Biol 301 Spring 2001
Assignment 4: Discrete time population dynamics
Due Wednesday, February 7, 2001

Use Mathematica to solve the following problems whenever it is convenient for you.

1. Consider the following model for a population that experiences intraspecific competition:

$$x(t+1) = \frac{\lambda \cdot x(t)}{[1 + x(t)]^b},$$

where $x(t)$ and $x(t+1)$ are population sizes in subsequent year, λ is the maximal number of offspring per individual, i.e. the number of offspring per individual when there is no competition, and b is an additional parameter. (Note that this model differs from the one I used in class!)

   (a) Plot the number of offspring *per individual* as a function of population size for different values of λ and b.

   (b) Plot $x(t+1)$ as a function of $x(t)$ for different values of λ and b.

   (c) Find the carrying capacity of this model.

   (d) Determine the stability conditions for this equilibrium in terms of the parameters λ and b.

   (e) Use the graphical method of cobwebbing to illustrate the dynamics of this population for different starting population sizes and for different values of λ and b.

2. Consider again the model used in problem 1. Fix $b = 10$. For the following parameter values of λ, start with a population size of $x(0) = 0.5$ in year 0 and plot the population sizes $x(t)$ in the next 20 years, i.e. for $t = 1, ...50$. Comment.

   (a) $\lambda = 2$

   (b) $\lambda = 5$

   (c) $\lambda = 10$

   (d) $\lambda = 20$

   (e) $\lambda = 40$

3. The model in problem 1 is usually interpreted in the following way: each year, each individual of the starting population $x(t)$ survives competition with a probability of $1/[1+x(t)]^b$, so that there are $x_s(t) = x(t)/[1+x(t)]^b$ survivors in year t. Each survivor then has λ offspring on average and dies after reproduction. The offspring form the starting population in the next year, so that $x(t+1) = \lambda \cdot x(t)/[1+x(t)]^b$. This allows one to actually measure the parameter b in the field, as follows. Consider the logarithm of the ratio between initial population size and survivors of competition:

$$\log\left(\frac{x(t)}{x_s(t)}\right).$$

Express this as a function with $\log(x(t))$ as the independent variable, and conclude that for large $x(t)$, $\log\left(\frac{x(t)}{x_s(t)}\right)$ is a linear function of $\log(x(t))$ with slope b. Based on this finding, describe how you would go about measuring b in the field or in the lab.

4. Based on the method of problem 3 and on measuring maximal reproductive rates λ, Hassell et al. (*J. Anim. Ecol.* 45, 471-486, 1976) give the following estimates for the parameters λ and b in the model of problem 1 for several insect populations:

	λ	b
Moth: *Zeiraphera diniana*	1.3	0.1
Bug: *Leptoterna dolobrata*	2.2	2.1
Mosquito: *Aedes aegypti*	10.6	1.9
Potato Beetle: *Lepinotarsa decemlineata*	75.0	3.4
Parasitoid Wasp: *Bracon hebetor*	54.0	0.9

   (a) Plot these values on a λ, b-parameter plane. (Recommendation: use a log scale for the λ-axis.)

   (b) Use your results from problem 1 to determine which of these species will have a stable equilibrium, and which ones won't.

5. Consider the following model for a population that experiences intraspecific competition:

$$x(t+1) = \lambda \cdot x(t) \cdot \exp[-q \cdot x(t)],$$

where $x(t)$ and $x(t+1)$ are population sizes in subsequent year, λ is the maximal number of offspring per individual, i.e. the number of offspring per individual when there is no competition, and q is an additional parameter. This model is called the *Ricker model*.

   (a) Plot the number of offspring *per individual* as a function of population size for different values of λ and q.

   (b) Plot $x(t+1)$ as a function of $x(t)$ for different values of λ and q.

   (c) Find the carrying capacity of this model.

   (d) Determine the stability conditions for this equilibrium in terms of the parameters λ and q.

   (e) Use the graphical method of cobwebbing to illustrate the dynamics of this population for different starting population sizes and for different values of λ and q.

6. Consider again the Ricker model introduced in problem 5. Fix $q = 1$. By plotting the dynamics for various values of λ,find values of this parameter for which the Ricker model exhibits

   (a) a stable equilibrium

   (b) a 2-cycle

(c) a 4-cylce

(d) a 8-cycle

(e) chaos.

7. (See Hastings, Fig. 4.11, p. 98.) In this problem we want to visualize the unpredictability in chaotic systems. For this purpose, choose values for λ and q for which the Ricker model (problem 5) exhibits chaos (e.g. $\lambda = \exp(3.5)$ and $q = 3.5/100$, which would correspond to the parameter values that Hastings uses for Fig. 4.11). Run the dynamics for a sufficient number generations and keep a record (in a list) of pairs $(x(t), x(t+1))$ during the run. Then plot $x(t+1)$ versus $x(t)$ from that list. The result should be similar to the upper panel in Fig. 4.11 in Hastings' book. Now do the same thing, but keep a record of pairs $(x(t), x(t+10))$. Then plot the various $x(t+10)$ versus the corresponding $x(t)$. Now the result should be similar to the lower panel in Fig. 4.11. What can you conclude about long term predictions in chaotic systems? What about short term predictions?

8. Consider the following graph of $x(t+1)$ as a function of $x(t)$:

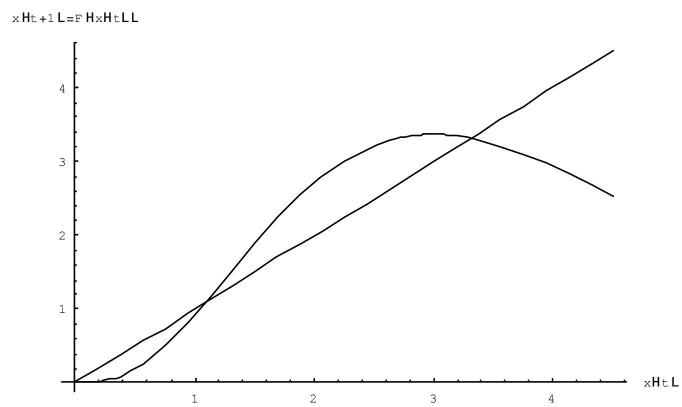

Figure 1: Allee Effect

This population is said to exhibit an *Allee effect*: For very small population sizes, the graph of F lies below the diagonal, which means that at very small population sizes the population is actually decreasing, e.g. because individuals have to spend too much time (and hence too many resources) looking for mates. This is in contrast to the models discussed in class, where populations are always growing exponentially when population sizes are low.

(a) How many non-zero equilibria are there for the dynamics of the population described by the graph in Figure 1?

(b) Which ones of those equilibria are stable?

(c) Suppose the population is at the lower of the equilibria found in (a), and is perturbed away from this equilibrium. What happens? (Hint: use cobwebbing; there are two cases to consider.)