Math 100 - Homework Set 4 (Curve Sketching-Optimization)

due date: Friday, April the 15th

1. Sketch the curve for \( f(x) \) defined for all \( x \), where we are given the following information:
   \( f(x) > 0 \) for all \( x \) and \( f(1) = 1, f(2) = 2, f(3) = 3, \lim_{x \to \infty} f(x) = 5, \lim_{x \to -\infty} f(x) = 0 \).
   Also
   \[
   f'(x) \begin{cases} 
   > 0 & \text{for all } x \neq 2 \\
   = 0 & \text{for } x = 2 
   \end{cases}, \quad f''(x) \begin{cases} 
   > 0 & \text{for all } x \in (-\infty, 1) \text{ and } x \in (2, 3) \\
   = 0 & \text{for } x = 1, 2, 3 \\
   < 0 & \text{for all } x \in (1, 2) \text{ and } x \in (3, \infty) 
   \end{cases}
   \]

   **Solution:** See the sketch at the end.

2. Let \( f(x) = \cos(2x) + 2x \) be a function defined on the interval \([0, 2\pi]\). Determine the critical numbers, and the intervals on which \( y = f(x) \) is increasing or decreasing. Use the First Derivative Test to classify those critical numbers as local maxima, local minima, or neither.

   **Solution:** We have \( f'(x) = -2\sin(2x) + 2 \), which is defined everywhere. So, the critical numbers are those values of \( x \) in \([0, 2\pi]\) such that \( f'(x) = 0 \), that is,

   \[-2\sin(2x) + 2 = 0 \Rightarrow \sin(2x) = 1 \Rightarrow 2x = \frac{\pi}{2}, \frac{5\pi}{2} \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}.\]

   Note that we want \( x \) in \([0, 2\pi]\), so we want to solve for \( 2x \) in \([0, 4\pi]\). Then, we get the following intervals:

   - \((0, \frac{\pi}{4})\): \( f'(x) > 0 \) (for example, \( f'\left(\frac{\pi}{6}\right) = -2\sin(\pi/3) + 2 = -\sqrt{3} + 2 > 0 \)).
     So, \( f \) is increasing.
   - \((\frac{\pi}{4}, \frac{5\pi}{4})\): \( f'(x) > 0 \) (for example, \( f'(\pi) = -2\sin(2\pi) + 2 = 2 > 0 \)). So, \( f \) is increasing.
   - \((\frac{5\pi}{4}, 2\pi)\): \( f'(x) > 0 \) (for example, \( f'\left(\frac{3\pi}{2}\right) = -2\sin(3\pi) + 2 = 2 > 0 \)). So, \( f \) is increasing.

   Since \( f' \) does not change sign, all those critical numbers are neither local maxima nor minima.

3. For the function \( y = (x + 1)^2e^{-2x} \), identify the intervals on which \( f \) is increasing or decreasing, all local and absolute maxima and minima, the intervals of concavity and any inflection points. Then, sketch the graph.
Solution: We compute

\[ f'(x) = y' = \frac{2(x + 1) \cdot e^{2x} - (x + 1)^2 2e^{2x}}{e^{4x}} = \frac{-2x^2 - 2x}{e^{2x}} \]

\[ e^{2x} > 0 \text{ for all } x \text{ and so the sign chart for } f'(x) \text{ is determined by the sign chart of } -2x^2 - 2x \text{ and so } f'(x) = 0 \text{ for } x = -1, 0, \text{ and } f'(x) < 0 \text{ (f is decreasing) for } x < -1 \text{ or } x > 0. \]

We compute

\[ f''(x) = y'' = \frac{(-4x - 2) \cdot e^{2x} - (-2x^2 - 2x)e^{2x}}{e^{4x}} = \frac{4x^2 - 2}{e^{2x}} \]

As before, \( e^{2x} > 0 \) for all \( x \) and so the sign chart for \( f''(x) \) is determined by the sign chart of \( 4x^2 - 2 \). Thus \( f''(x) = 0 \) for \( x = \pm \sqrt{\frac{1}{2}} \) and \( f''(x) > 0 \) (f is concave up) for \( x < -\sqrt{\frac{1}{2}} \) and \( x > \sqrt{\frac{1}{2}} \), and \( f''(x) < 0 \) (f is concave down) for \( -\sqrt{\frac{1}{2}} < x < \sqrt{\frac{1}{2}} \). We note that \( x = \pm \sqrt{\frac{1}{2}} \) yield inflection points.

4. For the function \( f(x) = 3\sqrt{(x^2 - 1)^2} \), identify the domain, intervals on which \( f \) is increasing or decreasing, all local and absolute maxima and minima, the intervals of concavity and any inflection points. Then, sketch the graph. You may use, without verifying it, the formula \( f''(x) = \frac{4(x^2 - 3)}{9(x^2 - 1)^{4/3}} \).

Solution: The domain is all of \( \mathbb{R} \). We can simplify \( f(x) \) to \( f(x) = (x^2 - 1)^{2/3} \). We compute \( f'(x) = y' = \frac{2}{3}(x^2 - 1)^{-1/3}(2x) = \frac{4x}{3(x^2 - 1)^{1/3}} \). The critical points are \( x = 0, -1, 1 \). The function is increasing where \( f(x) > 0 \), which is for \( -1 < x < 0 \) and \( x > 1 \). The function is decreasing where \( f(x) < 0 \), which is for \( x < -1 \) and \( 0 < x < 1 \). Thus \( x = 0 \) is a local maximum, and \( x = 1, -1 \) are both local minimums. Notice that the denominator of the second derivative is always positive (when \( x \neq -1, 1 \)). Thus the second derivative is positive (i.e. \( f(x) \) is concave up) when \( x < -\sqrt{3} \) or \( x > \sqrt{3} \). The second derivative is negative (i.e. \( f(x) \) is concave down) when \( -\sqrt{3} < x < -1, -1 < x < 1 \) or \( 1 < x < \sqrt{3} \). Therefore \( x = -\sqrt{3}, \sqrt{3} \) are the only inflection points.

Notice that \( f(x) \) approaches infinity as \( x \) approaches infinity and as \( x \) approaches negative infinity, and thus there will be no absolute maximum. The absolute minimum is 0, and occurs at \( x = -1, 1 \).

5. Sketch the graph of the function

\[ f(x) = \frac{x}{2x - 3} \]
over its natural domain. Make sure you find the intervals of increasing/decreasing, the intervals of concave up/concave down, and any asymptotes. Be sure to label local maxima and minima as well as any inflection points.

**Solution:**
Here are some facts we can deduce. The domain of a rational function of polynomials is all of \( \mathbb{R} \) except those values where the denominator is 0, namely here for \( x = \frac{3}{2} \). The sign chart is also relatively easy when you note that \( 2x - 3 > 0 \) for \( x > \frac{3}{2} \) and \( 2x - 3 < 0 \) for \( x < \frac{3}{2} \). The sign chart of \( x \) is easy.

Then the sign of the quotient \( f(x) = \frac{x}{2x-3} \) can be deduced as

\[
\begin{align*}
    f(x) &= \begin{cases} 
        \text{undefined} & \text{for } x = \frac{3}{2} \\
        0 & \text{for } x = 0 \\
        > 0 & \text{for } x > \frac{3}{2} \text{ or } x < 0 \\
        < 0 & \text{for } 0 < x < \frac{3}{2}
    \end{cases}
\end{align*}
\]

We also compute

\[
\begin{align*}
    \lim_{x \to \infty} \frac{x}{2x-3} &= \lim_{x \to \infty} \frac{1}{2 - 3/x} = \frac{1}{2} \\
    \lim_{x \to -\infty} \frac{x}{2x-3} &= \lim_{x \to -\infty} \frac{1}{2 - 3/x} = \frac{1}{2} \\
    \lim_{x \to 3/2^+} \frac{x}{2x-3} &= \infty \\
    \lim_{x \to 3/2^-} \frac{x}{2x-3} &= -\infty
\end{align*}
\]

So we have a horizontal asymptote \( y = 1/2 \) and a vertical asymptote \( x = 3/2 \).

We compute

\[
\frac{d}{dx} f(x) = f'(x) = \frac{1 \cdot (2x - 3) - 2 \cdot x}{(2x-3)^2} = \frac{-3}{(2x-3)^2}
\]

We then deduce \( f'(x) \) is defined for all \( x \) except \( x = \frac{3}{2} \). Moreover \(-3 < 0\) and \((2x-3)^2 > 0\) for all \( x \) except \( x = \frac{3}{2} \). So \( f'(x) < 0 \) for all \( x \) except \( x = \frac{3}{2} \). Thus \( f \) is decreasing. Now, compute

\[
\frac{d}{dx} f'(x) = f''(x) = \frac{0 \cdot (2x-3)^2 - (-3) \cdot 2 \cdot (2x-3) \cdot 2}{(2x-3)^4} = \frac{12}{(2x-3)^3}
\]

We then deduce \( f''(x) \) is defined for all \( x \) except \( x = \frac{3}{2} \). Moreover \( 12 > 0 \) and \((2x-3)^3 > 0\) for all \( x > \frac{3}{2} \) and \( f''(x) < 0 \) for all \( x < \frac{3}{2} \). So \( f \) is concave up for all \( x > \frac{3}{2} \) and \( f \) is concave down for all \( x < \frac{3}{2} \). Note that \( x = 3/2 \) is not an inflection point since it is not in the domain of \( f(x) \). We can form the graph from this information. The result is much like a scaled and shifted version of \( y = \frac{1}{x} \) which is hardly surprising when you realize that

\[
\frac{x}{2x-3} = \frac{1}{2} + \frac{3/2}{2x-3}
\]

You don’t need to see this reduction in order to get the correct curve.
6. Sketch the graph of the function

\[ f(x) = \frac{x^2}{2x - 3} \]

over its natural domain. Indicate any symmetry, asymptotes, local or global extreme points, and intercepts.

**Solution:** Note that this function is similar to the previous question. The domain of a rational function of polynomials is all of \( \mathbb{R} \) except those values where the denominator is 0, namely here for \( x = \frac{3}{2} \). The sign chart is also relatively easy when you note that \( 2x - 3 > 0 \) for \( x > \frac{3}{2} \) and \( 2x - 3 < 0 \) for \( x < \frac{3}{2} \). Since \( x^2 > 0 \) for all \( x \neq 0 \), then the sign of the quotient \( f(x) = \frac{x^2}{2x - 3} \) can be deduced as

\[
\begin{cases}
  \text{undefined} & \text{for } x = \frac{3}{2} \\
  0 & \text{for } x = 0 \\
  > 0 & \text{for } x > \frac{3}{2} \\
  < 0 & \text{for } x < \frac{3}{2}
\end{cases}
\]

We also compute

\[
\lim_{x \to 3/2^+} \frac{x^2}{2x - 3} = \infty
\]

\[
\lim_{x \to 3/2^-} \frac{x^2}{2x - 3} = -\infty,
\]

so we have a vertical asymptote at \( x = 3/2 \). Also,

\[
\lim_{x \to \infty} \frac{x^2}{2x - 3} = \lim_{x \to \infty} \frac{x}{2 - 3/x} = \infty
\]

\[
\lim_{x \to -\infty} \frac{x^2}{2x - 3} = \lim_{x \to -\infty} \frac{x}{2 - 3/x} = -\infty,
\]

so there could be a slant asymptote. Long division gives

\[
\frac{x^2}{2x - 3} = \frac{1}{2} x + \frac{3}{4} + \frac{9/4}{2x - 3}
\]

so \( \lim_{x \to \pm\infty} [f(x) - \frac{1}{2} x + \frac{3}{4}] = \lim_{x \to \pm\infty} \frac{9/4}{2x - 3} = 0 \). Thus the line \( y = \frac{1}{2} x + \frac{3}{4} \) is a slant asymptote. We compute

\[
\frac{d}{dx} f(x) = f'(x) = \frac{2x \cdot (2x - 3) - x^2 \cdot 2}{(2x - 3)^2} = \frac{2x^2 - 6x}{(2x - 3)^2}
\]

The critical points of \( f \) are \( x = 0, 3 \) where \( f'(x) = 0 \), and \( x = \frac{3}{2} \) where \( f' \) is undefined, however \( x = \frac{3}{2} \) is not in the domain of \( f \) so we omit it. Moreover \( 2x^2 - 6x > 0 \) for \( x < 0 \) and \( x > 3 \), while \( (2x - 3)^2 > 0 \) for all \( x \) except \( x = \frac{3}{2} \). So \( f'(x) > 0 \) for \( x < 0 \) and \( x > 3 \) and \( f'(x) < 0 \) for \( 0 < x < 3 \) except \( x = \frac{3}{2} \). Thus \( f \) is increasing for \( x < 0 \) and \( x > 3 \), and \( f \) is decreasing for \( 0 < x < 3/2 \) and
There is a local maximum at \( x = 0 \) whose value is \( f(0) = 0 \), and a local minimum at \( x = 3 \) whose value is \( f(3) = 3 \). To determine concavity, we study the second derivative.

\[
f''(x) = \frac{(4x - 6) \cdot (2x - 3)^2 - (2x^2 - 6x) \cdot 2 \cdot (2x - 3) \cdot 2}{(2x - 3)^4} = \frac{18}{(2x - 3)^3}.
\]

We then deduce \( f''(x) \) is defined for all \( x \) except \( x = \frac{3}{2} \). Moreover \( 18 > 0 \) and \( (2x - 3)^3 > 0 \) for all \( x > \frac{3}{2} \) and \( (2x - 3)^3 < 0 \) for all \( x < \frac{3}{2} \). So \( f \) is concave up for all \( x > \frac{3}{2} \) and \( f \) is concave down for all \( x < \frac{3}{2} \), and so there are no inflection points (\( x = 3/2 \) is not in the domain). We can form the graph from this information, see the figure below.

**Problem-Solving Strategy:** When you approach an optimization problem, try to follow these steps:

1. **Understand the problem:** Read the problem and make sure you understand it clearly.
   
   Ask yourself: • *What do I know?*
   
   → What information is given in the problem?
   
   • *What do I need to find?*
   
   → What is the problem asking?
   
   → Which quantity is to be optimized, and under what conditions?
   
   → Do I need to draw a sketch?
   
   → What are the relevant variables in the problem? (Use suitable notation!)

2. **Think of a plan:** Find the connection between the given information and the unknown.
   
   Ask yourself: • *What calculus concept(s) do I need to apply to optimize the quantity of interest?*
   
   → How can I express the quantity of interest as a certain function \( Q \)?
   
   → How can I express in a mathematical form the conditions given in the problem?
   
   → How can I simplify the function \( Q \) to be a function of only one variable?
   
   → What calculus technique do I need to apply?

3. **Carry out your plan:** Do your computations!

4. **Reflect on what you are doing:** Does your answer make sense?

7. Find the maximum area of a rectangle that can be drawn inside a circle of radius 2.
Solution: Draw the circle of radius 2 with the origin at \((0,0)\). Then the equation of the circle is \(x^2 + y^2 = 4\). If we drew a rectangle that wasn’t touching the circle at all 4 corners, we could make it bigger. Thus we can assume the rectangle is touching the circle at all four corners. Let \((t,z)\) be the coordinate of the top-right corner. Then \(z = \sqrt{4-t^2}\), and the area of the rectangle is \(2t \cdot 2z = 4t\sqrt{4-t^2}\). Let \(A(t) = 4t\sqrt{4-t^2}\). Then \(A(t)\) is our objective function and our constraint is \(0 \leq t \leq 2\).

We compute
\[
A'(t) = 4\sqrt{2-t^2} + \frac{4t^2}{\sqrt{2-t^2}}
\]
Setting \(A'(t) = 0\) gives:
\[
4\sqrt{4-t^2} = -\frac{4t^2}{\sqrt{4-t^2}}
\]
\[
\sqrt{4-t^2} = -\frac{t^2}{\sqrt{4-t^2}}
\]
\[
4-t^2 = t^2
\]
This gives \(t = \pm \sqrt{2}\). We take the positive one, \(t = \sqrt{2}\), and compute \(z = \sqrt{2}\) (so the rectangle we found is actually a square). This gives an area of \(A(\sqrt{2}) = 8\). Since \(A(0) = A(2) = 0\), by the extreme value theorem the maximum value is 8.

8. Find the dimensions of the rectangle of maximum area that can be inscribed in a right triangle with sides of length 5 and 12 and hypotenuse of length 13, if two sides of the rectangle lie along the two sides of the triangle. Make sure you justify that your answer is a maximum.

Solution: Imagine the right triangle drawn with the \(x\)-axis and \(y\)-axis forming the two perpendicular sides and the line \(y = -\frac{5}{12}x + 5\) forms the hypotenuse (it intersects the \(x\)-axis at \((12,0)\) and intersects the \(y\) axis at \((0,5)\)). Choose a point \((a,b)\) on the line so that \(b = -\frac{5}{12}a + 5\). The area of the rectangle is
\[
f(a) = a(-\frac{5}{12}a + 5) = -\frac{5}{12}a^2 + 5a
\]
Then \(f'(a) = -\frac{5}{6}a + 5\). We compute \(f'(a) = 0\) for \(a = 6\) and so for \(b = 5/2\). Does this rectangle of dimensions \(6 \times \frac{5}{2}\) have maximum area? We check that \(f'(a) > 0\) for \(0 < a < 6\) and \(f'(a) < 0\) for \(6 < a < 12\). Thus \(f(a)\) is maximized at \(a = 6\).

9. A shoebox, complete with overlapping lid, has a base with length \(5/3\) times its width. The rim of the lid overlaps the side of the main box by \(1/5\) of the height
of the box. Assuming the thickness of the box is negligible and of uniform density, and the total volume of the box is 3000 cubic centimetres, what dimensions will result in the minimum weight of the empty box? Make sure you follow the steps outlined in the problem-solving strategy above. Draw a picture of the box and remember that if your computations lead to large numbers you can always express such numbers as a product of their factors: this will help you to carry out algebraic calculations by hand.

Solution: The problem is more about accounting carefully for the weight. Let \( l \) denote the length, let \( w \) denote the width, and let \( h \) denote the height. We have been told the length is \( \frac{5}{3} \) times the width, namely

\[
l = \frac{5}{3} w
\]

Also the volume is 3000 and so \( whl = 3000 \) and so we can substitute for \( l \) and so

\[
\frac{5}{3} w^2 h = 3000
\]

From this we can express \( h \) in terms of \( w \) and deduce that

\[
h = \frac{1800}{w^2}
\]

The amount or area of cardboard is proportional to the weight (from the uniform density hypothesis). The area of the bottom of the box is

\[
wl + 2lh + 2wh
\]

the area of the lid is

\[
wl + 2\frac{h}{5} + 2w \frac{h}{5}
\]

and so the total area is

\[
2wl + (2l + 2w) \frac{6}{5} h
\]

If we substitute for \( l, h \) we get that total area is

\[
A(w) = \frac{10}{3} w^2 + \left( \frac{16}{3} w \right) \frac{6}{5} \frac{1800}{w^2} = \frac{10}{3} w^2 + \frac{11520}{w}
\]

We wish to minimize the area and so we differentiate to get

\[
A'(w) = \frac{20}{3} w - \frac{11520}{w^2}
\]

Thus \( A'(w) = 0 \) for \( w^3 = 11520 \times \frac{3}{20} = 1728 = (12)^3 \) and so for \( w = 12 \). Thus we have \( A'(w) = 0 \) for \( w = 12 \) and so \( l = 20 \) and \( h = 12.5 \). The lid will have depth \( \frac{1}{5} 12.5 = 2.5 \).

Now we might not be sure that this is the minimum area but can verify that our solution is a local minimum. The possible values for \( w \) are \( 0 < w \). For \( w \to 0 \) we have \( l \to 0 \) and so \( h \to \infty \) but in order to have a volume of 3000, we can show that \( A(w) \to \infty \). For \( w \to \infty \) we have \( l \to \infty \) and \( h \to 0 \) and so as \( w \to \infty \) we have \( A(w) \to \infty \) just based on the top and bottom surfaces.
10. You are in a dune buggy at point P in the desert, 12 km due south of the nearest point A on a straight east-west road. You want to get to a town B on the road 18 km east of A. If your dune buggy can travel at a speed of 15 km/h through the desert, and 39 km/h along the road, toward what point on the road should you head to minimize your travel time from P to B?

**Solution:** Again a picture would make this clearer. Let C be the first point on the east west road that the dune buggy arrives at. Let x denote the distance from A. Thus the dune buggy will travel $\sqrt{x^2 + 12^2}$ km in the desert and $18 - x$ on the road. Thus the travel time in the desert is $\frac{\sqrt{x^2 + 12^2}}{15}$ and the time on the road is $\frac{18 - x}{39}$. Let $f(x)$ be the time $f(x) = \frac{\sqrt{x^2 + 12^2}}{15} + \frac{18 - x}{39}$.

We compute

$$f'(x) = \frac{2x}{30\sqrt{x^2 + 12^2}} - \frac{1}{39}.$$ 

Now $f'(x) = 0$ for $\frac{2x}{30\sqrt{x^2 + 12^2}} = \frac{1}{39}$ and so $78x = 30\sqrt{x^2 + 12^2}$ and so $13x = 5\sqrt{x^2 + 12^2}$ and so $13^2 x^2 = 5^2(x^2 + 12^2)$ and so $12^2 x^2 = 5^212^2$ and so $x = 5$ (we may discard $x = -5$ which must be worse than $x = 5$). We check the signs of the derivative to ensure that $x = 5$ yields a minimum for $f(x)$, the travel time.

11. A piece of wire 10 cm long is cut into two pieces. One piece is bent into a circle while the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is minimal? Maximal? (You can use a calculator at the very end of this problem to help you apply the extreme value theorem).

**Solution:** Divide the wire into pieces of length $x$ and $10 - x$ forming equilateral triangle and circle respectively. The side length of the triangle is $x/3$ giving area $\frac{1}{2} \cdot x/3 \cdot x\sqrt{3}/6$. For the circle $2\pi r = 10 - x$ giving $\pi r^2 = \pi(10 - x)^2/(2\pi)$. The total enclosed area is the sum $A(x) = x^2\sqrt{3}/36 + (10 - x)^2/(4\pi)$. Thus $A'(x) = x\sqrt{3}/18 - (10 - x)/(2\pi)$. $A'(x) = 0 \Rightarrow x = 90/(\sqrt{3}\pi + 9) \approx 6.23$. We find $A(0) = 25/\pi \approx 7.96$, $A(10) = 25\sqrt{3}/9 \approx 4.82$, and $A(90/(\sqrt{3}\pi + 9)) \approx 3.00$. The min is $3.00\text{cm}^2$ and the max is $7.96\text{cm}^2$.

12. Find the length of the shortest ladder than can extend from a vertical wall, over a fence 2m high located 1m away from the wall, to a point on the ground beyond the fence.

**Solution:** One can rephrase this as finding the shortest line extending from the $y$ axis to the $x$-axis passing through $(1, 2)$. Let the slope of the line be $m$.
so that the line is \( y - 2 = m(x - 1) \). The line hits the axes at \((0, 2 - m)\) and \((1 - \frac{2}{m}, 0)\). Thus the length of the ladder (or line segment) is

\[
f(m) = \sqrt{(2 - m)^2 + \left(1 - \frac{2}{m}\right)^2}
\]

but we might as well consider the squared length for convenience

\[
g(m) = (2 - m)^2 + \left(1 - \frac{2}{m}\right)^2
\]

We wish to minimize \( g(m) \) for \( m < 0 \). Then we compute

\[
g'(m) = -2(2 - m) + 2 \left(1 - \frac{2}{m}\right) \left(\frac{2}{m^2}\right)
\]

we have \( g'(m) = 0 \) when \( 2(2-m) = \frac{4(m-2)}{m^3} \). Now either we have \( m = 2 \) (which we can discard as nonsensical, clearly we have \( m < 0 \)) or we can divide by \( 2 - m \) to obtain \( 1 = -\frac{2}{m^3} \). Thus we have \( m = -\sqrt[3]{2} \). We need to consider \( g'(m) \) and find that \( g'(m) < 0 \) for \( m < -\sqrt[3]{2} \) and \( g'(m) > 0 \) for \( -\sqrt[3]{2} < m < 0 \). We substitute \( m = -\sqrt[3]{2} \) in our distance formula to get \( \sqrt{(2 + \sqrt[3]{2})^2 + (1 + (\sqrt[3]{2})^2)^2} = 4.16 \) metres. A bit of a challenge getting the signs correct!

One could also tackle this problem by choosing the point \((0,a)\) on the \( y \)-axis where \( a > 2 \) and then determining the line as \( y = -(a - 2)x + a \) and so the line hits \( x \)-axis at the point \((\frac{a}{a-2}, 0)\). Proceed as above to minimize the squared distance \( h(a) = (\frac{a}{a-2})^2 + a^2 \). The calculations are essentially the same.

\[
h'(a) = 2a + 2 \left(\frac{a}{a-2}\right) \left(\frac{a-2}{(a-2)^2}\right)
\]

and so \( h'(a) = 0 \) when

\[
2a = \frac{4a}{(a-2)^3}
\]

and so (since \( a > 2 > 0 \)) when \( a - 2 = \sqrt[3]{2} \) and so when \( a = 2 + \sqrt[3]{2} \) as above. As in our first solution, we have verified that the slope of the ladder is \( m = -\sqrt[3]{2} \).