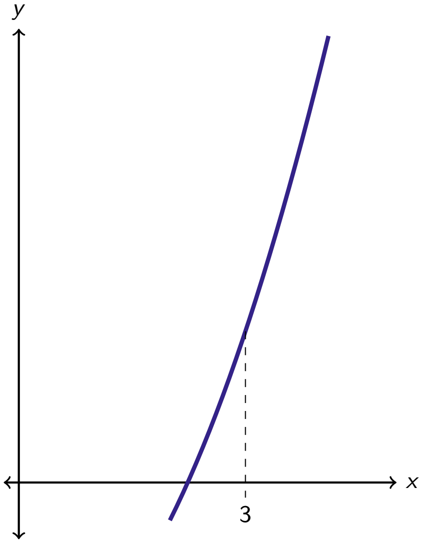
Where is $f'(x) = 0$?

Practice: Increasing and Decreasing



Where is $f'(x) = 0$? $f'(c) = 0$

Use the definition of the derivative to find the slope of the tangent line to $f(x) = x^2 - 5$ at the point $x = 3$.



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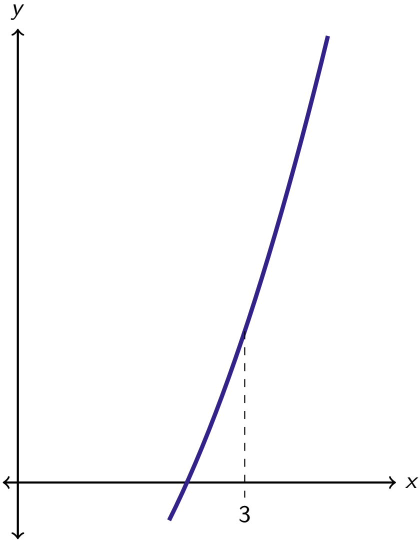

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

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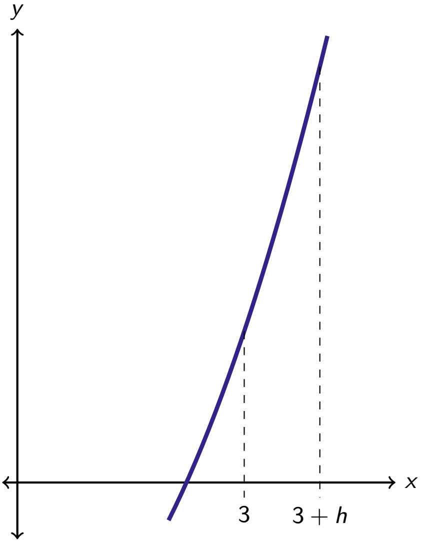

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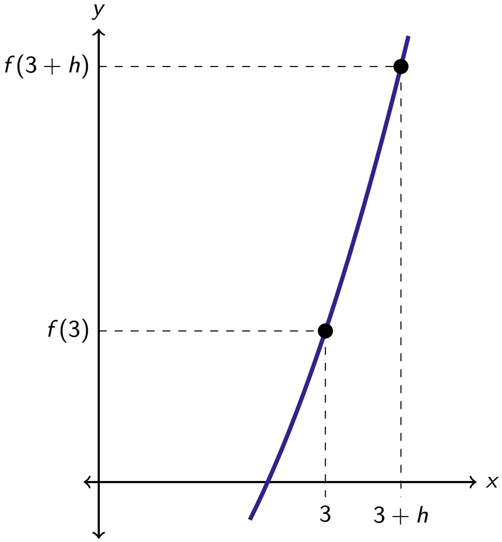

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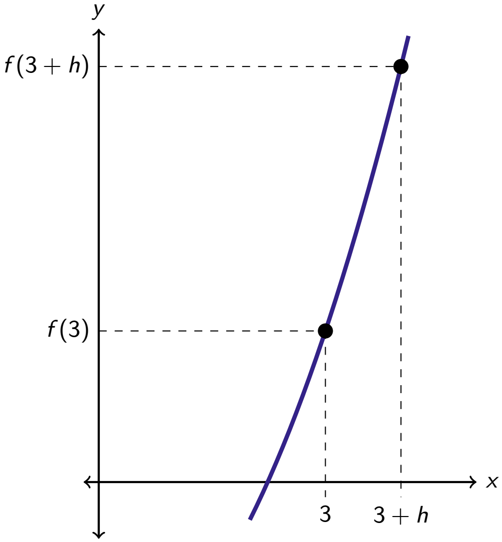

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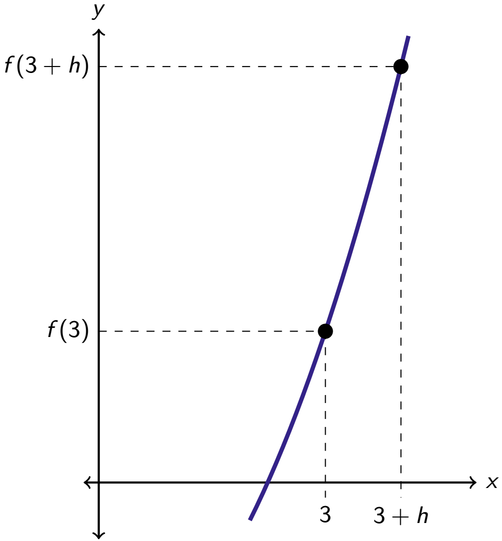

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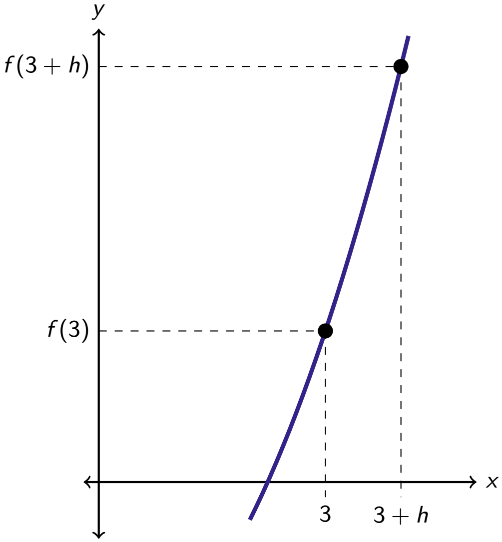

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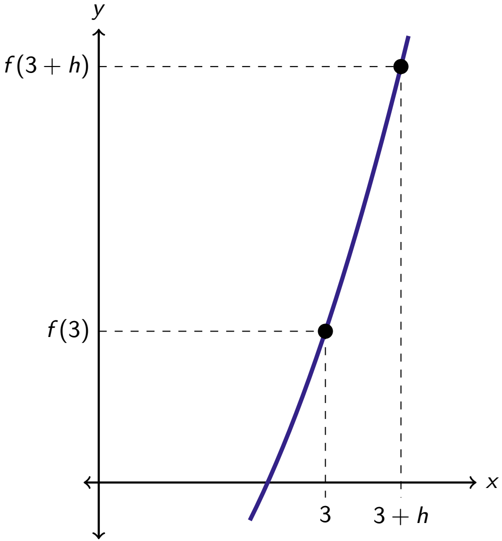

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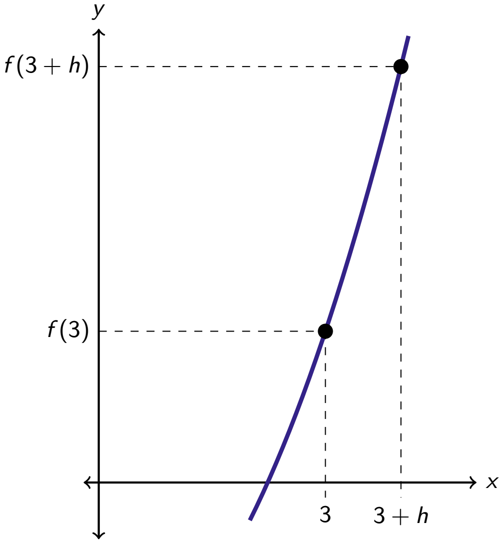

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Let's keep the function $f(x) = x^2 - 5$. We just showed $f'(3) = 6$.
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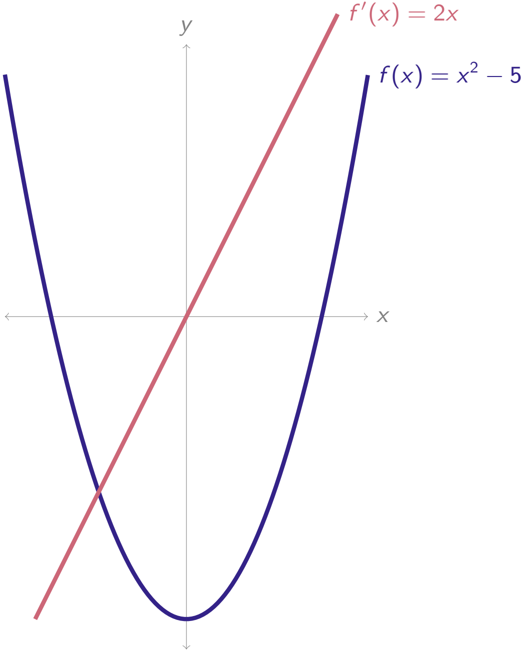
In particular, $f'(3) = 6$.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y
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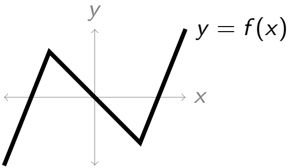
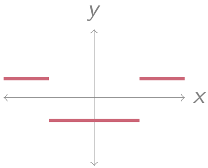
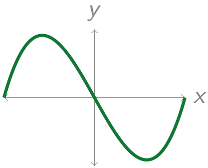
x

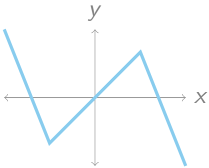
Increasing and Decreasing

In black is the curve $y = f(x)$. Which of the coloured curves corresponds to $y = f'(x)$?









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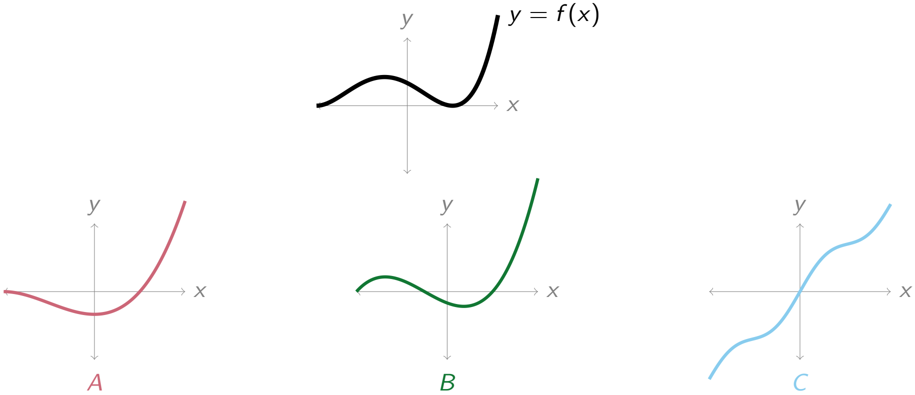

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provided the limit exists. Notice that x will be a part of your final expression: this is a function.

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Notation

The "prime" notation $f'(x)$ and $f'(a)$ is attributed to Newton. We will also use Leibniz notation:

$$\frac{df}{dx} \qquad \frac{df}{dx}(a) \qquad \frac{d}{dx}f(x) \quad \left.\frac{d}{dx}f(x)\right|_{x=a}$$

Derivatives

Let $f(x)$ be a function.
The derivative of $f(x)$ with respect to x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. Notice that x will be a part of your final expression: this is a function.

If $f'(x)$ exists for all x in an interval (a, b), we say that f is differentiable on (a, b).

Notation

The "prime" notation $f'(x)$ and $f'(a)$ is attributed to Newton. We will also use Leibniz notation:

$\frac{df}{dx}$	$\frac{df}{dx}(a)$	$\frac{d}{dx}f(x)$	$\frac{d}{dx}f(x)\Big\vert_{x=a}$
function	number	function	number

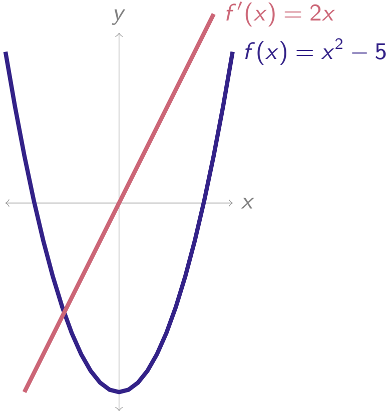
Alternate Notation

Newtonian Notation:

$$f(x) = x^2 + 5 \qquad f'(x) = 2x \qquad f'(3) = 6$$

Leibnitz Notation:

$\frac{df}{dx} =$ $\frac{df}{dx}(3) =$ $\frac{d}{dx}f(x) =$ $\frac{d}{dx}f(x)\Big|_{x=3} =$



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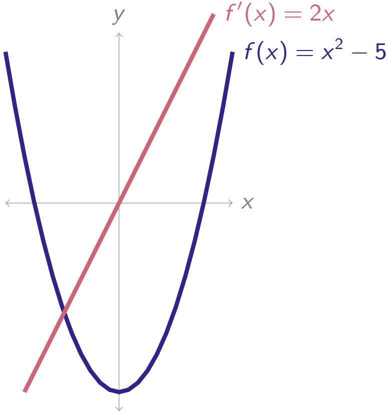

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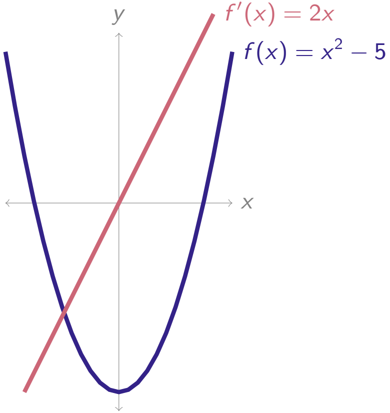

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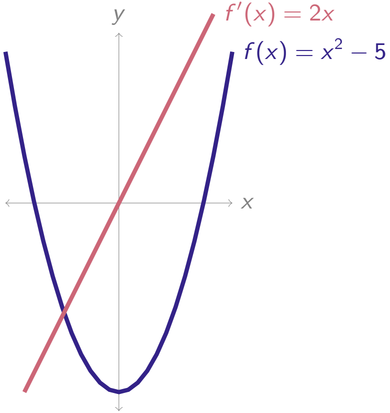

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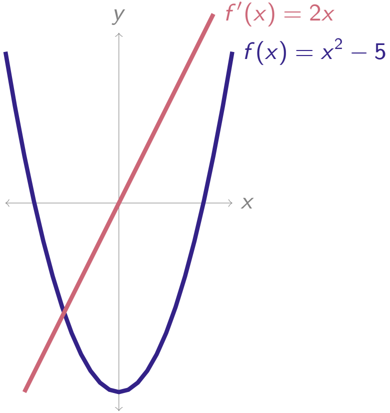


Alternate Definition

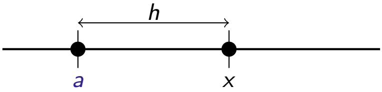
Calculating

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

is the same as calculating

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

Notice in these scenarios, $h = x - a$.



Using the Definition of a Derivative

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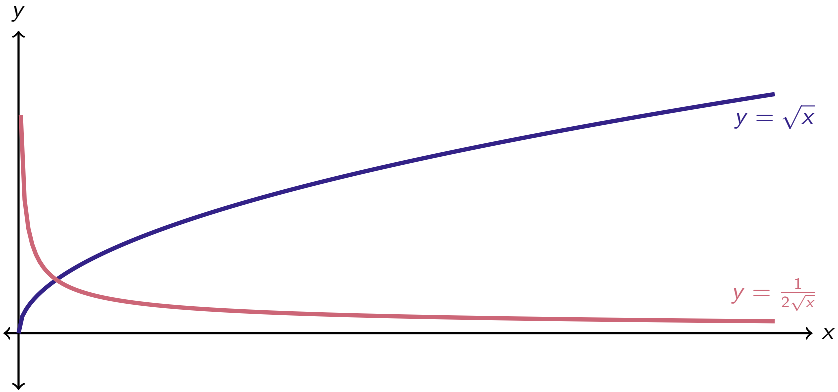
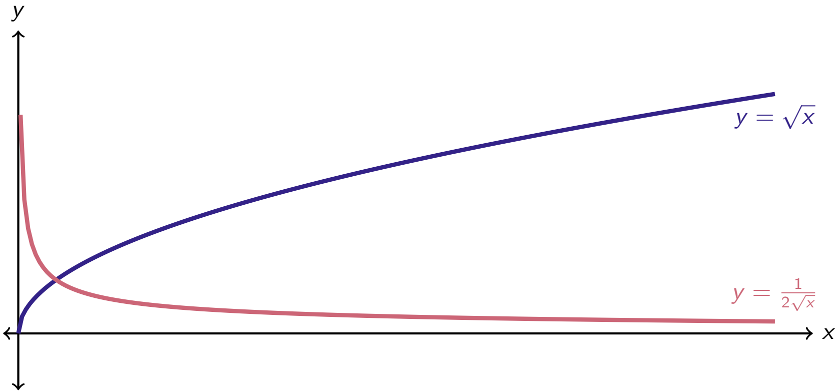
$$= \frac{1}{2\sqrt{x}}$$



y
x
$y = \sqrt{x}$
$y = \frac{1}{2\sqrt{x}}$



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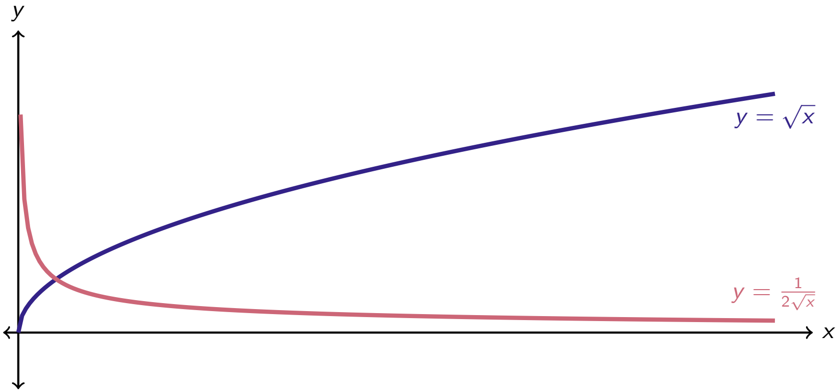

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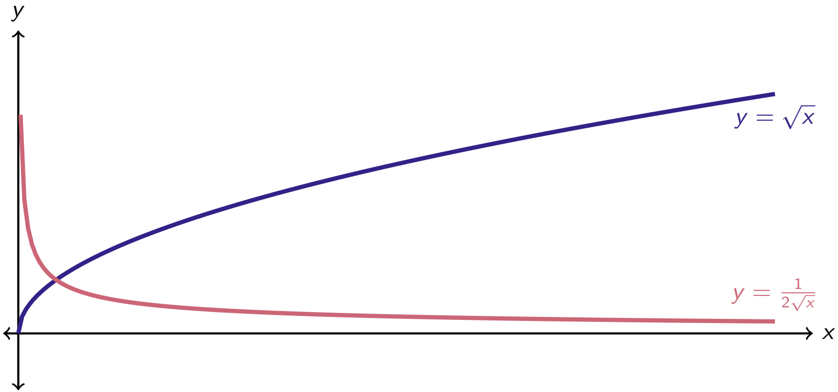

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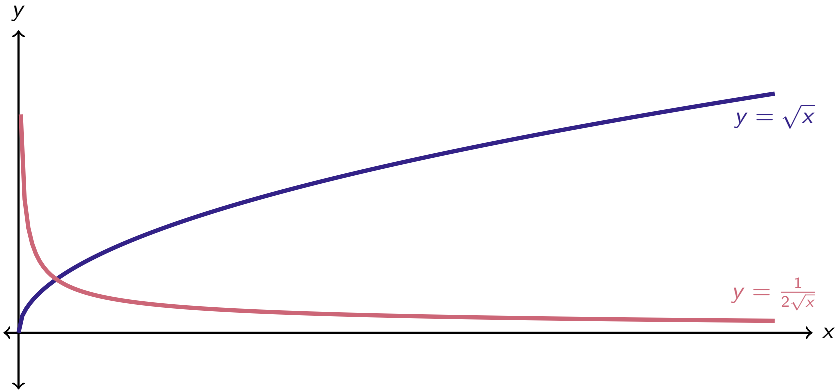

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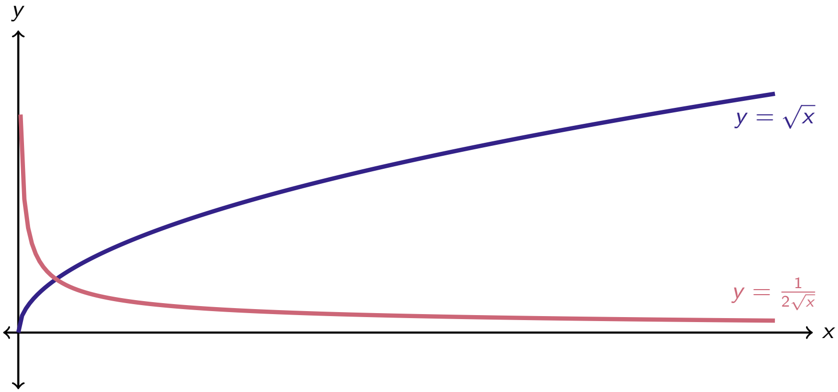

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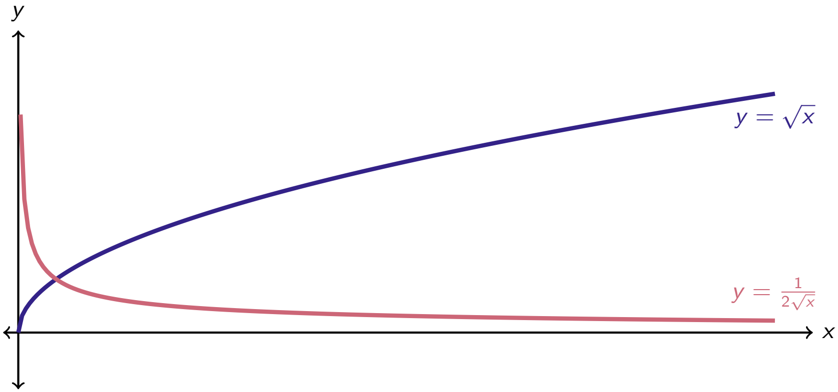

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$\lim_{x\to 0^+} \sqrt{x} = 0$ $\lim_{x\to 0^+} \frac{1}{2\sqrt{x}} = \infty$

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Using the definition of the derivative, calculate $\dfrac{d}{dx}\left\{\dfrac{1}{x}\right\}$.

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$$\frac{d}{dx}\left\{\frac{1}{x}\right\} = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

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Using the definition of the derivative, calculate $\dfrac{d}{dx}\left\{\dfrac{1}{x}\right\}$.

$$\begin{aligned}
\frac{d}{dx}\left\{\frac{1}{x}\right\} &= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{-1}{x(x+h)} \\
&= -\frac{1}{x^2}
\end{aligned}$$

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Using the definition of the derivative, calculate $\dfrac{d}{dx}\left\{\dfrac{2x}{x+1}\right\}$.

Using the definition of the derivative, calculate $\dfrac{d}{dx}\left\{\dfrac{1}{\sqrt{x^2+x}}\right\}$.

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Using the definition of the derivative, calculate $\dfrac{d}{dx}\left\{\dfrac{2x}{x+1}\right\}$.

$$\begin{aligned}
\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\frac{2(x+h)}{x+h+1}-\frac{2x}{x+1}}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{2(x+h)(x+1)}{(x+h+1)(x+1)}-\frac{2x(x+h+1)}{(x+1)(x+h+1)}\right)\\
&= \lim_{h\to 0}\frac{2}{h}\left(\frac{(x^2+x+xh+h)-(x^2+xh+x)}{(x+h+1)(x+1)}\right)\\
&= \lim_{h\to 0}\frac{2}{h}\left(\frac{h}{(x+h+1)(x+1)}\right)\\
&= \lim_{h\to 0}\frac{2}{(x+h+1)(x+1)} = \frac{2}{(x+1)^2}
\end{aligned}$$

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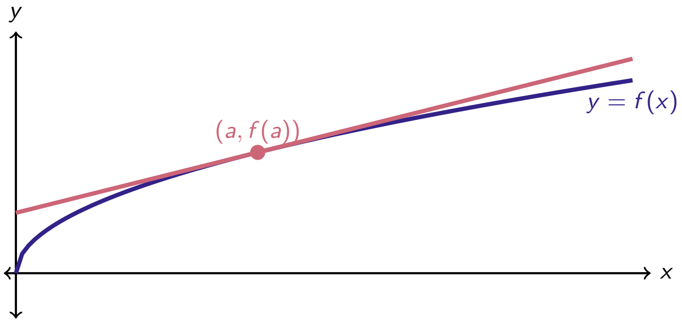
$$
\begin{aligned}
\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\frac{1}{\sqrt{(x+h)^2+x+h}}-\frac{1}{\sqrt{x^2+x}}}{h} \\
&= \lim_{h\to 0}\frac{1}{h}\left(\frac{\sqrt{x^2+x}}{\sqrt{(x^2+h)^2+x+h}\sqrt{x^2+x}}-\frac{\sqrt{(x+h)^2+x+h}}{\sqrt{(x^2+h)^2+x+h}\sqrt{x^2+x}}\right) \\
&= \lim_{h\to 0}\frac{1}{h}\left(\frac{\sqrt{x^2+x}-\sqrt{(x+h)^2+x+h}}{\sqrt{(x^2+h)^2+x+h}\sqrt{x^2+x}}\right)\left(\frac{\sqrt{x^2+x}+\sqrt{(x+h)^2+x+h}}{\sqrt{x^2+x}+\sqrt{(x+h)^2+x+h}}\right) \\
&= \lim_{h\to 0}\frac{1}{h}\left(\frac{(x^2+x)-[(x+h)^2+x+h]}{\sqrt{(x^2+h)^2+x+h}\sqrt{x^2+x}\left[\sqrt{x^2+x}+\sqrt{(x+h)^2+x+h}\right]}\right) \\
&= \lim_{h\to 0}\frac{1}{h}\left(\frac{-(2xh+h^2+h)}{\sqrt{(x^2+h)^2+x+h}\sqrt{x^2+x}\left[\sqrt{x^2+x}+\sqrt{(x+h)^2+x+h}\right]}\right) \\
&= \lim_{h\to 0}\frac{-(2x+h+1)}{\sqrt{(x^2+h)^2+x+h}\sqrt{x^2+x}\left[\sqrt{x^2+x}+\sqrt{(x+h)^2+x+h}\right]} = \frac{-(2x+1)}{2(x^2+x)^{3/2}}
\end{aligned}
$$

Equation of the Tangent Line

The tangent line to $f(x)$ at a has slope $f'(a)$ and passes through the point $(a, f(a))$.



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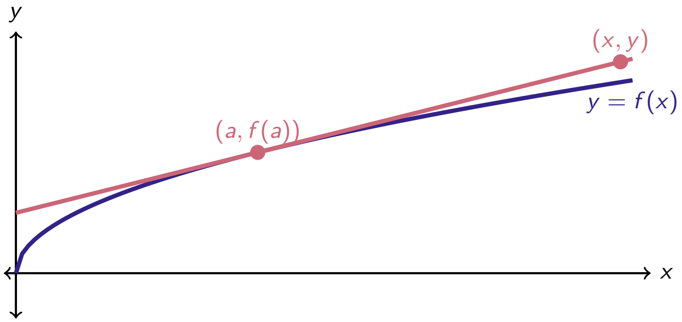

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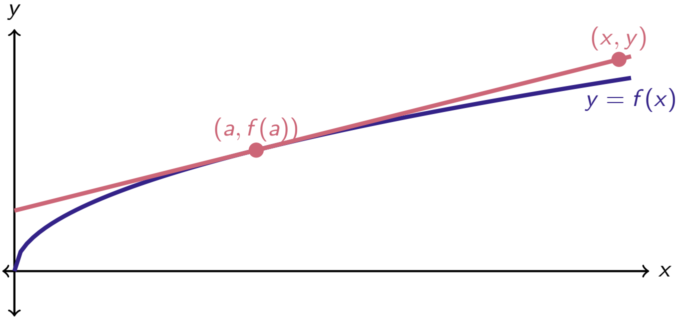

Slope of the tangent line:

$$f'(a) = \frac{\text{rise}}{\text{run}} = \frac{y - f(a)}{x - a}$$

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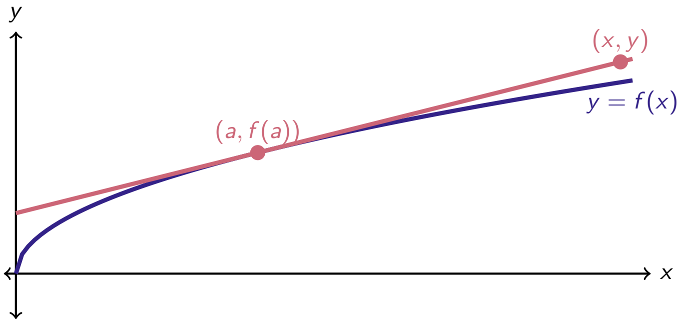

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Rearranging: $y - f(a) = f'(a)(x - a)$

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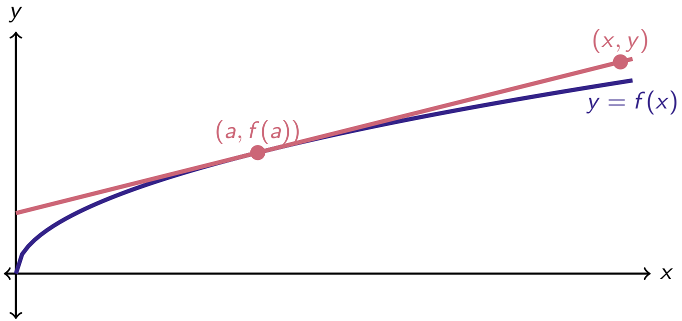

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Point-Slope Formula

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In general, a line with slope m passing through point (x_1, y_1) has the equation:

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Find the equation of the tangent line to the curve $f(x) = \sqrt{x}$ at $x = 9$.
Recall $\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}$.

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$f(a) =$

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$$(y - 3) = \frac{1}{6}(x - 9)$$

Now You

Things to have Memorized

The derivative of a function f at a point a is given by the following limit, if it exists:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The tangent line to the function $f(x)$ at point a is:

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Let $s(t) = 3 - 0.8t^2$. Then $s'(t) = -1.6t$. Find the equation for the tangent line to the function $s(t)$ when $t = 1$.

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Things to have Memorized

The derivative of a function f at a point a is given by the following limit, if it exists:

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The tangent line to the function $f(x)$ at point a is:

$$(y - f(a)) = f'(a)(x - a)$$

Let $s(t) = 3 - 0.8t^2$. Then $s'(t) = -1.6t$. Find the equation for the tangent line to the function $s(t)$ when $t = 1$.

$a = 1$

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$$f(x) = 2x - 15$$

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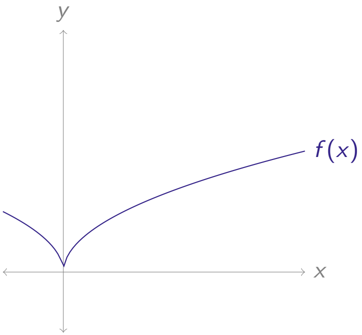
Adding a Constant



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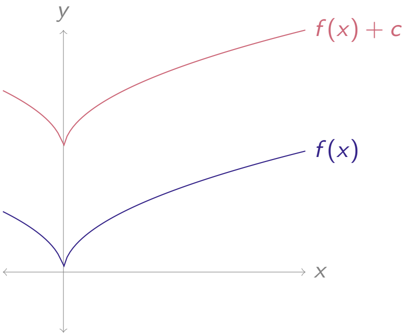

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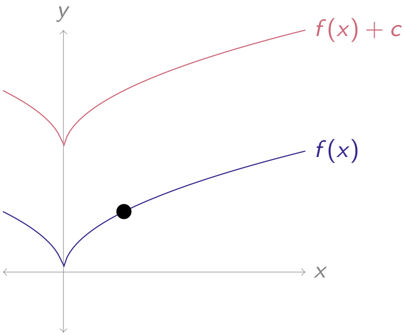

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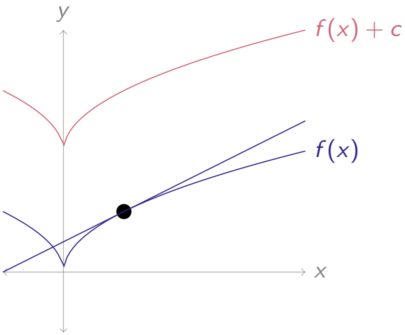

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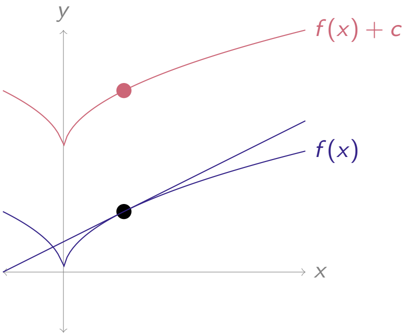

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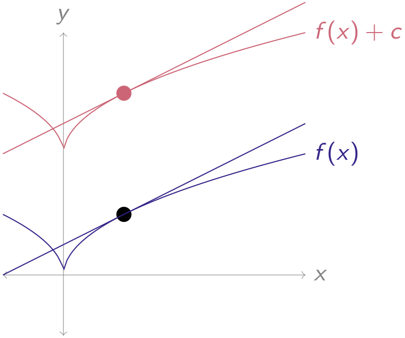

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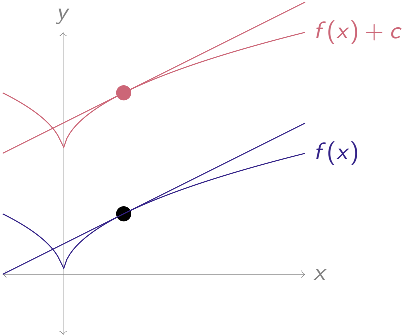
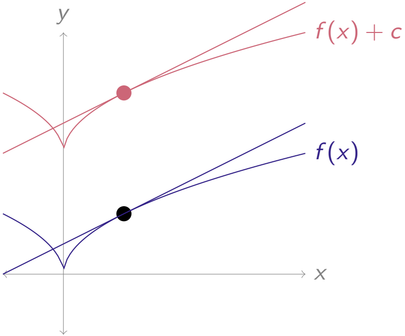


Adding or subtracting a constant to a function does not change its derivative.

Adding a Constant



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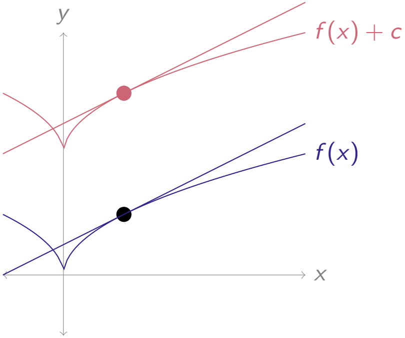

Adding or subtracting a constant to a function does not change its derivative.

$$\frac{d}{dx}\left(3 - 0.8t^2\right)\Big|_{t=1} = -1.6 \qquad \Rightarrow \qquad \frac{d}{dx}\left(10 - 0.8t^2\right)\Big|_{t=1} = -1.6$$

Rules

Suppose $f(x)$ and $g(x)$ are differentiable, and let c be a constant number. Then:

$$\frac{d}{dx}\{f(x)+g(x)\} = f'(x)+g'(x)$$
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Derivatives From Previous Calculations

$$\frac{d}{dx}\{2x-15\} = 2$$
$$\frac{d}{dx}\left\{x^2-5\right\} = 2x$$
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For instance: let $f(x)=10\left((2x-15)+13-\sqrt{x}\right)$. Then $f'(x)$

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For instance: let $f(x)=10\left((2x-15)+13-\sqrt{x}\right)$. Then $f'(x)=10\left((2)+0+\frac{1}{2\sqrt{x}}\right)$.

Now You

Suppose $f'(x) = 3x$, $g'(x) = -x^2$, and $h'(x) = 5$. Calculate:

$$\frac{d}{dx}\left\{f(x) + 5g(x) - h(x) + 22\right\}$$

A. $3x - 5x^2$

B. $3x - 5x^2 - 5$

C. $3x - 5x^2 - 5 + 22$

D. none of the above

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At midnight of January 1, 2012, the world population was increasing at a rate of 156 people each minute

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Suppose $P(n)$ gives the total profit, in dollars, earned by selling n widgets. How do you interpret $P'(100)$?

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Suppose $P(n)$ gives the total profit, in dollars, earned by selling n widgets. How do you interpret $P'(100)$?
How fast your profit is increasing as you sell more widgets, measured in dollars per widget, at the time you sell Widget #100. So, roughly the profit earned from the sale of the 100th widget.

Interpreting the Derivative

Suppose $h(t)$ gives the height of a rocket t seconds after liftoff. What is the interpretation of $h'(t)$?

Suppose $M(t)$ is the number of molecules of a chemical in a test tube t seconds after a reaction starts. Interpret $M'(t)$.

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The speed at which the rocket is rising at time t.

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The rate at which molecules of a certain type are being created or destroyed. Roughly, how many molecules are being added (or taken away, if negative) per second at time t.

Interpreting the Derivative

Suppose $G(w)$ gives the diameter in millimetres of steel wire needed to safely support a load of w kg. Suppose further that $G'(100) = 0.01$. How do you interpret $G'(100) = 0.01$?

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For the next batch of slides, look here.