Lab 4: Population dynamics in discrete time

For a population dynamical system of the form x(t+1)=F(x(t)), where x(t) and x(t+1) are population sizes in successive generations, and F(x(t)) is the function describing the population size in the next generation as a function of the population size in the present generation, one can easily obtain a graphic display of the dynamics by first creating a list of the population sizes in successive generations using the command NestList[], and then plotting this list using the command ListPlot[]. This is very convenient in understanding the effects of demographic parameters in the function F on the dynamic behaves that the system exhibits.

For the following, we use the model
x(t+1)=r*x(t)/(1+x(t)^b)
which should be familiar by now. Thus we have
x(t+1)=f(x(t))*x(t)
where f(x(t))=r/(1+x(t)^b) is the per capita reproductive output when the population size is equal to x(t).
To proceed, we first define the function F:

```
Clear@x, r, bD
F@x_D := r * x • H1 + x ^ bL
```

Note that we have first 'cleared' the parameters r and b as well as the variable x, just to make sure that we don't take over some old values that might have been assigned previously.
We then specify the parameters r and b:

```
r=3
b=1
```

3

1

Next, we use the command NestList[] to get a list of population sizes obtained from iterating the function F (i.e. exactly what we want!):

```
x0=Random[]
l=NestList[F,x0,100]
```

Here x0 is the initial population size we start out with, which in this case we have chosen to be random number between 0 and 1. The number 100 appearing in the command indicates how many iterations we want to have , and the function F specifies, of course, which function we want to iterate. Thus, the command above gives us a list l that contains the population sizes in 100 successive generations under the condition that we start out with the population size x0 (and under the assumption that the parameters r and b have the values previously assigned!!). Now we can plot this list to display the dynamic graphically:

```
ListPlot[l]
```

Task for this lab: Fix r at a value of 2, and display the dynamics of this model for different choices of the parameter b, and for different starting conditions x0.

For r=2, the model exhibits a stable equilibrium for b<4, independent of the starting condition x0 (why???). Asb is increased above 4, the model first exhibits a 2-cylce, then, as b is increased further, a 4-cycle, and in general a periodic 2^n cycles (use PlotJoined->True to clearly see the cycles). For this range of b-values you should see that this cyclic behavior is independent of the starting condition, i.e. occurs for any choice of x0 (except for one; which one?). As b is increased further, the system leaves the periodic regime, and the dynamics start to become very irregular: chaos sets in. Find the approximate value of b where you don't see cycles anymore. For a given value of b that induces chaos, plot the dynamics of the models in one and the same diagram for different starting conditions that are nevertheless very close to each other. For example:

```
r=2
b=20
x0=Random[]
l1=NestList[F,x0,100];
x0=x0+0.001
l2=NestList[F,x0,100];
ListPlot[l1,PlotJoined->True]
ListPlot[l2,PlotJoined->True]
Show[%,%%]
```

By zooming in on the x-axis you can more clearly see that even though exactly the same function is used in the two cases, and even though the starting conditions are almost identical, the trajectories of the two populations soon become very different. For example, in generation 63 one population is very large while the other is small. This sensitive dependence on initial conditions is the hallmark of chaos.

```
Show[%,PlotRange->{{60,80},{0,2}}]
```

Advanced topic: bifurcation diagrams

Now that you have seen that the dynamics change as the parameters of the model are changed, you might want to see this whole process in a single display. In such a BIFURCATION DIAGRAM the x-axis is the parameter that controls the dynamics, b in our case (we still assume that r is fixed at 2), and the y-axis shows what going on in terms of population dynamics for the given value of b. Can you come up with a procedure to produce such a bifurcation diagram?

```
Clear[r,x,b,l];
r=3;
l={};
l=Append[l,{0,0}];
n1=100+1;
n2=200;
n3=100;
For[
  i=1,i<n1,i++,b=1+i*6/(n1-1);x=Random[];
  For[j=1,j<n2,j++,x=F[x]];
  For[j=1,j<n3,j++,x=F[x];l=Append[l,{b,x}]];
  ]
```

```
ListPlot[l]
```