Biol 301 Spring 2001
Assignment 5: Discrete time predator-prey dynamics
Due Friday, March 9, 2001

Read the handout from 'Mathematical Models in Biology' by L. Edelstein-Keshet.
Read p. 181-188 in Hastings' book.

1. Consider the Nicholson-Bailey model for predator-prey interactions:

$$\begin{aligned} x(t+1) &= \lambda \cdot x(t) \cdot \exp\left[-ay(t)\right] = F\left(x(t), y(t)\right) \\ y(t+1) &= c \cdot x(t) \cdot \left(1 - \exp\left[-ay(t)\right]\right) = G\left(x(t), y(t)\right) \end{aligned}$$

   where $x(t)$ and $x(t+1)$ are prey population sizes in subsequent years, $y(t)$ and $y(t+1)$ are predator population sizes in subsequent years, λ is the maximal number of offspring per prey individual, a is the predator searching efficiency, and c is the conversion rate of captured prey into predator offspring.

   (a) Use Mathematica to plot $x(t+1)$ as a function of $x(t)$ and $y(t)$ for various values of λ and a. (Use the command Plot3D[ ].)

   (b) Use Mathematica to plot $y(t+1)$ as a function of $x(t)$ and $y(t)$ for various values of λ and a. (Use the command Plot3D[ ].)

   (c) Find the equilibrium population sizes x^* and y^*. (You can do that either by hand or using Mathematica.)

   (d) Calculate the partial derivatives $\partial F/\partial x$, $\partial F/\partial y$, $\partial G/\partial x$ and $\partial G/\partial y$.

   (e) Evaluate the four partial derivatives at the equilibrium (x^*, y^*) and write down the Jacobian matrix.

   (f) Write down the model for the dynamics of the distance to the equilibrium, $x(t) - x^*$ and $y(t) - y^*$, after a small initial perturbation away from the equilibrium.

   (g) Find the eigenvalues of the Jacobian using Mathematica. Plot these eigenvalues as a function of λ and convince yourself that the larger eigenvalue is always >1, which means that the equilibrium (x^*, y^*) is always unstable, independent of the parameter values of λ, a and c.

2. Consider the Nicholson-Bailey model with density dependence in the prey given by the Ricker model:

$$\begin{aligned} x(t+1) &= x(t) \cdot \exp[r(1 - x(t)/K] \cdot \exp\left[-ay(t)\right] = F\left(x(t), y(t)\right) \\ y(t+1) &= c \cdot x(t) \cdot \left(1 - \exp\left[-ay(t)\right]\right) = G\left(x(t), y(t)\right) \end{aligned}$$

   (see handout from Keshet's book 'Mathematical models in biology').

   Try to reproduce Figs. 3.5-3.8 in the handout using Mathematica (see computer lab 5).

3. Based on the section on heterogeneity of the environment in the handout (p. 87-89) derive a Nicholson-Bailey model with a prey refuge according to the following steps:

   (a) With a refuge of size EK, explain why $EK/x(t)$ is the chance that a given individual ends up in the refuge.

   (b) Based on (a), explain why for each prey individual, the chance of escaping predation in generation t is

$$\frac{EK}{x(t)} + \left(1 - \frac{EK}{x(t)}\right) \exp[-ay(t)]$$

   (c) Based on (b), write an equation for $x(t+1)$.

   (d) Explain why for each prey individual the chance of not escaping predation is

$$\left(1 - \frac{EK}{x(t)}\right) (1 - \exp[-ay(t)])$$

   (e) Based on (d), explain why the dynamic equation for the predator is

$$y(t+1) = c \cdot (x(t) - EK) \cdot (1 - \exp[-ay(t)]) .$$

4. Consider the Nicholson-Bailey model with a prey refuge derived in the previous problem. Assess the general effect of that the prey refuge has on the predator-prey dynamics by investigating the dynamics of this model for various parameter settings using Mathematica (see computer lab 5).

5. In this problem we consider the effect of interference among parasitoids on the equilibrium states in the Nicholson-Bailey model.

   (a) In the handout from Keshet's book, it is mentioned on p. 87 that due to interference the searching efficiency of the parasitoids may decrease, so that the chance that a prey individual escapes parasitism changes from $\exp[-ay(t)]$ to

$$\exp\left[-\{ay(t)\}^{1-m}\right].$$

   Explain why m should be chosen such that $0 < m < 1$.

   (b) Write a set of host-parasitoid equations that incorporates interference among parasitoids.

   (c) Find the equilibrium states of the new model. Compare those to the equilibrium states of the basic Nicholson-Bailey model. Comment.