Ÿ Lab 5: Population dynamics of the Nicholson-Bailey model

In this lab we will visualize the dynamics of the basic Nicholson-Bailey model, as well as of models that are obtained from the basic model by adding additional features such as density-dependence in the prey, interference among predators, and refuges. To visualize these dynamics, we can either plot the time series of each of the two populations separately, or we can plot them together in a so-called **phase diagram**, in which the two axis are the population sizes in the prey and the predator respectively, and in which pairs (x(t),y(t)) are plotted for a number of successive generations.

To iterate the Nicholson-Bailey model, we need to be able to follow tow variables at the same time, i.e. we need to be able to follow the dynamics of the prey population size and of the predator population size. Thus, we need to iterate two equations simultaneously. This can be done using the

```
For[i=1,i<n,i++,...]
```

construct.

This command tells *Mathematica* to do whatever is specified by the dots ... in the command above for i=1 to n.

For example, you could tell *Mathematica* to calculate the sum of the first 100 integers by using

```
x=0
For[i=1,i<101,i++,x=x+i]
```

The first command x=0 initializes the sum at 0, and the second command adds the numbers i from 1 to 100 successively to x. To view the result, we simply type the command

```
x
```

(Note that we actually don't need such a complicated procedure for the problem at hand, since there is an easy formula for the sm of the first N integers. Do you remember the formula?)

Use the help command to learn more about the For[] construct!

Using the For[ ] construct, we can iterate the Nicholson-Bailey dynamics. We start with defining the two functions used to describe the dynamics of the model:

```
Clear[x,y,r,a,c]
F[x_,y_]:=lambda*x*Exp[-a*y]
G[x_,y_]:=c*x*(1-Exp[-a*y])
```

These functions should be familiar by now. lambda is the maximal number of offspring per prey individual, a is the predator searching efficiency, c is the conversion rate of captured prey into predator offspring, and the dynamics of the Nicholson-Bailey model are given by:

x(t+1)=r*x(t)*exp[-a*y(t)]=F[x(t),y(t)]
y(t+1)=c*x(t)*(1-exp[-a*y(t)])=G[x(t),y(t)]

To iterate these functions numerically, we need to write a little program, in which we first specify numeric values for the parameters, and then iterate the functions F[x,y] and G[x,y] for a given number of times and store pairs (x,y) in successive generations a list l.

```
Clear[l,x,y,r,lambda,a,c]
l={}
lambda=Exp[2]
a=0.1
c=0.1
x=Random[]
y=Random[]
n=1000
For[i=1,i<n,i++,x1=F[x,y];y1=G[x,y];x1=x,y1=y;l=Append[l,{x,y}];]
```

There are several things we should note here:

- The parameter values assigned to lambda, a and c are just examples. We need to specify some values for the procedure to work.

- All the instructions within the For[] construct that come after the first three entries, i.e. after i=1,i<n,i++, are separated by semicolons ';' .

- We need to assign the values of F[x,y] and G[x,y] to intermediate variables x1 and y1, and then we reassign x1 and y1 to x and y after both calculations are completed. The reason for this is that if we assigned first x=F[x,y] and then y=G[x,y], then the new x-value, obtained from F[x,y], would be used in the calculation for the new y-value, i.e in G[x,y], which would correspond to using next year's prey density to calculate the outcome of this year's interactions!

- We end up with a list containing pairs of numbers (x,y) in successive generations. If we plot this list using ListPlot[], we get a phase diagram of the dynamics: we don't see the time explicitly, but we see pairs of prey and predator densities in successive generations. Phase diagrams are also shown in the handout from kissed's book '*Mathematica*l models in biology in Figures 3.5-3.8.

- To obtain series for the prey and the predator populations sizes separately, we could use a variant of the program above, e.g.

```
Clear[lprey,lpred,x,y,r,lambda,a,c]
lprey={}
lpred={}
lambda=Exp[2]
a=0.5
c=0.2
x=Random[]
y=Random[]
n=1000
```

```
For[i=1,i<n,i++,x1=F[x,y];y1=G[x,y];x1=x,y1=y;lprey=Append[lprey,x];lpred=Append[lpred,y];]
```

Ÿ Tasks for this lab:

Use the methods introduced above to visualize the dynamics of

1) The basic Nicholson-Bailey model
2) The extension of the Nicholson-Bailey model obtained by including density-dependent growth in the prey populations (see homework 5)
3) The extension of the Nicholson-Bailey model obtained by assuming that the prey has a partial refuge (see homework 5)