Basic Skills required to work through the problems:

- determining the domain of a function;
- applying the concept of derivative at a point as the instantaneous rate of change of a function to find the slope of the tangent line to the graph at that point;
- applying the concept of derivative as the slope of the tangent line to a curve at a point;
- relating the coordinates of a point that lies on a curve of given equation.
- identify a composite function and break it down into simpler functions using products, quotients, function composition;
- manipulating expressions and performing computations involving logarithms and exponentials.

Learning Goals: After completing this set, you should be able to:

- master the basic skills listed above;
- find inverse functions of commonly-used functions, explain their properties (such as domain and range), compute their derivatives;
- compute derivatives using implicit differentiation.
- compute the derivative of exponential and logarithmic functions;
- apply logarithmic differentiation, that is, compute the derivative of a complicated function by taking the logarithms and differentiating the resulting equation implicitly;

1. (Warm up) Find the missing entries in the table:

<table>
<thead>
<tr>
<th></th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( f(g(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>( x - 4 )</td>
<td>( x + 4 )</td>
<td>( \frac{1}{x^2} )</td>
</tr>
<tr>
<td>(ii)</td>
<td>( \frac{1}{(x+1)^2} )</td>
<td>( \frac{1}{x^2} )</td>
<td>( 2x + 3 )</td>
</tr>
<tr>
<td>(iii)</td>
<td>( x^\frac{1}{3} )</td>
<td>( \frac{1}{x^2} )</td>
<td>( 2x + 3 )</td>
</tr>
</tbody>
</table>

2. There is more than one way to find the derivative of the function

\[
f(x) = \frac{1}{2x^5 + 40x^2}.
\]
(a) Use the quotient rule to compute \( f'(x) \), then check your result by computing the same derivative using the chain rule.

(b) When is \( f'(x) = 0 \)? What is this telling you about the graph of \( f(x) \)?

3. Find the domain of each of the following functions. Then calculate the derivative of each. Use whichever methods you like.
   (a) \( f(x) = e^{\sqrt{\tan x}} \)
   (b) \( f(x) = e^x (\sin x)(\cos x) \)
   (c) \( f(x) = \frac{x^2}{x^2 + 4} \)
   (d) \( f(x) = \sin(\cos(e^x)) \)

4. Consider the function
   \[ F(x) = f(\sqrt{x} - 2x)g(h(x)^2) \]
   obtained from some functions \( f, g \) and \( h \). Given the following values,
   \[
   f(-6) = 2, \quad g(9) = 3, \quad h(4) = -3, \quad f'(-6) = 4, \quad g'(9) = 0, \quad h'(4) = 6;
   \]
   determine the slope of the tangent to the curve \( F(x) \) when \( x = 4 \).

5. Consider the curve given by the equation \( y = 2 \sin^2(\pi x/4) \).
   (a) Find a point on the curve at which the tangent line has slope \( \pi/2 \).
   (b) Is there a point on the curve at which the tangent line has slope \( \pi \)?

6. Find the derivative of \( f(x) = \frac{\cos \sqrt{x}}{2 + \sin \sqrt{x}} \). Simplify your answer.

7. (Warm up) Find an inverse function for each of the following functions. In each case, describe the inverse function as best as you can using the information you have. Whenever possible determine the domain of the inverse function.
   (a) \( f(x) = 7x + 2 \)
   (b) \( f(x) = \cos x, \) where \( 0 \leq x \leq \pi \)
   (c) \( f(x) = x^2 \) \( (x < 0) \)

8. Evaluate, when possible, the following expressions, or explain why they cannot be evaluated. NOTE: \( \sin^{-1} x \) and \( \cos^{-1} x \) denote the inverse sine and cosine functions, not the reciprocals of \( \sin x \) and \( \cos x \).
   (a) \( \cos^{-1}(\cos(7\pi)) \)
   (b) \( \sin(\sin^{-1}(\pi)) \)
   (c) \( \sin^{-1}(\sin(\pi)) \)

9. Sketch the graph of the function \( f(x) = |(x-1)^2 - 9| \). Determine on what intervals, if any, this function is one-to-one.

10. The following problem has two parts. Near the point \((0, -1)\), the equation \( y^2 - 2xy + 3x^2 = 1 \) defines \( y \) implicitly as a function of \( x \). You will use two different approaches to calculate \( \frac{dy}{dx} \) at a point.
(a) Perform implicit differentiation to find $\frac{dy}{dx}$ at $(0, -1)$.

(b) Solve for $y$ explicitly as a function of $x$ and then differentiate to find $\frac{dy}{dx}$ at $(0, -1)$. Compare this approach to your calculations in part (a). Hint: Use the quadratic formula.

11. (a) Compute $\frac{dy}{dx}$ if $y = \sin^{-1}(2 - x^2)$. For which values of $x$ does the derivative make sense?

(b) A careless mathematics professor asked his calculus class on their final examination to find $\frac{dy}{dx}$ if $y = \sin^{-1}(2 + x^2)$. What’s wrong with this problem?